Dualism of spacetime as the origin of the fermion mass hierarchy

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It is shown that hypothetical dualism of spacetime could give rise to the pattern observed in the properties of the fundamental fermions. By guessing at the basic symmetries of space that underlie the properties of the simplest possible (primitive) particle it is found that these symmetries imply the existence of a series of structures – equilibrium configurations of primitive particles – whith properties that cohere with those of the fundamental fermions, including their masses.

Keywords: Topology of space, fermion mass hierarchy

Introduction

Dualism is known to be one of the general properties of matter. Many successful ideas in physics were based on this property (the well-known examples are the concepts of wave-particle duality and supersymmetry in quantum field theories). It is conceivable that by understanding how spacetime dualism manifests itself on the sub-quark scale it would be possible to resolve some (if not all) of the puzzles that plague modern physics.

One of such long-standing puzzles is the hierarchy of masses of the fundamental fermions [1]. The masses of quarks and leptons (Table 1) span a very wide range ($\sim 10^5$). They are not predicted

Table 1: Experimental masses of the fundamental fermions (in proton mass units). The data are taken from [2] for the charged leptons and from [3] for quarks.

First generation		Second generation		Third generation	
e	0.0005446170232(12)	μ	0.1126095173(34)	τ	1.8939(3)
ν_e	$\leq 3 \cdot 10^{-9}$	ν_μ	$\leq 2 \cdot 10^{-4}$	ν_τ	$\leq 2 \cdot 10^{-2}$
u	0.0047	c	1.6	t	189
d	0.0074	s	0.16	b	5.2

from any application of first principles of the Standard Model of particle physics, nor has any analysis of the observed data (see e.g. [4]) yielded an explanation as to why they should have strictly the observed values instead of any others. Some theorists claim that this pattern could be randomly frozen in one of the early stages of the universe [5], much like the particular properties of the planets were determined by the detailed history of the formation of the solar system. But most people believe that these masses can be explained by different mechanisms of symmetry [6][7], hidden spatial dimensions [8][9], quantisation of energies of hypothetical exotic topological objects [10], or some other geometrical properties of space [11][12].

However, the history of particle physics shows that, whenever a regular pattern was observed in the properties of matter (for instance, the eight-fold pattern of mesons and baryons or the periodical table of chemical elements), this pattern could be explained by invoking some underlying structures. According to this experience, the very existence of the pattern in the properties of the fundamental fermions unambiguously points to their internal structures, as opposed to the Standard Model where all the fundamental particles are point-like objects. In this paper we shall follow this lead and assume that the fundamental fermions are bound states of smaller particles, which hereafter we shall call "primitive" (to distinguish them from the "fundamental" and "elementary" particles). The primitive particles, by definition, should not decay into other particles, nor should they be of more than one type. Their properties must be defined by the very basic symmetries of spacetime, dualism being one of them. Then, based on these symmetries, one would expect to derive the properties of all the known (supposedly composite) particles and, perhaps, to predict the existence of new, so far unknown, particles.

The Universe

Let us begin with the following conjectures (postulates) about the general properties of matter and the universe:

1. Matter is structured, and the number of its structural levels is finite;
2. The simplest (and, at the same time, the most complex) structure in the universe is the universe itself;
3. The universe is self-contained (by definition);
4. All objects in the universe spin (including the universe itself).

The postulate (1) is as old as science itself. The idea of everything consisting of elementary indivisible atoms originates from Leucippus and Democritus. Since then, various structural levels of matter have been found, from molecules and atoms to nucleons and quarks. Each time, when a lower level of matter has been revealed, it was thought that this level is the ultimate and the simplest one. But patterns in properties of the "elementary" particles always indicated that there are underlying structures responsible for these properties. Fortunately, these patterns became simpler on lower structural levels, suggesting that matter is structured down to the simplest possible level (entity).

The postulate (2), according to which the universe is considered as the simplest possible structure, is not very new. In fact, this idea lies at the heart of modern cosmology. Theories of general relativity and of the Big Bang consider the universe to be a simple uniform object with curved space. Unfortunately, the character of this curvature (shape of the universe) cannot be deduced from Einstein's equations without additional observations. For simplicity, A.Einstein and A.Friedmann assumed the universe being a three-dimensional hyper-sphere, S^3, of positive, negative or zero curvature. But if we take into account the definition of the universe as a self-contained object (postulate 3), the spherical shape becomes inappropriate because the sphere has two interfaces, "inner" and "outer" (of course, in the case of S^3 these interfaces are three-dimensional hyper-surfaces). This might give rise to some doubts about the universe's self-contained character. The topology avoiding this problem is the well-known Klein bottle [13] usually visualised with the use of a two-dimensional surface (Fig.1). Similarly to S^3, a three-dimensional Klein bottle, K^3, can be of positive, negative or zero curvature. But K^3 has a unique hyper-surface, and its sym-

Figure 1: Two-dimensional representation of the Klein-bottle.

metries are different from those of S^3. For simplicity, in Fig.2 we visualise the hipothetical K^3-topology of the universe in the form of a one-dimensional scheme. An important feature of K^3 is the unification of its “inner” and “outer” hyper-surfaces. In the case of the universe, the unification might well occur on the sub-quark level, giving rise to the properties of elementary particles. In Fig.2 the unification region is marked with the symbol Π (primitive particle). Of course, in order to form structures there should be more than one primitive particle in the universe. This makes a difference between the topology of the universe and the conventional Klein bottle. One can call such an object a multi-connected hyper-Klein bottle. But its main features remain the same: dualism and unification of two opposite manifestations of space, which result in identification of the global cosmological scale with the local scale of elementary particles (supposedly, the distances comparable with the Planck-length scale).

Primitive particle

Let us suppose that space is smooth and continuous, i.e., that

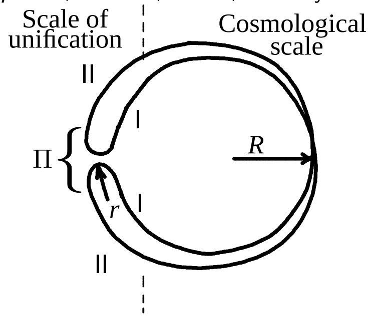

Figure 2: One-dimensional scheme of the Klein-bottle topology of the universe. The "inner" (I) and "outer" (II) manifestations of space are unified through the region Π (which can be considered as a primitive particle). R is the radius of curvature of the universe; r is the local curvature of the inverted space.

the local curvature of the medium cannot exceed some finite value ($r \neq 0$). Then, within the region Π space can be considered as being locally curved "inside-out", which is also visualised in Fig.3 in the form of a two-dimensional surface.

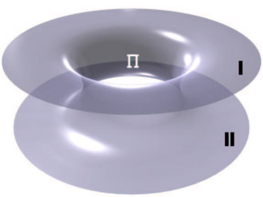

Figure 3: Two-dimensional visualisation of the "inversion" region Π (the primitive particle).

In this case, the primitive particles do not exist independently of the medium, but rather they are made of it. Then, the postulate (4) about the spinning universe gives us an insight into the possible origin of the particle masses. This postulate is not obvious, but it comes from the common fact that so far non-rotating objects have never been observed. The universe spinning with its angular velocity ω (if considered from the embedding space) would result in the linear velocity $\pm\omega R$ of the medium in the vicinity of the primitive particle, where R is the universe's (global) radius of curvature. It is seen in Fig.2 that the opposite signs in this expression correspond to two opposite manifestations of space, I and II. Thus, the spinning universe gives rise to an acceleration, a_g, of the primitive particle because of the local curvature, $1/r$, of space in the vicinity of Π (Fig.2). According to Newton's second law, this could be viewed as an action of a force, $F_g = m_g a_g$, proportional to this acceleration. The coefficient of proportionality between the acceleration and the force can be regarded as inertial mass of the primitive particle. However, for an observer in the coordinate frame of the primitive particle this mass is perceived as gravitational (m_g) because the primitive particle is at rest in this frame. Thus, the spinning universe implies the accelerated motion of the primitive particle (together with the observer) along its world line (time-axis). If now the primitive particle is forced to move along spatial coordinates with an additional acceleration a_i, it resists this force in exactly the same way as it does when accelerating along the time-axis. A force $F_i = m_i a_i$, which is required in order to accelerate the primitive particle, is proportional to a_i with the coefficient of proportionality m_i, and the observer will conclude that the primitive particle possesses an inertial mass m_i. But, actually, in our model the particle's inertial, m_i, and gravitational, m_g, masses are generated by the same mechanism of acceleration. Hence, they are essentially the same thing; $m_i \equiv m_g$, that is, mass is inertial.

Positive and negative signs of ωR do not affect the sign of the gravitational force $F_g = m_g a_g$ because the second derivative $a_g = \frac{\partial^2(ict)}{\partial t^2}$ has always the same sign (the local curvature, $1/r$, is the property of space and it does not depend on the direction of motion). However, the first derivative $\frac{\partial(ict)}{\partial t}$ (which is just another way of expressing ωR) can be either positive or negative, and the corresponding force, $F_e = q_\Pi \frac{\partial(ict)}{\partial t}$, should be either repulsive or attractive (depending on the choice of the trial particle). It would be natural here to identify F_e with the electric force, and the coefficient of proportionality, q_Π, — with the charge of the primitive particle. Although q_Π is positive in this expression, one can equivalently consider that it is this quantity that changes its sign, but not $\frac{\partial(ict)}{\partial t}$. Within our consideration, m_g and q_Π are of the same magnitude. For simplicity, hereafter we shall regard these quantities as being normalised to unity and denote them as $m_\circ$ and $q_\circ$. In fact, the above simple mass acquisition scheme should be modified because, besides the curvature, one must account for torsion in the vicinity of the inversion region (corresponding to the Weyl tensor). In the three-dimensional case, torsion has three degrees of freedom and the corresponding field splits into three components (six, if both manifestations of space are taken into account). It is reasonable to identify these three components with three polarities (colours) of the strong interaction [14].

As we can see, dualism of space gives rise to a specific symmetry of SU(3)/U(1)-type, which means that the primitive particles must possess both electric and colour charges ($\mathbf{\Pi}, \overline{\mathbf{\Pi}} \in \mathbf{3}_c$). We assume that by rotational transformations (SU(3)-symmetry) the primitive particle can be translated into one or another colour-charge state.

The existence of the inversion region of space around the primitive particle means also that, together with distances ρ measured from the particle centre, one can equivalently use the reciprocal distances, $\tilde{\rho} = \frac{1}{\rho}$, say, for the reciprocal manifestation of space (see, e.g., [3], p.p.239-249). Thus, any potential, which is proportional to $\frac{1}{\rho}$, in the reciprocal manifestation of space will be proportional to $\frac{1}{\tilde{\rho}}$ (or simply ρ) . For instance, the Coulomb-like potential, $\phi_e \propto \frac{1}{\rho}$, in the reciprocal manifestation of space should manifest itself strong-likely, $\phi_e \to \phi_s \propto \rho$, and vice versa. Note that due to the inversion of space the potentials cannot grow indefinitely when $\rho \to 0$ and, unlike the Coulomb potential, both ϕ_e and ϕ_s should decay to zero at the origin. The equilibrium distance, $\rho_\circ$, where $|\ \rho\ |=|\ \tilde{\rho}\ |$ and $|\ \phi_e\ |=|\ \phi_s\ |$, establishes a characteristic scale (breakes the symmetry of scale invariance) on which the primitive particles form structures (we assume the coupling constants to coincide, $\alpha = \alpha_s = 1$). A simple example of the split equilibrium field, which decays at the origin, is the field $F = \phi_s + \phi_e$, where $\phi_s = F_\circ(\rho)$, $\phi_e = -F'_\circ(\rho)$, and $F_\circ = s\exp(-\rho^{-1})$. Here the signature $s = \pm 1$ indicates the sign of the interaction (attraction or repulsion); the derivative of $F_\circ$ is taken with respect to the radial coordinate ρ. Far from the source, the second component of F mimics the Coulomb gauge, whereas the first component extends to infinity, being almost constant (similarly to the strong field). Here we are not going into details of the split field; what does matter for our consideration is the anti-symmetric character of two reciprocal fields due to the dual nature of space.

In order to formalise the use of the tripolar charges we shall introduce a set of auxiliary 3×3 singular matrices Π^i with the following elements:

$$^{\pm}\pi^i_{jk} = \pm\delta^i_j(-1)^{\delta^k_j}, \tag{1}$$

where δ^i_j is the Kronecker delta-function; the $\pm$-signs correspond to the sign of the charge; and the index i stands for the colour ($i =$

$1, 2, 3$ or red, green and blue). The diverging components of the field can be represented by reciprocal elements: $\tilde{\pi}_{jk} = \pi_{jk}^{-1}$. Then, we can define the (unit) charges and masses of the primitive particles by summation of these matrix elements:

$$q_\Pi = \mathbf{u}^\intercal \Pi \mathbf{u} \, , \quad \tilde{q}_\Pi = \mathbf{u}^\intercal \tilde{\Pi} \mathbf{u} \tag{2}$$

and

$$m_\Pi = \mid \mathbf{u}^\intercal \Pi \mathbf{u} \mid , \quad \tilde{m}_\Pi = \mid \mathbf{u}^\intercal \tilde{\Pi} \mathbf{u} \mid \tag{3}$$

($\mathbf{u}$ is the diagonal of a unit matrix; $\tilde{q}_\Pi$ and $\tilde{m}_\Pi$ diverge).

Assuming that the strong (colour) and electric interactions are opposite manifestations of the same geometrical feature of space and, taking into account the known pattern [14] of the coloure interaction (two like-charged but unlike-coloured particles are attracted, otherwise they repel), we can write the signature, s_{ij}, of the combined "chromoelectric" interaction between two primitive particles, say of the colours i and j, as:

$$s_{ij} = -\mathbf{u}^\intercal \Pi^i \Pi^j \mathbf{u}. \tag{4}$$

This expression is illustrated by Fig.4: the left part of this Figure shows the known pattern of attraction and repulsion between two strongly interacting particles. The right part corresponds to the hypothetical chromaticism of the electromagnetic interaction, antisymmetric (in our case) to the strong interaction. According to this pattern, under the electric force, two like-charged particles are attracted to each other if they are of the same colour charge and repel otherwise.

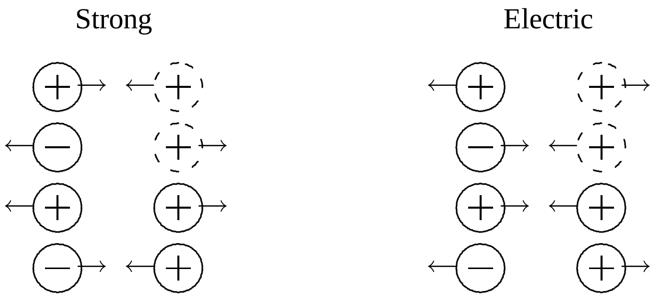

Figure 4: Pattern of the chromoelectric interaction between two primitive particles. Dashed and solid circles represent primitive particles with opposite colour-charges.

Colour dipoles

Obviously, the simplest structures allowed by the tripolar field are the monopoles, dipoles and tripoles, unlike the conventional bipolar (electric) field, which allows only the monopoles and dipoles. For example, two like-charged particles with unlike colours should form an equilibrium configuration – a charged dipole, $g^{\pm}$. The particles in such a dipole will be mutually attracted because of the strong force (increasing with distance). At the same time, the electric repulsive force, increasing when distance decreases, necessarily implies the existence of an equilibrium distance, $\rho_\circ$, at which the attraction caused by the strong force will be cancelled. Similarly, a neutral dipole g^0 of two unlike-charged primitive particles can also be formed.

Based on the analogy between the accelerated medium and the field flow in the vicinity of the primitive particle we can define the mass of a system containing, say, N particles, as being proportional to the number of these particles, wherever their field flow rates are not cancelled. For this purpose, we shall consider the total field flow rate, v_N, of such a system as a superposition of the individual volume flow rates of its N components. Then, the net mass can be calculated (to a first-order approximation) as the number of particles, N, times the normalised to unity field flow rate v_N:

$$m_N = \mid N v_N \mid \quad \text{where} \quad v_i = \frac{q_i + v_{i-1}}{1 + \mid q_i v_{i-1} \mid}. \tag{5}$$

Here v_N is computed recursively as a superposition of the individual flow rates, v_i, where $i = 2, \ldots, N$; and $v_1 = q_1$. The normalisation condition (5) expresses the common fact that the superposition flow rate of, say, two antiparallel flows ($\uparrow\downarrow$) with equal rate magnitudes $\mid \mathbf{v}_\uparrow \mid = \mid \mathbf{v}_\downarrow \mid = v$ vanishes ($v_{\uparrow\downarrow} = 0$), whereas, in the case of parallel flows ($\uparrow\uparrow$) it cannot exceed the magnitudes of the individual flow rates ($v_{\uparrow\uparrow} \leq v$).

When two oppositely charged particles combine (say red and antigreen), their oppositely directed velocities along the time-axis, $\frac{\partial(ict)}{\partial t}$, cancel each other (resulting in a neutral system). The corresponding, acceleration, $\frac{\partial^2(ict)}{\partial t^2}$, also becomes almost zero, which is implicit in (5). This formula implies the complete cancellation of masses in the systems with vanishing electric field, but this is just an approximation because the complete cancellation is possible only when the centres of the interacting particles coincide, which is not exactly our case (the primitive particles are separated at least by the equilibrium distance $\rho_\circ$). However, for the sake of simplicity, we shall neglect the small residual masses of electrically neutral systems.

In the matrix notation, the positively charged dipole, g_{12}^+, formed of the red and green colour charges (indices 1 and 2) can be represented as a sum of two matrices, Π^1 and Π^2:

$$g_{12}^{+} = \Pi^1 + \Pi^2 = \begin{pmatrix} -1 & +1 & +1 \\ +1 & -1 & +1 \\ 0 & 0 & 0 \end{pmatrix}, \tag{6}$$

with $q_{g^+} = +2\,[q_\circ]$. The mass and the reciprocal mass of the charged colour dipole will be $m_{g^+} \approx 2$ and $\tilde{m}_{g^+} = \infty$. If two components of the dipole are oppositely charged, say, $g_{12}^{0} = \Pi^1 + \overline{\Pi}^2$ (of whatever colour combination), then their electric fields cancel each other ($q_{g^0} = 0$). The neutral colour dipole will be massless ($m_{g^0} \approx 0$), but still $\tilde{m}_{g^0} = \infty$ due to the null-elements in the matrix g^0 (the dipole lacks, at least, one colour charge to make it colour-neutral). The infinities in the reciprocal masses of the dipoles imply that neither $g^{\pm}$ nor g^0 can exist in free states (because of their infinite energies).

In an ensemble of a large number of neutral dipoles g^0, not only electric but all the chromatic components of the field can be cancelled (statistically). Then, any additional charged particle Π^l belonging to this ensemble, if coupled to a neutral dipole g_{ik}, will restore the mass of the system:

$$m(\Pi^i, \overline{\Pi}^k, \Pi^l) = 1, \quad \text{but still} \quad \tilde{m}(\Pi^i, \overline{\Pi}^k, \Pi^l) = \infty. \tag{7}$$

According to Stokes' theorem, the charge of the new system will coincide with the charge of the additional particle Π^l.

Colour tripoles

Obviously, three complementary colour charges will tend to cohere and form a Y-shaped structure with the distance of equilibrium, $\rho_\circ$, between its components. Thus, by completing the set of colour-charges in the charged dipole (adding, for example, the blue-charged component to the system g^+_{12}, one would obtain a colour-neutral (but electrically charged) tripole:

$$\mathsf{Y} = \Pi^1 + \Pi^2 + \Pi^3 = \begin{pmatrix} -1 & +1 & +1 \\ +1 & -1 & +1 \\ +1 & +1 & -1 \end{pmatrix} ,$$

which is colour-neutral at infinity but colour-polarised in the vicinity of its constituents. Both m and $\tilde{m}$ of the particle Y are finite, $m_{\mathsf{Y}} = \tilde{m}_{\mathsf{Y}} = 3\ [m_\circ]$, since all of the diverging components of its combined chromofield are mutually cancelled (converted into the binding energy of the tripole). Locally, the red, green, and blue colour-charges of the tripole's constituents will be distributed in a plane forming a ring-closed loop. Thus, a part of the strong field of these three particles is closed in this plane, whereas another is extended (over the ring's poles).

Doublets of tripoles Two distant like-charged Y-particles will combine pole-to-pole with each other and form a charged doublet $\delta^- =$ Y⅄. Here we use the rotated symbol ⅄ in order to indicate the fact that the second tripole is turned through $180°$ with respect to the first one. The enhanced arm of Y marks one of the colours, say, red, in order to visualise the orientation of tripoles with respect to each other in the structural diagrams. The charged doublet will have the charge $q_\delta = -6\ [q_\circ]$ and a mass $m_\delta = \tilde{m}_\delta = 6\ [m_\circ]$. Likewise, if two unlike-charged Y-particles are combined, they will form a neutral structure $\gamma =$ Y⅄̲ (with $q_\gamma = 0$, $m_\gamma = \tilde{m}_\gamma = 0$). One can show that the shape of the potential in the vicinity of the pair allows a certain degree of freedom for the components of the doublet to rotate oscillating within the position anlge $\pm 120°$ with respect to each other. The corresponding states of equilibrium, δ and γ, can be written as

$$\delta^-_{\circlearrowleft} = \mathsf{YY}, \ \delta^-_{\circlearrowright} = \mathsf{YY}, \text{ and } \gamma_{\circlearrowleft} = \overline{\mathsf{Y}}\mathsf{Y}, \ \gamma_{\circlearrowright} = \overline{\mathsf{Y}}\mathsf{Y},$$

which correspond to two possible directions of rotation, clockwise ($\circlearrowright$) and anticlockwise ($\circlearrowleft$).

Triplets of tripoles The $\frac{2}{3}\pi$-symmetry of the tripole Y allows up to three tripoles to combine if all of them are like-charged. Necessarily, they will combine into a closed-loop (triplet), which can be in one of its eight possible states, four of which correspond to Y:

$$e^-_{\circlearrowleft} = \mathsf{YYY}, \ e^-_{\circlearrowright} = \mathsf{YYY}, \ \tilde{e}^-_{\circlearrowleft} = ⅄⅄⅄, \ \tilde{e}^-_{\circlearrowright} = ⅄⅄⅄,$$

and four others correspond to $\overline{\mathsf{Y}}$. The upside-down position of the symbol ⅄ in the last two diagrams is used to indicate that the vertices of the tripole are directed outwards of the centre of the structure (see Fig.5b). Another possibility is the orientation of the vertices towards the centre (Fig.5a). These configurations can be viewed as

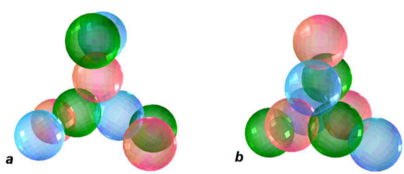

Figure 5: (a): Three like-charged Y-particles joined with their vertices directed towards the centre of the structure and (b): outwards the centre.

two different phase states of the same structure with its components spinning around its ring-closed axis and, at the same time, translating along this axis. The corresponding trajectories of colour-charges (currents) are shown in Fig.6. The structure e^- is charged, with $q_e = -9\ [q_\circ]$, and have a mass, $m_e = \tilde{m}_e = 9\ [m_\circ]$, corresponding to its nine constituents. The colour charges spinning around the ring-closed axis of e will generate a toroidal (ring-closed) magnetic

Figure 6: Trajectories of colour charges in the structure $3\mathsf{Y}{=}e$. The charges spin around the ring-closed axis of $3\mathsf{Y}$ and, at the same time, synchronously translate along this axis.

field which, at the same time, will force these charges to move along the torus. This circular motion of charges will generate a secondary (poloidal) magnetic field, contributing to the spin of these charges around the ring-axis, and so forth. The strength of the magnetic field will be covariant with the radius r_e. The interplay of the varying toroidal and poloidal magnetic fields, oscillating radius r_e, and varying velocities of the rotating charges converts this system into a complicated harmonic oscillator with a series of eigenfrequences and oscillatory modes.

Hexaplets. The pairs of unlike-charged Y-particles can form chains with the following colour patterns:

$$\nu_{e\circlearrowright} = \mathsf{Y}\overline{\mathsf{Y}} + \underline{\curlywedge}\curlywedge + \mathsf{Y}\overline{\mathsf{Y}} + \underline{\curlywedge}\curlywedge + \mathsf{Y}\overline{\mathsf{Y}} + \underline{\curlywedge}\curlywedge + \ldots \qquad (8)$$

or

$$\nu_{e\circlearrowleft} = \mathsf{Y}\overline{\mathsf{Y}} + \underline{\curlywedge}\curlywedge + \mathsf{Y}\overline{\mathsf{Y}} + \underline{\curlywedge}\curlywedge + \mathsf{Y}\overline{\mathsf{Y}} + \underline{\curlywedge}\curlywedge + \ldots \qquad (9)$$

(depending on the chosen direction of rotation of the constituents with respect to each other). The patterns repeat after each six consecutive links. The sixth link is compatible (attractive) by the configuration of its colour-charges with the first link, which allows closure of the chain 6Y$\overline{\mathrm{Y}}$ in a loop (hexaplet). The trajectories of the spinning colour charges in the structure are clockwise ($\nu_{e\circlearrowright}$) or anticlockwise ($\nu_{e\circlearrowleft}$) helices. The hexaplet consists of twelve tripoles ($n_{\nu_e} = 36$ primitive particles). It is electrically neutral and almost massless, according to (5).

Combinations of the structures Y, 3Y, and 6Y$\overline{\mathrm{Y}}$

Unlike the structure Y, with its partially closed strong field (in one plane), the strong fields of 3Y (e) and 6Y$\overline{\mathrm{Y}}$ (ν_e) are closed. Thus, these particles can be found in free states. They can combine with each other, as well as with Y, due to residual chromaticism of their potentials. The configuration of colour charges in the particle 6Y$\overline{\mathrm{Y}}$ matches (attracts) that of 3Y if both of these particles have helices of the same sign.

The repulsive and attractive forces between 6Y$\overline{\mathrm{Y}}$ and 3Y are of short range: far from the particle the colour potentials mix together into colourless electric potential. Separated from other particles, the structure 6Y$\overline{\mathrm{Y}}$ behaves like a neutral particle. But, if two particles 6Y$\overline{\mathrm{Y}}$ approach one another, they will be either attracted or repelled. The sign of the interaction depends on the compatibility of the colour patterns of both particles, that is, on the helicities of their currents.

The particle 6Y$\overline{\mathrm{Y}}$, which has its red, green, and blue colour constituents $120°$-symmetrically distributed along its closed loop, can combine with the particles Y or 3Y. The combined structure Y_1=ν+Y will have mass and be charged, with its charge $q_{\mathrm{Y}_1} = \pm 3$, corresponding to the charge of a single Y-particle, and a mass $m_{\mathrm{Y}_1} = 39$ (in units of $m_\circ$), corresponding to $n_{\nu_e} + m_{\mathrm{Y}} = 36 + 3$. The structure 3Y6Y$\overline{\mathrm{Y}}$ $= e\nu_e$ has a charge corresponding to its charged component,

e (± 9), and a mass of 45 mass units $(m_{e\nu_e} = m_e + n_{\nu_e} = 9 + 36)$.

The structure Y_1 cannot be free (similarly to Y). It will couple further with other Y_1-particles (through an intermediate ν_e of the opposite helicity) forming links $\mathsf{Y}_1\nu_e$⅄$_1$. A single link $\overline{\mathsf{Y}}_1\nu_e$ $\underline{⅄}_1$, which can be identified here with the *u*-quark, will have the charge derived from two Y-particles, $q_u = +6\ [q_\circ]$, and a mass of 78 mass units, $m_u = m(2\mathsf{Y}_1) = 2 \times 39\ [m_\circ]$. The positively charged u-quark can combine with the negatively charged structure $e^-\nu_e$ (with its 45-units mass), forming the *d*-quark of a 123-units mass, $m_d = m_u + m_{e\nu_e} = 78 + 45 = 123\ [m_\circ]$, and with its charge $q_d = q_u + q_e = +6 - 9 = -3\ [q_\circ]$ (see Fig.7). Expressing the

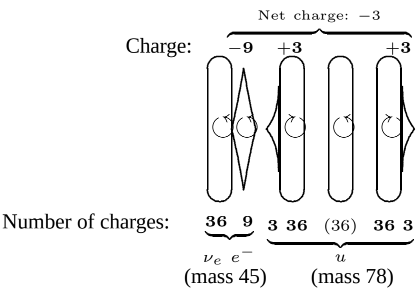

Figure 7: Scheme of the d-quark. The symbol ◇ is used for the electron, the symbols ⟨| and |⟩ stand for the tripoles (Y-particles), and the symbols 0 represent neutrinos. The symbols ↻ and ↺ denote the clockwise and anticlockwise helicities of the colour currents in the particle components.

charges $q_u = +6\ [q_\circ]$ and $q_d = -3\ [q_\circ]$ in units of q_e (dividing these values by nine) one would obtain the commonly known fractional charges of the u- and d-quarks $(+\frac{2}{3}$ and $-\frac{1}{3})$.

The second and third generations of particles

It is natural to suppose that the particles of higher (heavier) generations should be composed of simpler structures belonging to lower (lighter) generations. For example, the muon neutrino (a neutral particle) can be formed of two unlike-charged structures Y_1 and $\overline{\mathsf{Y}}_1$ intermediated by a neutral particle ν_e:

$$\nu_\mu = \mathsf{Y}6(\mathsf{Y}\overline{\mathsf{Y}})_{\circlearrowleft}\, 6(\mathsf{Y}\overline{\mathsf{Y}})_{\circlearrowright}\, 6(\mathsf{Y}\overline{\mathsf{Y}})_{\circlearrowleft}\overline{\mathsf{Y}} = \mathsf{Y}_1\nu_e\overline{\mathsf{Y}}_1. \quad (10)$$

The intermediate particle has the helicity sign opposite to that of the charged components of the structure, which creates a repulsive potential maintaining the system in equilibrium. The structure of the muon (Fig.8) can be written as

$$\mu = [6(\mathsf{Y}\overline{\mathsf{Y}})_{\circlearrowright} + 3\mathsf{Y}_{\circlearrowright}\overline{\mathsf{Y}}][6(\mathsf{Y}\overline{\mathsf{Y}})_{\circlearrowleft}6(\mathsf{Y}\overline{\mathsf{Y}})_{\circlearrowright}6(\mathsf{Y}\overline{\mathsf{Y}})_{\circlearrowleft}\mathsf{Y}] = \overline{\nu}_e e^- \nu_\mu. \quad (11)$$

Some of the binding forces between the components of the structure can be weaker than others, depending on whether there exist an extra electric repulsion between the components. Due to this factor, the second and third generations of particles can be considered as oscillating clusters, rather than rigid structures. In (11) the clustered components of the structure are enclosed in brackets. Computing the mass-energies of such "nonrigid" systems with oscillating components is not a straighforward task. However, these masses are computable in principle, which could be shown with the use of the following empirical formula:

$$m = \frac{m_1 + m_2 + \cdots + m_N}{1/\tilde{m}_1 + 1/\tilde{m}_2 + \cdots + 1/\tilde{m}_N}, \quad (12)$$

where m_i and $\tilde{m}_i$ are the masses and reciprocal masses of the components (all unit conversion coefficients are assumed to be set to unity). Alternatively, we shall use the following brief notation for the above summation rule:

$$m = \overline{m_1 + m_2 + \cdots + m_N}.$$

The fermion masses computed with the use of (12) are summarised in Table 2. As an example, let us compute the muon's mass. The masses of the muon's components, according to its structure (Fig.8), are: $m_1 = \tilde{m}_1 = 48$, $m_2 = \tilde{m}_2 = 39$ (in units of $m_\circ$). And the muon's mass is

$$m_\mu = \overline{m_1 + m_2} = \overline{48 + 39} = \frac{48 + 39}{1/48 + 1/39} = 1872\ [m_\circ]$$

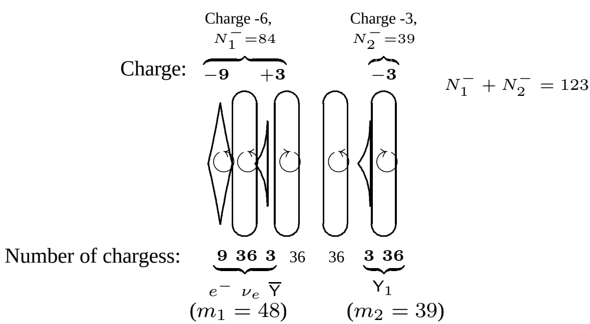

Figure 8: Scheme of the muon.

For the τ-lepton (Fig.9), $m_1 = \tilde{m}_1 = 201$, $m_2 = \tilde{m}_2 = 156$, and its mass is $m_\tau = \overline{201 + 156} = 31356\ [m_\circ]$.

For the proton, the positively charged fermion consisting of two *up* ($N_u = 2$), one *down* ($N_d = 1$) quarks and surrounded by a cloud of gluons g^0, the masses of its components are $m_u = \tilde{m}_u = 78$,

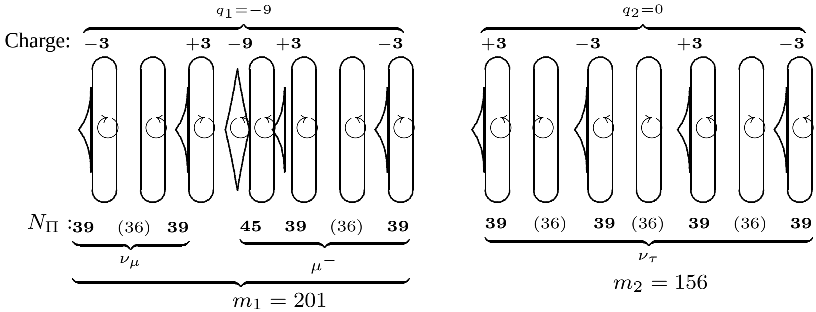

Figure 9: Scheme of the tau-lepton.

$m_d = \tilde{m}_d = 123$. The total number of primitive charges comprising the proton's structure is

$$N_p = 2 \times m_u + m_d = 2 \times 78 + 123 = 279,$$

which would correspond to the number of gluons (N_g) interacting with each of these charges (that is, $N_g = N_p = 279$). The masses of these gluons, according to (7), are $m_{g^0} = 1$, $\tilde{m}_{g^0} = \infty$. The resulting proton mass is

$$m_p = \overline{N_u m_u + N_d m_d + N_g m_g} = 16523\ [m_\circ]. \tag{13}$$

With this value of m_p one can convert m_e, m_μ, m_τ, and the masses of all other particles from units $m_\circ$ into proton mass units, m_p, thus enabling these masses to be compared with the experimental data (Table 1). The computed fermion masses are listed in Table 2. The value $m_p = 16523\ [m_\circ]$ also explains the well-known but not yet explained proton-to-electron mass ratio, since $\frac{m_p}{m_e} = \frac{16523\ m_\circ}{9\ m_\circ} \approx 1836$.

In this Table, the symbols Y_1, Y_2 and Y_3 denote the structures containing the triplets Y coupled to the ring-shaped neutral particles ν of increasing complexity. For example, the electron-neutrino,

Table 2: Predicted rest masses of quarks and leptons. The values given in the fourth column are converted into proton mass units dividing them by m_p=16523, Eq.(13); they can be compared with the experimental data, see Table 1. The overline notation in the third column correspond to the shorten form of Eq. (12)

Particle and its structure (components)		Number of charges in the components with non-cancelled mass	Predicted masses (in units of $m_\circ$)	Masses converted into m_p
First family				
ν_e	$6\mathsf{Y}\overline{\mathsf{Y}}$	≈ 0	≈ 0	≈ 0
e^-	$3\overline{\mathsf{Y}}$	9	9	0.0005447
u	$\overline{\mathsf{Y}}_1 \quad \nu_e \quad \overline{\mathsf{Y}}_1$	78	78	0.004720
d	$u \quad \nu_e \quad e^-$	123	123	0.007443
Second family				
ν_μ	$\mathsf{Y}^* \quad \nu_e \quad \overline{\mathsf{Y}}^*$	≈ 0	≈ 0	≈ 0
μ^-	$\nu_\mu + \overline{\nu}_e e^-$	$\overline{48+39}$	1872	0.1133
c	$\overline{\mathsf{Y}}_2 + \overline{\mathsf{Y}}_2$	$\overline{165+165}$	27225	1.6477
s	$c + e^-$	$\overline{165+165+9}$	2751	0.1665
Third family				
ν_τ	$u \quad \nu_e \quad \overline{u}$	≈ 0	≈ 0	≈ 0
τ^-	$\nu_\tau + \overline{\nu}_\mu \mu^-$	$\overline{156+201}$	31356	1.8977
t	$\overline{\mathsf{Y}}_3 + \overline{\mathsf{Y}}_3$	$\overline{1767+1767}$	3122289	188.94
b	$t + \mu^-$	$\overline{1767+1767+48+39}$	76061.5	4.603

$\nu_e = 6\mathsf{Y}\overline{\mathsf{Y}}$, gives rise to a particle $\mathsf{Y}_1 = \nu_e\mathsf{Y}$. Cyclic structures similar to that of the electron neutrino, may also appear in the form of "heavy neutrinos", $\nu_h = 6\mathsf{Y}_1\overline{\mathsf{Y}}_1$. They can further form "ultra-heavy" neutrinos $\nu_{uh} = 3(\overline{\mathsf{Y}}_1\nu_h u)e^-$ and so on, with the number of constituents increasing with the complexity of the structure. In Table 2 the components Y_2 and Y_3 of c and t have the following structures: $\mathsf{Y}_2 = u\nu_e u\nu_e e^-$, consisting of 165 primitive particles, and $\mathsf{Y}_3 = \nu_{uh}\mathsf{Y}$, consisting of 1767 primitive particles.

Table 2 and Fig.7-9 illustrate the family-to-family similarities between the particle structures. For example, in each family, the d-like quark appears as a combination of the u-like quark, with a charged lepton belonging to the lighter family. Thus, according to this scheme, the *s*-quark is composed of the *c*-quark and the electron, $s = c + e^-$, then, its mass is $m_s = \overline{165 + 165 + 9} = 2751$, derived from $m_c = \overline{165 + 165} = 27225$ and $m_e = 9$. Similarly, the mass of the *b*-quark, $b = t + \mu^-$, is the combination of $m_t = \overline{1767 + 1767} = 3122289$ with $m_\mu = \overline{48 + 39} = 1872$, resulting in $m_b = \overline{1767 + 1767 + 48 + 39} = 76061.5$. The overline notation here corresponds to the abbreviated form of (12). Each charged lepton is a combination of the neutrino from the same family with the neutrino and the charged lepton from the lighter family: $\mu^- = \nu_\mu + \overline{\nu}_e e^-$, $\tau^- = \nu_\tau + \overline{\nu}_\mu \mu^-$. Their masses computed according to the same scheme are also in a good agriment with experiment.

Conclusions

Based on the conjecture of spacetime dualism in the form of two reciprocal manifestations of space unified through an "inversion" region, our approach provides a reasonable explanation for the pattern of the fermion masses. Being only a framework for building the models of the composite fundamental fermions, our approach however already yields the quantum numbers and masses of these particles that agree with the experimental values to an accuracy better then 0.5%. The masses are derived without using any experimental input parameters (in this sense our model is self-contained) The proposed framework can be used for exploring the details of the early universe evolution. Such things as inflation, dark matter, dark energy, and others can easily be explained within this framework.

References

[1] Kobzarev, I. Yu.; Manin, Yu. I. *Elementary particles*; 2nd Ed., "Phasis": Moscow, 2000; 208 p.

[2] Hagiwara, K., et al.(Particle Data Group), Phys. Rev. D**66** (2002) 010001

[3] Greene, B.R., *The elegant universe*, Vintage, London (2000) 448p.

[4] Pesteil, P., *Numerical analysis of particle mass: the measure of the universe*, Apeiron **1**-No.11 (1991) 15-29

[5] Donoghue, J.F.. *The weight formation quark masses*, Phys.Rev. D**57** (1998) 5499-5508

[6] Golfand, Yu.A., Likhtman, E.P., *Extension of the algebra of Poincaré group generators and violation of P-invariance*, JETP Lett., **13** (1971), 323

[7] Likhtman, E.P., *Around SuSy 1970*, Nucl.Phys.Proc.Suppl., **101** (2001), 5-14

[8] Randall, L., Sundrum, R., *A large mass hierarchy from a small extra dimension*, Phys.Rev.Lett., **83** (1999) 3370

[9] Libanov, M., Nougaev E., *Towards the realistic fermion masses with a single family in extra dimensions*, JHEP 0204, (2002) 55

[10] Hoidn, P., Kusner, R.B., Stasiak, A., *Quantization of energy and writle in self-repelling knots* (2002) 16p., arXiv:physics/0201018

[11] Spaans, M., *On the topological nature of fundamental interactions*, arXiv:gr-qc/9901025 (2000) 18 p.

[12] Huber, S.J., Shafi, Q., *Fermion masses, mixing and proton decay in a Randall-Sundrum model*, Phys.Lett, B**498** (2001) 256-262

[13] Kastrup, H.A., *The contributions of Emmy Noether, Felix Klein and Sophus Lie to the modern concept of symmetries in physical systems*, in "Symmetries in physics (1600-1980)", Barcelona (1987), 113-163

[14] Suisso, E.F., de Melo, J.P.B.C., Frederico, T., *Relativistic dynamics of Qqq systems*, Phys.Rev. D**65** (2002) 094009