

Scale Expanding Cosmos IV- A possible link between General Relativity and Quantum Mechanics

C. Johan Masreliez
Redmond, WA 98052
USA

Email: jmasreliez@estfound.org

Three previous papers in this series introduce and discuss the Scale Expanding Cosmos (SEC) theory. This paper proposes a connection between the theory's discrete, stepwise, cosmological scale expansion and quantum mechanics. Very high frequency, small amplitude, temporal excitation in the metrical coefficients of the Minkowski spacetime is modelled in general relativity. DeBroglie type matter-waves are shown to result from motion of spatially confined metrics oscillating at the Compton frequency. The momentum "guiding function" of Bohm and de Broglie naturally follows from the geodesic equations of general relativity. A clear physical explanation to the double-slit interference experiment is given. Setting part of the Ricci scalar equal to zero gives a wave equation from which the Schrödinger equation is derived. This possible link between general relativity theory and quantum theory explains the particle-wave duality and suggests that quantum mechanical wave functions are amplitude and phase

modulations of very high frequency oscillations in general relativity's metrical coefficients.

Keywords: Quantum Ontology, Quantum Mechanics vs. General Relativity, Oscillating spacetime metrics, Particle-wave Duality, Double-slit experiment

1. Introduction.

Three previous papers in this journal have presented properties of a new cosmological model, the Scale Expanding Cosmos (SEC). The first paper showed that the SEC model resolves several cosmological puzzles and agrees better with observational data. The second paper investigated cosmic drag predicted by the SEC theory and suggested ways of confirming this new phenomenon. The third paper dealt with gravitation in the SEC showing that the gravitational potential is modified by the scale expansion and rolls off at the Hubble distance. It also appears that the modified gravitational potential might prevent the formation of black holes. In this paper I show that the discrete scale expansion mode of the SEC theory might provide an ontological explanation to quantum mechanics.

Anyone first encountering quantum theory is puzzled by the fact that no ontological explanation to the quantum world is given. This article shows that quantum mechanics follows naturally if the metrical coefficients (metrics) of spacetime oscillate at very high frequencies. It appears that this could explain the particle-wave duality and help resolve other quantum theoretical puzzles, for example the double slit experiment.

It is shown that oscillating metrics treated in General Relativity (GR) naturally leads to Quantum Mechanics (QM) if particles always were accompanied by oscillation of the spacetime metrics at the Compton frequency. In this view the quantum mechanical wave

functions are modulations of this Compton “carrier” oscillation of the spacetime metrics. I derive the de Broglie matter-wave, the de Broglie/Bohm “pilot wave” momentum relation, and the Schrödinger equation from the GR line element assuming that the metrics oscillate. A detailed physical explanation for the double slit particle interference is also given.

2. The SEC expansion mode.

In my first paper in this series (Masreliez, 2004a) the SEC theory was introduced as a way of achieving cosmological expansion without cosmological aging. Scale expansion implies that all epochs are equivalent and provides a direction of time via cosmic drag, which over time diminishes relative motion. Equations of motion are not time-symmetrical in the SEC.

GR occupies a prominent position in modern cosmology and any new cosmos theory ignoring GR would be unacceptable. Yet, the universe is what it is and we should not expect that it necessarily confirms to science at our present level of understanding. This is an ancient dilemma; we are always constrained by the level of contemporary science, even if we suspect it to be inadequate. But, we have no choice; cosmology without science is metaphysics.

In my opinion Parmenides’ conclusion that existence rules out non-existence is crucial to any theory of the universe, because it is inconceivable the anything ever could emerge from non-existence. We must conclude that existence is eternal and accept that the universe as a whole does not age.

If we try to use GR to model eternal scale expansion, all epochs ought to be equivalent and be modeled by the same line element. However, we find that epochs with identical expanding line elements cannot be connected via continuous variable transformation. This is

disappointing since line elements related by continuous variable transformation are physically equivalent in GR. Thus, by GR different epochs would not be physically equivalent in the SEC. Since there seems to be no reason why a certain cosmological scale should take preference, it appears that GR may fall short, at least when it comes to modeling cosmological scale expansion.

However, if we allow discrete scale transformation in addition to continuous variable transformation it would be possible to model the SEC using GR, since the GR is “blind” to discrete scale increments. This suggests a novel expansion mode whereby continuous scale expansion is complemented by discrete scale adjustment. One might visualize this process as a piecewise continuously expanding cosmological scale at all levels from galaxies to elementary particles, with everything participating in the expansion, repeatedly and incrementally “jumping into” slightly larger scales at very high frequencies. This process of incremental scale expansion could be what we perceive as the progression of time.

In this paper I will show that this incremental nature of the scale expansion would explain why the world is “quantum mechanical” and provide the missing link between GR and QM. To further illustrate the expansion process, let us investigate the SEC expansion mode at a certain fixed frequency and assume the following steps:

1. Starting at a scale $\exp(t/T)$ the scale expands continuously from t to $t+\Delta t$.
2. At $t+\Delta t$ the scale is $\exp(\Delta t/T)\cdot\exp(t/T)$ and the GR equations are identical to those of scale $\exp(t/T)$ since GR is blind to discrete scale increments.
3. The discrete scale increment $\exp(\Delta t/T)$ may therefore be ignored, which will restore the original line element in step 1. Another way to see this novel step would be to admit a discrete change in the pace of proper time $ds \Rightarrow \exp(\Delta t/T)\cdot ds$.

The scale increment appears on both sides of the line element and cancels out, restoring the original line element.

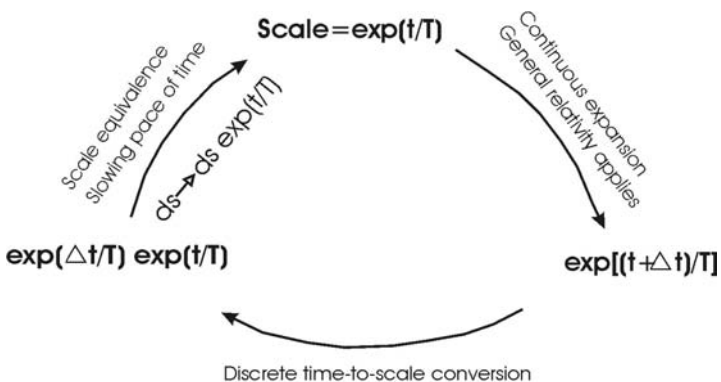


Figure 1: Proposed expansion cycle

This expansion loop is illustrated in Figure 1. Admittedly this is a very simplified picture since this expansion process might occur simultaneously at many different frequencies creating the vacuum zero point field.

Below I will show that if particles are associated with oscillating metrics at the Compton frequency, QM follows naturally from the GR line element.

3. The modulated line element and the matter-wave.

The SEC theory implies that there exists a cosmological reference frame and that the cosmological scale expansion at any epoch might be modeled in this frame by GR using the SEC line element:

$$ds^2 = e^{2t/T} (dt^2 - dx^2 - dy^2 - dz^2) \quad (3.1)$$

We will investigate very high frequency modulation of the four metrical coefficients in this line element. In this treatment the exponential scale factor $\exp(2t/T)$ will be replaced by high frequency oscillation of the metrics.

Consider the line element:

$$ds^2 = e^{2p(t)} (dt^2 - dx^2 - dy^2 - dz^2) \quad (3.2)$$

Assume a periodic modulating function,

$$p(t), \quad (3.3)$$

period modulation with angular velocity ω $\omega \sim 1/t_p$, t_p being period of modulation $\ll T$. In particular with $p(t) = C \cdot \cos(\omega t) = \text{Re}\{C \exp(-i\omega t)\}$ where $C = \text{constant}$:

$$ds^2 = \exp\left\{\text{Re}\left(2Ce^{-i\omega t}\right)\right\} (dt^2 - dx^2 - dy^2 - dz^2) \quad (3.4)$$

In the following I will omit the label $\text{Re}(\cdot)$. The use of a complex exponent is to be interpreted as the real part, for example $i \cdot \exp(-i\omega t)$ means $\sin(\omega t)$. This will be justified below.

Motion of a spatially confined region with the line element (3.4) at a constant velocity v in the x direction may be modeled by the Lorentz transformation:

$$\begin{aligned} x &= \gamma(x' - vt') \\ t &= \gamma(t' - vx') \\ \gamma &= (1 - v^2)^{-1/2} \end{aligned} \quad (3.5)$$

The modulating part of the exponent in the metric then becomes:

$$2Ce^{-i\omega t} \rightarrow 2Ce^{i\gamma\omega(vx'-t')} \quad (3.6)$$

If the modulation is confined to spatial region (particle) the metric modulation $\exp(i\gamma\omega vx')$ is reminiscent of the quantum mechanical wave function of a single moving particle with wave number:

$$k = \gamma\omega v \quad (3.7)$$

Thus, motion of a locally confined spatial region with oscillating metrics has the effect of spatially modulating the phase of the excitation.

It appears that this spatial modulation could be related to the quantum mechanical wave function.

The relationship between the momentum and the wave number is:

$$p = \hbar k = mv \quad (3.8)$$

which from (3.7) implies $m = \hbar\gamma\omega$. This relation suggests that every particle is associated with a metric excitation frequency that corresponds to its energy as given by (3.8), i.e. the relativistic Compton frequency. Motion causes this oscillation to be “phase modulated” in the form of a spatial wave, $\exp(ikx)$ that modulates the Compton oscillation. This could be the de Broglie “matter-wave”. Thus, if Compton oscillation accompanies each particle as modeled by (3.4) this oscillation will in motion be modulated by a de Broglie type spatial matter-wave. In this interpretation the quantum mechanical matter-wave is a spatial wave pattern in the metrics of spacetime formed by a *relativistic* effect due to time dilation according to (3.6). The very high Compton frequency corresponding to the particle matter energy makes this small relativistic temporal effect significant even at relatively low velocities. With this interpretation the matter-wave is a purely relativistic phenomenon generated by motion.

To explore this possible connection between GR and QM a bit further, consider the line element:

$$ds^2 = \exp\left[2C \cdot h(x, y, z) e^{-i\gamma\omega t}\right] \left[dt^2 - dx^2 - dy^2 - dz^2\right] \quad (3.9)$$

C is a constant. Here a possibly complex valued wave function $h(x, y, z)$ provides amplitude and phase modulation of the Compton oscillation. The geodesic equation of GR for motion in the x-direction becomes at low velocities:

$$\frac{d^2x}{ds^2} = -\Gamma_{00}^1 \left(\frac{dt}{ds}\right)^2 - 2\Gamma_{10}^1 \frac{dt}{ds} \frac{dx}{ds} + \text{small terms with the velocity squared.}$$

The two Christoffel symbols corresponding to the line element (3.4) are:

$$\begin{aligned} \Gamma_{00}^1 &= C \cdot \frac{\partial h}{\partial x} e^{-i\varpi t} = h_x e^{-i\varpi t} \\ \Gamma_{10}^1 &= -C \cdot i h \varpi e^{-i\varpi t} \end{aligned} \quad (3.10)$$

where $\varpi = \gamma\omega$

Since all velocities are small we have:

$$\begin{aligned} \frac{dt}{ds} &\approx \exp(-C \cdot h \cdot e^{-i\varpi t}) \text{ and} \\ \frac{d^2x}{ds^2} &\approx \frac{d}{ds} \left(\frac{dx}{dt} \frac{dt}{ds}\right) \approx \frac{d}{ds} \left[\frac{dx}{dt} \exp(-C \cdot h \cdot e^{-i\varpi t})\right] = \end{aligned} \quad (3.11)$$

$$\left[\frac{d^2x}{dt^2} + (C i \varpi h e^{-i\varpi t}) \frac{dx}{dt}\right] \left[\exp(-C h \cdot e^{-i\varpi t})\right]^2$$

The geodesic equation becomes:

$$\frac{d^2x}{dt^2} + (Ci\varpi h e^{-i\varpi t}) \frac{dx}{dt} = -Ch_x e^{-i\varpi t} + 2(Ci\varpi h e^{-i\varpi t}) \frac{dx}{dt} \quad (3.12)$$

Setting terms modulated by $e^{-i\varpi t}$ equal:

$$v_x = \frac{dx}{dt} = -i \frac{1}{\varpi} \left(\frac{h_x}{h} \right) = \frac{1}{\varpi} \text{Im} \left(\frac{h_x}{h} \right); \text{Im} = \text{imaginary part}$$

In general using (3.8):

$$\mathbf{p} = \hbar \varpi \mathbf{v} = \hbar \cdot \text{Im} \frac{\nabla h}{h} \quad (3.13)$$

$$\frac{d^2 \mathbf{x}}{dt^2} = \text{Re} \left(-C \cdot \Delta h \cdot e^{-i\varpi t} \right) \quad (3.14)$$

If the function $h(x,y,z)$, which modulates the Compton oscillation, is proportional to the quantum mechanical wave function ψ , relation (3.13) is the de Broglie/Bohm [Bohm, 1952] momentum relation, i.e. the “pilot guiding function”.

This shows that de Broglie’s and Bohm’s momentum relation follows directly from the geodesic equation of GR if the spacetime metrics of a particle oscillate at the Compton frequency. Thus, the previously mysterious guiding function finds its physical explanation if a particle always is accompanied (or sustained) by oscillation of the spacetime metrics at the Compton frequency.

4. Deriving a metric wave equation from General Relativity.

Einstein’s GR equations are:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = K \cdot T_{\mu\nu} \quad (4.1)$$

As usual $G_{\mu\nu}$ is Einstein's tensor, $R_{\mu\nu}$ is the Ricci tensor, $g_{\mu\nu}$ the metric tensor. K is Einstein's constant and $T_{\mu\nu}$ is the energy-momentum tensor. These ten equations reduce to four if the matrix $g_{\mu\nu}$ is diagonal.

Consider the line element:

$$\begin{aligned} ds^2 &= e^{2Ch(x,y,z)\cdot p(t)} \left(dt^2 - dx^2 - dy^2 - dz^2 \right) \\ g_{00} &= e^{2Ch(x,y,z)\cdot p(t)} \\ g_{\mu\mu} &= -e^{2Ch(x,y,z)\cdot p(t)}, \quad \mu = 1, 2, 3 \\ g_{\mu\nu} &= 0; \quad \mu \neq \nu \end{aligned} \tag{4.2}$$

The Ricci tensor is as usual:

$$\begin{aligned} R_{\mu\nu} &= \Gamma_{\mu\nu,\alpha}^{\alpha} - \Gamma_{\mu\alpha,\nu}^{\alpha} + \Gamma_{\mu\nu}^{\alpha} \Gamma_{\alpha\beta}^{\beta} - \Gamma_{\mu\beta}^{\alpha} \Gamma_{\alpha\nu}^{\beta} = R^{lin} + R^{quad} \\ R^{lin} &= \Gamma_{\mu\nu,\alpha}^{\alpha} - \Gamma_{\mu\alpha,\nu}^{\alpha} \\ R^{quad} &= \Gamma_{\mu\nu}^{\alpha} \Gamma_{\alpha\beta}^{\beta} - \Gamma_{\mu\beta}^{\alpha} \Gamma_{\alpha\nu}^{\beta} \end{aligned} \tag{4.3}$$

Here the Christoffel symbols are:

$$\Gamma_{\mu\nu}^{\alpha} = \frac{1}{2} g^{\alpha\beta} \left(g_{\beta\mu,\nu} + g_{\beta\nu,\mu} - g_{\mu\nu,\beta} \right) \tag{4.4}$$

Einstein's summation convention for repeated indices applies and all Christoffel symbols contain first derivatives of the metrical coefficients as indicated by the index following the comma.

The first two terms in (4.3) are *linear* in the second derivatives of $h(x,y,z)p(t)$ and will average to zero assuming that both $p(t)$ and its time derivatives average to zero. However, the amplitude could be large if the frequency is high. The last two terms, which are quadratic in the derivatives of the metrics, typically do not average to zero. I will treat these two contributions separately.

The contributions from the two linear terms are:

$$\begin{aligned} R_{00}^{lin} &= \Gamma_{00,1}^1 + \Gamma_{00,2}^2 + \Gamma_{00,3}^3 - \Gamma_{01,0}^1 - \Gamma_{02,0}^2 - \Gamma_{03,0}^3 \\ &= (h_{xx} + h_{yy} + h_{zz}) \cdot p - 3h \cdot \ddot{p} \end{aligned} \quad (4.5)$$

$$\begin{aligned} R_{11}^{lin} &= \Gamma_{11,0}^0 + \Gamma_{11,2}^2 + \Gamma_{11,3}^3 - \Gamma_{10,1}^0 - \Gamma_{12,1}^2 - \Gamma_{13,1}^3 \\ &= -(h_{yy} + h_{zz} + 3h_{xx}) \cdot p + h \cdot \ddot{p} \end{aligned} \quad (4.6)$$

$$\begin{aligned} R_{22}^{lin} &= \Gamma_{22,0}^0 + \Gamma_{22,1}^1 + \Gamma_{22,3}^3 - \Gamma_{20,2}^0 - \Gamma_{21,2}^1 - \Gamma_{23,2}^3 \\ &= -(h_{xx} + h_{zz} + 3h_{yy}) \cdot p + h \cdot \ddot{p} \end{aligned} \quad (4.7)$$

$$\begin{aligned} R_{33}^{lin} &= \Gamma_{33,0}^0 + \Gamma_{33,1}^1 + \Gamma_{33,2}^2 - \Gamma_{30,3}^0 - \Gamma_{31,3}^1 - \Gamma_{32,3}^2 \\ &= -(h_{xx} + h_{yy} + 3h_{zz}) \cdot p + h \cdot \ddot{p} \end{aligned} \quad (4.8)$$

The Ricci scalar based on the linear terms in (4.3) is given by:

$$R^{lin} = g^{\mu\nu} R_{\mu\nu}^{lin} = 6 \cdot \left[(h_{xx} + h_{yy} + h_{zz}) \cdot p - h \cdot \ddot{p} \right] \quad (4.9)$$

In my third paper (Masreliez, 2004c) Hilbert's action integral was modified to include the scale:

$$\begin{aligned} I_{SEC} &= \int S^2 \cdot (G - K \cdot T_{SEC}) \sqrt{-g} \cdot dV = \\ &= - \int S^2 \cdot (R + K \cdot T_{SEC}) \sqrt{-g} \cdot dV \end{aligned} \quad (4.10)$$

Setting the variation of the scale S equal to zero implies $R + K \cdot T_{SEC} = 0$. However, the amplitude of the oscillatory linear Ricci scalar is very much larger than the term T_{SEC} since $C \cdot \omega^2 \gg 1/T^2$, see below. Therefore, for all practical purposes:

$$R^{lin} = 0 \quad (4.12)$$

From (4.9):

$$\left(h_{xx} + h_{yy} + h_{zz}\right) \cdot p - h \cdot \ddot{p} = 0 \quad (4.13)$$

Or:

$$\nabla^2 \Phi - \frac{\partial^2 \Phi}{\partial t^2} = 0; \quad \Phi = h \cdot p \quad (4.14)$$

This is a standard wave equation with known solutions expressing resonating spatial metrics in response to the scale excitation.

For example, if $p(t) = \cos(\omega t)$ one possible solution is:

$$h(x, y, z) = C \cdot \cos(\pm k_x x + \varphi_x) \cos(\pm k_y y + \varphi_y) \cos(\pm k_z z + \varphi_z) \quad (4.15)$$

$$k_x^2 + k_y^2 + k_z^2 = \omega^2$$

This is a family of three-dimensional standing waves. Waves moving in the positive or negative direction at the speed of light are also solutions to (4.14).

The contribution to the Ricci scalar from the two last terms in relation (4.3) given by R^{quad} typically does not average to zero. These terms contain “real” positive or negative energy density. However, the linear and quadratic parts of the Ricci scalar can be treated separately due to their different characters.

The wave solution $h(x,y,z)$ of (4.14), which could be proportional to the quantum mechanical wave function ψ , may be interpreted as amplitude and phase modulation of a high frequency carrier oscillation $p(t)$ in the metrics of spacetime.

The remaining quadratic part R^{quad} of the Ricci tensor could generate a particle's matter energy. The energy density expressed by terms in R^{quad} is proportional $K^{-1} \cdot (C \cdot \omega)^2$. If the diameter of the particle were in the order of $1/\omega$ the energy would be proportional to $K^{-1} \cdot C^2 /$

ω . Setting this equal to a particle's energy $(h\bar{h}) \cdot \omega$ the oscillation amplitude becomes:

$$C \sim L_{pl} \cdot \omega \quad (4.16)$$

where L_{pl} is the Planck length. For the electron C is in the order of 10^{-23} since $\omega = 10^{12} / \text{m}$ and $L_{pl} = 1.6 \cdot 10^{-35} \text{ m}$. Clearly $C \cdot \omega^2 \gg 1/T^2$ with $T = 10^{26} \text{ m}$.

Note that second derivatives of the modulating factor in the metric appear linearly in the linear wave equation (4.14). This justifies the use of a complex exponent to express amplitude and phase modulation and permits superposition of wave functions.

5. The approach of Louis de Broglie and David Bohm.

Over the years several attempts have been made to interpret QM. Louis de Broglie suggested at the Solvay conference in 1927 that a particle might be guided by a "pilot wave" directly related to the wave function. At this meeting Wolfgang Pauli challenged him to explain what happens to his pilot wave at scattering, which causes a single wave function to split up into a superposition of many different components. A single pilot wave corresponding to this superposed wave function cannot explain the different possible trajectories taken by the scattered particle. David Bohm, who independently revived de Broglie's idea in the 1950s, countered this challenge by speculating that decoherence quickly occurs between the different branches of the scattered wave function and that the scattered particle selects only one of the possible branches leaving the other branches "empty".

The validity of this explanation is supported by our interpretation where scattered contributions to the wave function at difference energies *are uncorrelated due to their different Compton frequencies*.

The various possible trajectories appear as different solutions to the geodesic equation, one for each Compton frequency. After scattering, the particle will take one of several possible trajectories corresponding to its energy.

David Bohm further developed de Broglie's proposal in his hidden variable theory of 1952-1954 [Bohm, 1952 and 1954]. He was able to show that QM may be developed in a straightforward manner based on the assumption that there exists in Nature a "quantum potential" of the form:

$$\text{Quantum potential} = -\frac{\hbar^2}{2m} \frac{\nabla^2 Q}{Q} \quad (5.1)$$

The real valued function Q is related to the amplitude of the QM wave function by:

$$\psi = Q \cdot e^{iU/\hbar} \quad (5.2)$$

The phase function U is real valued and related to the momentum by:

$$\mathbf{p} = \nabla U \quad (5.3)$$

From relation (4.14) we have:

$$\frac{\hbar^2}{2m} \frac{\nabla^2 h}{h} \propto \frac{\hbar^2}{2m} \frac{\ddot{p}}{p} \quad (5.4)$$

This is (5.1) if h is real valued. Thus, according to GR oscillating metrics could generate a "quantum potential" of the form proposed by Bohm. This is an important observation, since Bohm demonstrates that QM can be derived based on the assumption that this quantum potential actually exists. However, in the past the source of the quantum potential has been a mystery and generally Bohm's theory has not been well accepted.

More recent versions of Bohm's theory championed by John Bell [Bell 1986], and others [for example Dürr, Goldstein and Zanghi 1996, Holland 1993], show that a consistent quantum mechanical theory can be derived based on just two assumptions:

There exists a function, ψ (of unspecified ontology), which satisfies Schrödinger's wave equation and for small velocities, $v \ll c$, the motion of particles satisfies the relation:

$$\mathbf{p} = \hbar \cdot \text{Im} \frac{\nabla \psi}{\psi} \quad (5.5)$$

This is (5.3) expressed in a different form. We have shown in (3.13) that (5.5) follows directly from the geodesic equation of GR assuming oscillating spacetime metrics and David Bohm and his followers have shown that (3.13) together with the Schrödinger equation may be used to construct a theory that in all respects is equivalent to classical QM.

One puzzling aspect of Bohm's theory is the non-local character of the momentum relation (5.5). Since it contains the ratio between two functions it could exert influence over vast distances even at very low amplitudes. In the past it was difficult to understand how this might be possible and how distant wave functions of negligible power could influence the local motion of particles. Bohm called this property "active information" [Bohm, 1993] proposing that (5.5) somehow informs each particle how to move without exerting any physical force. This mysterious long-range action might have discouraged more substantial support for Bohm's theory since it appears rather speculative.

However, with the present interpretation this "pilot wave" action finds its natural explanation; relation (5.5) expresses modulation of the spacetime metrics without carrying energy. A particle moves on a geodesic without being subjected to any external force. However, if

the local spacetime is curved relative to a stationary observer, the particle trajectory might be curved and regions of resonance might be preferred, see below. In this way the motion of a particle might be influenced without energy transfer, i.e. without any external force.

6. The generalized geodesic.

The proposition that the QM wave functions might be modulations of the metrics of spacetime and that the pilot function is the GR geodesic permit us to generalize the geodesic relation (3.13) in a straightforward manner using a more general line element:

$$ds^2 = \exp\left[2C \cdot h(x, y, z, t) \cdot e^{-i\Phi(x, y, z, t)}\right] \left[dt^2 - dx^2 - dy^2 - dz^2\right] \quad (6.1)$$

Proceeding as in Section 3 assuming velocities much lower than the speed of light we get the *generalized geodesic*:

$$\mathbf{v} = \text{Im} \frac{\nabla h - i \nabla \Phi \cdot h}{i \frac{\partial h}{\partial t} + \frac{\partial \Phi}{\partial t} \cdot h} \quad (6.2)$$

As an example, consider a particle moving with wave number k in a wave field h :

$$\begin{aligned} h &= h(x, y, z) \\ \Phi &= -\mathbf{k} \cdot \mathbf{x} + \gamma\omega t \\ \text{where } \mathbf{x} &= (x, y, z) \text{ and } \mathbf{k} = (k_x, k_y, k_z) \end{aligned} \quad (6.3)$$

Applying (6.2) we again get:

$$\mathbf{v} = \frac{\mathbf{k}}{\gamma\omega} \quad (6.4)$$

This is trivially true if \mathbf{k} is constant. However, if the wave field interacts with the motion, \mathbf{k} is not constant. In this case we could

transform the line element (5.1) into the particle's instantaneous inertial frame at (x_b, y_b, z_b) by applying the (inverse) Lorentz transformation.

Let $\mathbf{x}_f = \mathbf{x}' + \mathbf{x}''$ where \mathbf{x}' is a vector parallel to the velocity and \mathbf{x}'' is perpendicular to it.

Define new variables:

$$\begin{aligned} \mathbf{x} &= \mathbf{x}'' + \gamma(\mathbf{x}' + \mathbf{v} \cdot t') \\ t &= \gamma(t' + \mathbf{v} \cdot \mathbf{x}') \end{aligned} \quad (6.5)$$

We then get the same form of the line element (6.1) but with transformed functions:

$$\begin{aligned} h &= h\left[x'' + \gamma(x' + v_x t'), y'' + \gamma(y' + v_y t'), z'' + \gamma(z' + v_z t')\right] \\ \Phi &= -\mathbf{k} \cdot \mathbf{x}'' + \omega t' \end{aligned} \quad (6.5)$$

The geodesic (5.2) evaluated at \mathbf{x}_f with differentiations with respect to \mathbf{x}' and t' becomes:

$$\delta v = \text{Im} \left[\frac{\gamma \cdot \nabla h}{i\gamma(\nabla h \cdot \mathbf{v}) + \omega h} \right] = \frac{-\nabla h \cdot (\nabla h \cdot \mathbf{v})}{(\nabla h \cdot \mathbf{v})^2 + \left(\frac{\omega h}{\gamma}\right)^2} \quad (6.6)$$

This holds if h is real valued. By Lorentz transforming (6.3) into the particle's instantaneous inertial rest frame we get (6.6) according to which the particle still has a velocity relative to the inertial frame. In classical physics this velocity would be zero; instead there might be acceleration between the inertial frame and the particle frame. The difference here is due to the geodesic (6.2) that deals with velocity rather than acceleration. How this should be interpreted is not

obvious. However, (6.6) might be seen as giving a velocity correction between consecutive discrete temporal frames. Perhaps the particle trajectory evolves according to (6.6) with time increments comparable to the period of the Compton frequency.

The geodesic may then be evaluated from the iteration:

$$\begin{aligned}\delta \mathbf{v}_{k+1} &= \frac{-\nabla h_k \cdot (\nabla h_k \cdot \mathbf{v}_k)}{(\nabla h_k \cdot \mathbf{v}_k)^2 + (\omega \cdot \mathbf{h}_k / \gamma_k)^2} \\ \mathbf{v}_{k+1} &= \mathbf{v}_k + \delta \mathbf{v}_{k+1} \\ \mathbf{x}_{k+1} &= \mathbf{x}_k + \mathbf{v}_{k+1} \cdot \Delta t \\ t_{k+1} &= t_k + \Delta t \\ \nabla h_{k+1} &= \nabla h(\mathbf{x}_{k+1}, t_{k+1})\end{aligned}\tag{6.7}$$

Relation (6.7) gives a velocity change induced by the wave function h that counteracts positive or negative motion in the direction of the gradient of the wave function striving to align a particle's trajectory perpendicular to the gradient vector or at an extreme of h . The geodesic trajectory depends on the locally changing wave function relative to the moving particle permitting both feedback interactions by self-interference and response to external influences.

7. The double slit interference experiment.

Consider the double slit experiment in which a particle moves parallel to the x-axis toward a screen at $x=0$ with two narrow slits located at $y=y_0$ and $y=-y_0$. After passing the slits the particle strikes a second screen located at $x = D$, where an interference pattern develops even when particles arrive one at a time.

According to the standard interpretation the particle somehow passes through both slits at the same time and “interferes with itself”. David Bohm and others have shown that an interference pattern

develops if the momentum relation (3.13) guides the particle, assuming that a wave function associated with the particle simultaneously passes through both slits. However, this does not explain the physical mechanism at work. This problem is addressed here.

When the particle passes through one of the two slits, it might become slightly deflected and move in the y-direction as well as the x-direction. The matter-wave it generates interferes with the double slit geometry. Assume that the particle after passing the screen has the wave vector (k_x, k_y) and that its matter-wave interferes with the two slits generating a standing wave pattern :

$$\begin{aligned} f &= C \cdot \left\{ e^{i \cdot k_x \cdot x + i \cdot k_y \cdot (y - y_0)} + e^{i \cdot k_x \cdot x + i \cdot k_y \cdot (y + y_0)} \right\} = \\ &= C \cdot e^{i \cdot (k_x \cdot x + k_y \cdot y)} \cdot 2 \cdot \cos(k_y \cdot y_0) \end{aligned} \quad (7.1)$$

Defining the wave vector by:

$$k_x = \gamma \cdot \omega \cdot v_x; \quad k_y = \gamma \cdot \omega \cdot v_y \quad (7.2)$$

Making the approximation:

$$v_y \approx y \cdot v_x / x$$

We have:

$$k_y = \gamma \cdot \omega \cdot v_y = k_x \cdot y / x = 2\pi \cdot y / (x \cdot \lambda) \quad (7.3)$$

λ is the wave length of the matter wave in the x-direction.

So that:

$$\begin{aligned} f &= C \cdot e^{i \cdot (k_x \cdot x + k_y \cdot y)} \cdot 2 \cdot \cos(k_x \cdot y \cdot y_0 / x) \\ |f|^2 &= 4 \cdot C^2 \cos^2(k_x \cdot y \cdot y_0 / x) = 4 \cdot C^2 \cos^2 \left[2\pi \cdot y \cdot y_0 / (x \cdot \lambda) \right] \end{aligned} \quad (7.4)$$

This implies:

$$\begin{aligned} h &= \cos(k_x \cdot y \cdot y_0 / x) \\ \Phi &= -(k_x \cdot x + k_y \cdot y) + \gamma \omega t \end{aligned} \quad (7.5)$$

The resonance pattern is fixed in space and is determined by the geometry of the two slits. It expresses a *potential* that is activated by the particle's matter-wave, which resonates with varying amplitude in response to the particle's position and velocity. The wave function h represents the amplitude modulation of the Compton excitation *if the particle is present at* (x, y) .

Following the procedure above by Lorentz transforming (7.5) into the instantaneous rest frame we get:

$$\begin{aligned} x &= x'' + \gamma(x' + v_x t') \\ y &= y'' + \gamma(y' + v_y t') \\ h &= \cos[k_x \cdot (y'' + \gamma(y' + v_y t')) \cdot y_0 / (x'' + \gamma(x' + v_x t'))] \\ \phi &= \omega \cdot t' \end{aligned} \quad (7.6)$$

We have:

$$\begin{aligned} \frac{\partial h}{\partial x'} &= -\sin[k_x \cdot y \cdot y_0 / x] \cdot [-k_x \cdot \gamma \cdot y \cdot y_0 / x^2] \\ \frac{\partial h}{\partial y'} &= -\sin[k_x \cdot y \cdot y_0 / x] \cdot [k_x \cdot \gamma \cdot y_0 / x] \\ \frac{\partial h}{\partial t'} &= -\sin[k_x \cdot y \cdot y_0 / x] \cdot [k_x \cdot \gamma \cdot y_0 / x] [-v_x (y/x) + v_y] \end{aligned} \quad (7.7)$$

This holds true if k_x is constant, i.e. if v_x is constant. The geodesic (7.6) becomes:

$$\delta v_x = \frac{[k_x y_0 \gamma y / x^2] \cdot \sin^2(k_x y_0 y / x) [k_x \gamma y_0 / x] [-v_x (y/x) + v_y]}{\sin^2(k_x y_0 y / x) [k_x \gamma y_0 / x]^2 [-v_x (y/x) + v_y]^2 + [\omega \cdot h]^2} = \quad (7.8)$$

$$= \frac{[y/x] \cdot \sin^2(k_x y_0 y / x) [k_x \cdot y_0 / x]^2 [-v_x (y/x) + v_y]}{\sin^2(k_x y_0 y / x) [k_x \cdot y_0 \cdot x]^2 [-v_x (y/x) + v_y]^2 + [\omega \cdot \cos(k_x y_0 \cdot y / x) / \gamma]^2}$$

$$\delta v_y = -\delta v_x \frac{x}{y} \quad (7.9)$$

The velocity correction for v_x is much smaller than for v_y if $y \ll x$, which justifies the assumption that k_x is constant. The particle's direction changes as long as v_y/v_x differs from y/x , i.e. as long as the particle does not move on a straight line from the origin at $x=y=0$. Since the particle passes through one of the two slits it does not come from the origin and its direction will change to align itself with one of the resonance ridges of the wave function. Between these ridges where the wave function is zero we have:

$$k_x \cdot y_0 \cdot y / x = \pi / 2 + n \cdot \pi \quad (7.10)$$

$$\delta v_y = (-x/y) \cdot \delta v_x = -1 / [-v_x \cdot (y/x) + v_y]$$

Thus, the velocity in the y -direction becomes large where the wave function (7.4) is close to zero even if v_y/v_x is close to y/x and the particle avoids these regions.

Where $y_0 \ll x$ we have for $k_x \cdot y_0 \cdot y / x \neq \pi / 2 + n \cdot \pi$:

$$\delta v_y \approx \frac{-\tan^2(k_x y_0 \cdot y / x) [k_x y_0 / x]^2 [-v_x (y/x) + v_y]}{(\omega / \gamma)^2} \quad (7.11)$$

The direction changes rapidly close to the slits but slower with increasing distance, x , from the slits.

This explains the interference fringes around each (positive and negative) resonance crest of the wave function and provides a physical explanation to the double slit particle interference phenomenon. The particle interacts with its own matter-wave occupying regions where the matter-wave resonates with the geometry. Simulated trajectories based on (7.8) and (7.9) using the approach of (6.7) show that v_y/v_x converges rapidly to y/x , which means that the particle after an initial adjustment travels in a straight line from the origin. The direction y/x changes when the initial velocities v_y change as shown in Figure 2. The particles prefer certain directions and form the fringe pattern on the screen. This gives a physical explanation for the double slit experiment.

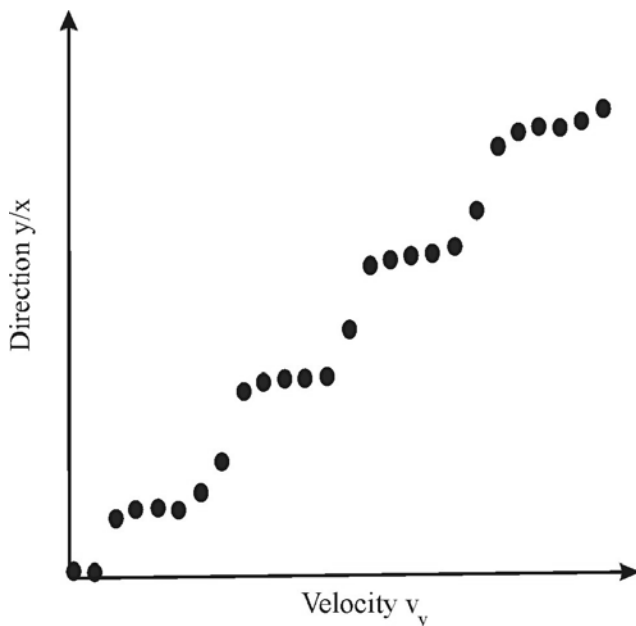


Figure 2: Simulated double slit dispersion.

Self-interference might also explain the stable states for electrons in an atom. Resonance defines regions (corresponding to different energy states) that confine the electrons. Motion in the radial direction is counteracted by the geodesic (6.6). The trajectories line up perpendicular to the radial gradient resulting in circular orbits. These regions are created when the matter-wave resonates with the geometry and with externally imposed energy potentials, for example the electrostatic field. The self-interfering matter-wave automatically confines the electron. Stable states are sustained by feedback since the electron's motion defines the matter-wave, which resonates with the geometry and controls the motion. The electron moves on a geodesic, which explains why it does not radiate electromagnetic energy.

8. Deriving the Schrödinger equation from GR assuming oscillating spacetime metrics

Consider the modulated Minkowski line element:

$$ds^2 = \exp[2 \cdot C \cdot \psi(x, y, z, t) \cdot e^{-i\sigma t}] (dt^2 - dx^2 - dy^2 - dz^2) \quad (8.1)$$

As before, setting the Ricci scalar corresponding to this line element equal to zero results in the linear wave equation (4.14) and justifies the use of the complex notations. I will assume that the temporal oscillation is associated with a particle, for example an electron, and that it is confined to a small spatial, region.

Now assume that the phase of this oscillation depends on the location as modeled by a scalar energy potential V and that its Compton frequency might change slightly, which corresponds to changing the energy by E , which is assumed to be constant.

Assume further that the wave function ψ that modulates the metric coefficients has the following form:

$$\psi(x, y, z, t) = h(x, y, z) \cdot e^{-iEt/\hbar} \cdot e^{i\varpi(\oint(1+V/m)ds \cdot \mathbf{n})} \quad (8.2)$$

ds is the path increment vector, \mathbf{n} a unit vector and $V=V(x,y,z)$.

Carrying out the differentiations in (4.14) and separating terms:

Terms not containing \mathbf{n} :

$$\nabla^2 h - \varpi^2 \left[\left(1 + \frac{V}{m}\right)^2 - \left(1 + \frac{E}{\hbar\varpi}\right)^2 \right] h = 0 \quad (8.3)$$

Terms containing \mathbf{n} :

$$\varpi \left[2\left(1 + \frac{V}{m}\right) \cdot \frac{\nabla h}{h} \cdot \mathbf{n} + \frac{\nabla V \cdot \mathbf{n}}{m} + \left(1 + \frac{V}{m}\right) \nabla \mathbf{n} \right] = 0 \quad (8.4)$$

Assuming that the velocity field is divergence free this becomes:

$$\left[2\left(1 + \frac{V}{m}\right) \cdot \frac{\nabla h}{h} + \frac{\nabla V}{m} \right] \cdot \mathbf{n} = 0 \quad (8.4a)$$

Considering (8.3) we have if $V \ll m$:

$$\left[1 + \frac{V}{m} \right]^2 = 1 + \frac{2V}{m} + \left(\frac{V}{m} \right)^2 \approx 1 + 2 \frac{V}{m} \quad (8.5)$$

Also, if $E \ll \varpi \hbar$:

$$\left[1 + \frac{E}{\varpi \hbar} \right]^2 = 1 + 2 \frac{E}{\varpi \hbar} + \left(\frac{E}{\varpi \hbar} \right)^2 \approx 1 + 2 \frac{E}{\varpi \hbar} \quad (8.6)$$

From (8.3), (8.5) and (8.6) we get the Schrödinger equation:

$$-\frac{\hbar^2}{2m} \nabla^2 h + (V - E) \cdot h = 0 \quad (8.7)$$

where $m = \varpi \hbar$

This derivation may easily be generalized to the situation where h also depends on time. We then get the additional terms:

$$2i\left(\varpi + \frac{E}{\hbar}\right)\frac{\partial h}{\partial t} - \frac{\partial^2 h}{\partial t^2} \approx 2i\varpi \frac{\partial h}{\partial t} \quad (8.8)$$

$$-\frac{\hbar^2}{2m}\left(2i\varpi \frac{\partial h}{\partial t}\right) = -i\hbar \frac{\partial h}{\partial t}$$

Moving this term to the right hand side of (8.7):

$$-\frac{\hbar^2}{2m}\nabla^2 h + (V - E) \cdot h = i\hbar \frac{\partial h}{\partial t} \quad (8.9)$$

This is the time dependent Schrödinger equation.

We saw that the vector \mathbf{n} does not influence our derivation of the Schrödinger equation; it only depends on the location of the particle and its relativistic energy as given by its Compton frequency. This suggests that the Schrödinger equation models resonance conditions of the spacetime metrics that only depend on the location and Compton frequency of a particle, see further section 10.

I will not analyze the imaginary part here since it is not needed to derive the Schrödinger equation.

The development above demonstrates that the Schrödinger equation can be obtained directly from the GR line element (8.1) by setting the linear part of the Ricci scalar, which contains second derivatives of the metric, equal to zero. Lorentz transformation retains the form of the wave equation (4.14) facilitating transformation between inertial frames. Although the form of the Schrödinger equation generally will change with coordinate transformation, the line element representation (8.1) allows the corresponding wave equation to be derived in any coordinate system simply by setting the Ricci scalar equal to zero.

The function h , which is a solution to the Schrödinger equation, models resonance conditions in the metrics of spacetime. This wave function depends on the surrounding geometry and on the applied

field potential V . When a particle moves, the wave field surrounding it creates resonance regions, which guide the particle via the generalized geodesic of section 6. If other particles also are present accompanied by their own wave fields, the resulting wave field is a superposition of all individual fields. However, since a particular particle's wave function is modulated by its own very high frequency Compton excitation, the wave functions from other sources, for example from different particles with different excitation frequencies (motions), will not interfere with a particle's motion.

With this interpretation the Schrödinger equation models the passive response of spacetime to the presence of a particle at a certain location. The resonating wave field determines the resulting trajectory subject to energy and momentum constraints. External influences, for example the presence of other particles with their associated wave fields, might influence the motion if their Compton excitations match. In this way distant particles may interact via their wave functions. Furthermore, when the particle moves in a field potential its kinetic energy might change, which changes the resonance conditions of the metric field and as a result the trajectory might change via feedback action. This could influence the motion and confine the particle to resonating regions. The energy state determines the Compton frequency with its corresponding resonance field, which might confine the particle in a certain energy state by geodesic feedback.

9. Schrödinger equation for the electromagnetic field.

For completeness I will also derive the Schrödinger equation for the electromagnetic field, which is obtained by changing the scalar potential V to the electromagnetic field vector potential A and adding the electric field-potential Φ :

$$\psi(x, y, z, t) = h(x, y, z) \cdot e^{i(e\Phi/\hbar)t} \cdot e^{i\varpi \oint (\mathbf{n} - e\mathbf{A}/m) \cdot d\mathbf{s}}$$

$$\mathbf{A}(x, y, z) = (A_x, A_y, A_z) \quad (9.1)$$

$\Phi = \text{Constant field potential}$

Proceeding as above using this modulation instead of (8.1), separating terms containing the vector \mathbf{n} from those without this vector, we get:

Terms without \mathbf{n} :

$$\nabla^2 h - i \cdot 2 \frac{e}{\hbar} \mathbf{A} \cdot \nabla h - i \frac{e}{\hbar} \nabla \mathbf{A} \cdot h - \left(\frac{e\mathbf{A}}{\hbar} \right)^2 h - \frac{2e\Phi\varpi}{\hbar} h = 0 \quad (9.2)$$

This may be put into the form:

$$\left(\nabla - i \frac{e\mathbf{A}}{\hbar} \right)^2 h - \frac{2e\Phi\varpi}{\hbar} h = 0 \quad (9.3)$$

Where I have used:

$$\left(\nabla - i \frac{e\mathbf{A}}{\hbar} \right)^2 \triangleq \nabla^2 h - i \nabla \left(\frac{e\mathbf{A}}{\hbar} h \right) - i \frac{e\mathbf{A}}{\hbar} \nabla h - \left(\frac{e\mathbf{A}}{\hbar} \right)^2 h \quad (9.4)$$

The electromagnetic Schrödinger equation now follows directly:

$$\left\{ -\frac{\hbar^2}{2m} \left(\nabla - i \frac{e\mathbf{A}}{\hbar} \right)^2 + e \cdot \Phi \right\} h = 0 \quad (9.5)$$

The remaining terms containing \mathbf{n} are:

$$\varpi \left\{ i \left[2 \cdot \mathbf{n} \frac{\nabla h}{\hbar} + \nabla \mathbf{n} \right] + 2 \frac{e}{\hbar} \mathbf{n} \cdot \mathbf{A} \right\} \quad (9.6)$$

If the velocity field is divergence-free:

$$\left\{ i \left[\frac{\nabla h}{h} \right] + \frac{e}{\hbar} \cdot \mathcal{A} \right\} \cdot \mathbf{n} = 0 \quad (9.6a)$$

10. The wave function for multiple particles.

For a single particle I assumed that the metric modulation function h is proportional to the QM wave function. In a situation with many particles we might expect that the modulation of the spacetime metrics at a certain location will depend on the cumulative influences from all particles. Every particle is surrounded by its own carrier modulation field and generates a matter wave that depends on its velocity. Therefore, the net modulation at a certain location ought to depend on the relative positions as well as on the individual matter waves generated by the particle motions.

However, if similar particles move with different velocities they will have different Compton carrier frequencies due to the relativistic factor γ appearing in (3.6). This means that they cannot interfere. Therefore, similar particles at different velocities will move independently without interaction, except possibly via some random disturbance, which is characteristic for quantum motion. If multiple particles interference they must be in the same energy state with identical Compton carrier frequencies, for example a beam of photons, electrons or neutrons. In this situation the modulation function h simply depends on the various locations of the particles; the (magnitude of the) velocity may be suppressed since it is the same for all particles. This explains why the metric modulation function h (and the wave function of quantum mechanics) is a function of the configuration space of the particles. In other words, the use of a multidimensional wave function will only be of interest in situations where interference is possible. Since this implies identical carrier

frequencies, the configuration space (locations) for the particles suffice when modeling the metric excitation, i. e. the wave function.

Thus, the multidimensional wave function of QM, which traditionally is perceived partly as a physical wave, partly as a probability distribution, expresses modulation of the metrics in response to the geometry and the shifting locations of particles. This provides a natural ontological explanation to the multidimensional QM wave function.

11. The probability interpretation.

Born's interpretation, by which the squared magnitude of the wave function is a probability density, is central to the standard interpretation of QM. However, the fact that Bohm's momentum relation is identical to the geodesic equation of GR suggests that the statistical interpretation of the wave function might be secondary. Bohm and Vigier [Bohm & Vigier, 1954], Belinfante [Belinfante, 1973] and Valentini [Valentini, 1991]] have demonstrated that the probability interpretation for the wave function is a direct consequence of the Schrödinger equation and Bohm's momentum relation (the geodesic) if one assumes that some additional excitation also is present. It is apparent from (3.14) that this is the case; excitation in the form of high frequency acceleration is always present if the metrics oscillate.

Several authors have investigated the connection between the Schrödinger equation and stochastic processes that modulate particle motion subject to random disturbances. A recent example is the strictly mathematical treatment by Nelson [Nelson, 1982] who derives the Schrödinger equation from Brownian motion with three assumptions:

1. Bohm's momentum relation applies.

2. The energy is preserved.
3. A diffusion constant ν is defined by:

$$\text{Expectation of } \Delta x_{\mu} \Delta x_{\nu} = 2 \cdot \nu \cdot \delta_{\mu\nu} \Delta t$$

$$\nu = \frac{\hbar}{m}$$

Thus, it appears that random motion induced by oscillating spacetime metrics, constrained by the GR geodesic and the uncertainty relation characterizes the quantum world.

12. Summary

General Relativity might be closely related to Quantum Mechanics if the metrics of spacetime oscillate. If a particle always is accompanied by oscillation of the spacetime metrics at the Compton frequency the de Broglie's matter-wave arises as a relativistic effect when the particle moves. In addition, setting the linear part of the Ricci scalar equal to zero leads to the Schrödinger equation.

In this interpretation the matter-wave is a purely relativistic effect created by moving oscillating metrics. The particle moves on the GR geodesic, which at low velocities is identical to the de Broglie/Bohm momentum relation.

Furthermore, the quantum mechanical wave functions might be amplitude and phase modulations of high frequency oscillations in the spacetime metrics. If the QM wave functions are modulations of the spacetime metrics, they are not propagating waves in the ordinary sense but resonance conditions intimately depending on the local geometry and field potentials. This explains how particles suddenly can jump between energy levels when shifts between different resonance modes occur. Spacetime oscillations could resonate with the motion of electrons in atoms. The electron automatically finds one

of several possible regions of resonance and motion on a geodesic sustains this resonance.

This also could explain the enigmatic double slit experiment. The geometry of the two slits in the screen creates a spacetime resonance pattern that guides the particle by self-interference. Changing the geometry instantly changes the interference pattern. After passing through either one of the two slits in the screen the particle is guided by its own resonating matter-wave as determined by the double slit geometry.

The wave functions often are complex having both amplitude and phase. In the present interpretation this represents phase shift of the modulated “carrier” oscillation. Therefore, the complex nature of the QM wave functions has clear physical meaning.

Particles might be created as oscillatory resonance modes in spacetime with their energies sustained by the cosmological scale expansion. Individual modulation of all four metrics would provide additional degrees of freedom beyond the four coordinates of GR and could explain the success of string theory.

Summarizing, if the metrics of spacetime oscillate, quantum mechanics may be derived directly from general relativity in a straightforward manner by setting the linear part of the Ricci scalar equal to zero. In this interpretation the quantum mechanical wave functions are amplitude and phase modulations of carrier oscillation at the Compton frequency associated with a particle and the de Broglie/Bohm pilot wave is the geodesic of general relativity. This connection with general relativity may be used to derive a generalized guiding function (the geodesic), which provides a simple and direct physical explanation to the previously enigmatic double-slit experiment.

References

- Aspect A., 1986, *Quantum Concepts in Space and Time*, Oxford University Press. 1-15
- Belinfante L.E., 1973, *A survey of hidden variable theories*, Oxford, Pergamon Press
- Bell J. S., 1987, *Speakable and unspeakable in quantum mechanics*, Cambridge University Press, Cambridge.
- Beller M., 1999, *Quantum Dialogue*, University and Chicago Press
- Bohm D., 1952, A suggested interpretation of quantum theory in terms of “hidden” variables, I and II, *Phys. Rev.* 85, 166-193
- Bohm D. and Vigier J.V., 1954, Model of the Causal Interpretation of Quantum Theory in terms of Fluid with Irregular Fluctuations, *Phys. Rev.* 96, 208-216
- Bohm D., 1957, *Causality and Chance in Natural Law*, D. Van Nostrand, Princeton New Jersey
- Bohm D., 1980, *Wholeness and the Implicate Order*, Routledge, London
- Bohm D. and Hiley B.J., 1993, *The Undivided Universe*, Routledge, London and New York
- Dirac, P. A. M., 1958, *Quantum Mechanics*, 4th ed. London, Oxford University Press.
- Dürr J. S., Goldstein S. and Zanghi N., 1992, Quantum Equilibrium and the Origin of Absolute Uncertainty, *J. Stat. Phys.* 67, 843-907
- Dürr J. S., Goldstein S. and Zanghi N., 1996, Bohmian Mechanics as the Foundation of Quantum Mechanics, in *Bohmian Mechanics and Quantum Theory, An Appraisal*, Kluwer Academic Publishers, Dordrecht
- Holland P., 1993, *The Quantum Theory of Motion. An Account of the de Broglie-Bohm Causal Interpretation of Quantum Mechanics*. Cambridge University Press, New York, 1993.xx, 598 pp.
- Home D., 1997, *Conceptual Foundations of Quantum Physics*, Plenum Press, New York and London
- Kolesnik Y., 2000, *Proceedings IAU 2000*
- Kolesnik Y, and Masreliez C. J., 2004, “Secular trends in the mean longitudes of the planets derived from optical observations” *AJ*, Vol. 128, No. 2., p. 878
- Masreliez C.J., 1999, *The Scale Expanding Cosmos Theory*, *Astroph. & Space Science*, 266, Issue 3, p. 399-447

- Masreliez C.J., 2004a, “Scale Expanding Cosmos Theory I – An Introduction”,
Apeiron, July
- Masreliez C.J., 2004b, “Scale Expanding Cosmos Theory II – Cosmic Drag”,
Apeiron, October
- Masreliez C.J., 2004c, “Scale Expanding Cosmos Theory III – Gravitation”,
Apeiron, October
- Nelson E., 1979, Connection between Brownian motion and quantum mechanics,
Lecture notes in Physics, vol.100, Springer, Berlin
- Penrose R., 1986, Quantum Concepts in Space and Time, Oxford University Press.
129-146
- Popper K. R., 1982, Quantum Theory and the Schism in Physics, Hutchinson & Co.
Ltd.
- Valentini A., 1991, Phys. Lett. A, 111, 274-276