Derivation of the Relativistic Doppler Effect from the Lorentz Force

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This paper demonstrates that calculation and interpretation of the relativistic Doppler effect is possible using only the Lorentz force and relativity theory. This method eliminates the need for the Lorentz transformation and time dilation. It also demonstrates the proper link between the invariance of the light speed c and the Doppler relations, $c = \lambda \nu = \lambda' \nu'$.

Keywords: Doppler Effect, speed of light; time dilation.

I Introduction

Einstein [1] presented the relativistic Doppler effect (RDE) in terms of Lorentz transformations (LT) in order to incorporate relativistic dynamics and time dilation. This led many physicists to attempt equivalent (RDE) theories without (LT) and its kinematical effects [2, 3]. Since there is no propagating medium in (RDE), only the relative velocity u between the source and observer is considered. Therefore, the difference between the classical Doppler effect and (RDE) is the introduction of time dilation and the transverse Doppler effect (TDE)

Assume two moving inertial frames S and S', a source with frequency ν_0 in frame S and an observer in frame S', Source and observer are receding from each other with the relative velocity u . We then have two cases:

If the observer is receding from the source, the frequency observed is classically,

$$\nu' = \nu_0\left(1 - \frac{u}{c}\right) \tag{1a}$$

Conversely, if the source is receding from the observer,

$$\nu' = \frac{\nu_0}{1 + \frac{u}{c}} \tag{1b}$$

In either case, the frequency decrease due to recessional motion is called the 'classical red shift'.

If source and observer are approaching one another, u is replaced with $-u$ in Eqs. (1a) and (1b),

$$\nu' = \nu_0\left(1 + \frac{u}{c}\right) \tag{1c}$$

$$\nu' = \frac{\nu_0}{1 - \frac{u}{c}} \tag{1d}$$

This is the 'classical blue shift'. In SRT's formalism, this effect is represented by time dilation.

From the observer's point of view, if the source is receding, its clock runs slower by $\sqrt{1-\frac{u^2}{c^2}}$. This reduces the frequency by $\sqrt{1-\frac{u^2}{c^2}}$ Hence, Eq. (1b) becomes

$$v' = \sqrt{1-\frac{u^2}{c^2}}\frac{v_0}{1+\frac{u}{c}} = v_0\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}} \tag{2a}$$

This represents the relativistic red shift frequency. As in Eq. (1d) (approaching), we replace u with $-u$ in Eq. (2a) to provide,

$$v' = \sqrt{1-\frac{u^2}{c^2}}\frac{v_0}{1-\frac{u}{c}} = v_0\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}} \tag{2b}$$

This represents the relativistic blue shift. When the light wave is at an angle θ relative to the ox axis, then Eqs. (2a) and (2b) become

$$v' = \sqrt{1-\frac{u^2}{c^2}}\frac{v_0}{1+\frac{u}{c}\cos\theta} \tag{3}$$

If the motion is normal to the line conecting source and observer, then $\theta = 90^\circ$ and $\cos\theta = 0$, and we have the "red-shifted" (TDE) relative to the observer; *i.e.,*

$$v' = v_0\sqrt{1-\frac{u^2}{c^2}} = \frac{v_0}{\gamma} \tag{4}$$

Experiments [4,5] show that the observed frequency ν' is reduced by γ [Eq.(4)]. This is considered evidence of time dilation and proves the validity of Eq.(4). Therefore, time dilation plays an important part in modern physics.

Eq. (4) is considered a unique feature of SRT and is related to the dilation of time only for the moving source. Therefore it was assumed in some textbooks that "TDE should not occur in classical physics". However, it was recognized that time dilation was essential to (RDE) as without it, absolute motion could be detected.

These statements are invalid in some representations [6a,6b], and incorrect if (TDE) is derived without resorting to an ether or relativistic modifications. This is demonstrated in the following:

II Energy, Mass, Momentum, Velocity and Electromagnetic Transformations.

As determined in [7] and contrary to what is often claimed in SRT, relativity theory was derived from its fundamental postulates and the classical laws, *i.e.,*

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}\ , \ \frac{d\varepsilon}{dt} = \mathbf{F}\mathbf{v} \tag{5}$$

This method is used in the case of a charged particle q moving with velocity $\mathbf{v}$ in frame S, subject to an electric field $\mathbf{E}$ and a magnetic flux density $\mathbf{B}$. We find that Eqs. (5) yield:

$$\frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})\ , \ \frac{d\varepsilon}{dt} = q\mathbf{E}\mathbf{v} \tag{6}$$

The Cartesian components of Eqs. (6) in frame S are

$$\frac{dp_x}{dt} = q\left(E_x + V_y B_z - V_z B_y\right) \tag{7a}$$

$$\frac{dp_y}{dt} = q\left(E_y + V_z B_x - V_x B_z\right) \tag{7b}$$

$$\frac{dp_z}{dt} = q\left(E_z + V_x B_y - V_y B_x\right) \tag{7c}$$

$$\frac{d\varepsilon}{dt} = q\left(E_x V_x + E_y V_y + E_z V_z\right) \tag{7d}$$

Starting from Eqs. (7a,7d), we multiply Eq. (7d) with $\frac{u}{c^2}$ and then subtract the result from Eq. (7a). Following the approach used in [8], we obtain

$$P'_x = \gamma\left(P_x - \frac{u}{c^2}\varepsilon\right) \tag{8a}$$

$$V'_y = \frac{V_y}{\gamma\left(1 - \frac{uV_x}{c^2}\right)} \tag{9b}$$

$$V'_z = \frac{V_z}{\gamma\left(1 - \frac{uV_x}{c^2}\right)} \tag{9c}$$

and

$$E'_x = E_x \ , \ B'_y = \gamma\left(B_y + \frac{u}{c^2}E_z\right) \ ,$$

$$B'_z = \gamma\left(B_z - \frac{u}{c^2}E_y\right) \ , \ dt' = \gamma dt\left(1 - \frac{uV_x}{c^2}\right)$$

We start once again with Eqs. (7a,7d), but now multiply Eq. (7a) with – u and add it to (7d). Following the approach used in [8], we get

$$\varepsilon' = \gamma\left(\varepsilon - uP_x\right) \text{ (8b)},\ V'_x = \frac{V_x - u}{1 - \frac{uV_x}{c^2}} \tag{9a}$$

and

$$E'_y = \gamma\left(E_y - uB_z\right),\ E'_z = \gamma\left(E_z + uB_y\right)$$

The scalar factor γ can be fixed by applying relativity theory to Eq. (9b) to get

$$\gamma^2\left(1 - \frac{u^2}{c^2}\right) = 1 \text{ (a) or } \gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \text{ (b)} \tag{10}$$

Now starting from Eq. (7b) and using Eq. (10a). Following the approach in [8], we have

$$\begin{aligned} P'_y &= P_y \\ B'_x &= B_x \end{aligned} \tag{8c}$$

In a similar way, if we start with Eq. (7c), we have

$$P'_z = P_z \tag{8d}$$

Now the conventional definition of momentum in frames S and S', *i.e.,* $\mathbf{P} = m\mathbf{v}$ and $\mathbf{P}' = m'\mathbf{v}'$, will lead to the true relativistic expression for mass and energy.

As we know, Eqs. (9) are equivalent to

$$\frac{1}{\sqrt{1-\frac{V'^2}{c^2}}}=\frac{\left(1-\frac{uV_x}{c^2}\right)}{\sqrt{1-\frac{u^2}{c^2}}\sqrt{1-\frac{V^2}{c^2}}} \qquad (11a),$$

$$\frac{V'_x}{\sqrt{1-\frac{V'^2}{c^2}}}=\frac{V_x-u}{\sqrt{1-\frac{u^2}{c^2}}\sqrt{1-\frac{V^2}{c^2}}} \qquad (11b)$$

Multiplying Eqs. (11) with m_0, and comparing both Eqs. (11), with (8a) and (8b), we deduce

$$m=\frac{m_0}{\sqrt{1-\frac{V^2}{c^2}}}\ ,\ \varepsilon=mc^2 \qquad (12)$$

and

$$m'=\frac{m_0}{\sqrt{1-\frac{V'^2}{c^2}}}\ ,\ \varepsilon'=m'c^2$$

It is simple to show that Eqs. (8) and (12) lead to

$$\varepsilon'^2-c^2\boldsymbol{P}'^2=\varepsilon^2-c^2\boldsymbol{P}^2=m_0^2c^4 \qquad (13)$$

Based on the above formulation, we derive the Doppler relations for light waved. No ether or relativistic assumptions are required [9a,9b].

III Doppler Effect and Aberration

The constancy of light speed, *i.e.,* that $V = c$ in a specific reference frame, was discovered by Hertz. Using this in the relation $V = \lambda \nu$, we have

$$c = \lambda v \tag{14}$$

Applying relativity principle to Eq. (14) means that Eq. (14) is valid for all observers, *i.e.,*

$$c = \lambda v \text{ (a)}, \; c = \lambda' v' \text{ (b)} \tag{15}$$

SRT's assumption of the invariance of light speed is based completely on the concept of time dilation. If time is not delayed as presented by SRT, then light speed invariance cannot be maintained. According to Eqs. (15), one may conclude that the light speed postulate is an inevitable consequence of the relativity principle. Now assume that particle q in frame S, as an ion (source), emits a light wave (photon) that moves at an angle θ with the positive x axis. Light is received by the observer in frame S' at angle θ' relative to the x' axis. The momentum of the photon in frame S has components

$$P_x = P\cos\theta \;,\; P_y = P\sin\theta \;,\; P_z = 0 \tag{16}$$

For a photon with rest mass $m_0 = 0$, we have from Eq. (13)

$$\varepsilon^2 - c^2 \boldsymbol{P}^2 = 0 \text{ or } \varepsilon = cp \tag{17}$$

If we insert $P_x = \frac{\varepsilon}{c}\cos\theta$ [Eqs. (16) and (17)] in formula (8b) we find

$$\varepsilon' = \gamma\varepsilon\left(1 - \frac{u}{c}\cos\theta\right)$$

For a photon we can let $\varepsilon = h\nu, \varepsilon' = h\nu'$

$$\nu' = \gamma\nu\left(1 - \frac{u}{c}\cos\theta\right) \tag{18a}$$

But the connection between the two frequencies in frames S and S' is also given by

$$\nu = \gamma\nu'\left(1 + \frac{u}{c}\cos\theta'\right) \tag{18b}$$

If we consider frame S to be co-moving with the source and receding from /approaching the observer, (18a) becomes

1) for $\theta = \theta' = 0^\circ$ ..

$$\nu' = \nu_0\gamma\left(1 - \frac{u}{c}\right) = \nu_0\sqrt{\frac{1 - \frac{u}{c}}{1 + \frac{u}{c}}} \tag{19a}$$

2) for $\theta = \theta' = 180^\circ$ ..

$$\nu' = \nu_0\gamma\left(1 + \frac{u}{c}\right) = \nu_0\sqrt{\frac{1 + \frac{u}{c}}{1 - \frac{u}{c}}} \tag{19b}$$

Eqs. (19a,19b) do not have relativistic analogues, but have classical analogues in Eqs. (1a,1c).

According to the relativity principle, we can also consider the frame S' to be co-moving with the observer and receding/approaching the source. Then Eq. (18b) could be written,

$$v' = \frac{v_0\sqrt{1-\frac{u^2}{c^2}}}{1+\frac{u}{c}\cos\theta'} \tag{18c}$$

Hence we have Eqs. (19c,19d) which are, similar to Eqs. (19a,19b):

$$v' = \frac{v_0\sqrt{1-\frac{u^2}{c^2}}}{1+\frac{u}{c}} = v_0\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}} \tag{19c}$$

$$v' = \frac{v_0\sqrt{1-\frac{u^2}{c^2}}}{1-\frac{u}{c}} = v_0\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}} \tag{19d}$$

Eqs. (19c,19d) are identical to Eqs. (2a,2b) in SRT , and the classical analogues are (1b,1d).

If the velocity of the observer/ source is perpendicular to the line of sight, then we obtain from Eq. (18a)

$$v' = \frac{v_0}{\sqrt{1-\frac{u^2}{c^2}}} = \gamma v_0 \tag{20a}$$

Formula (20a) has been confirmed in Mossbauer experiments [10].

From Eq. (18c), we get

$$v' = v_0\sqrt{1-\frac{u^2}{c^2}} = \frac{v_0}{\gamma} \tag{20b}$$

As is known, the relativistic formalism of (RDE) applied only to a moving source vs. a stationary observer. The case of rest source and moving observer for (RDE) is absent. For this reason the two expressions, (19a) and (19b) do not have relativistic analogues. According to the principle of relativity, co-moving source and moving observer is conceptually the same as a moving source and co-moving observer *i.e.,* the change of reference frame (observer or source considered at rest) does not change the physics. Therefore, we have to consider the case where the source is stationary and the observer is moving.

As indicated above, if we consider that the frame S to be co-moving with the source and receding from/approaching the observer, we have Eqs. (19a) and (19b). If we consider frame S' to be co-moving with the observer and receding/approaching the source, we are led to the same result as Eqs. (19a) and (19b), *i.e.,* Eqs.(19c) and (19d). Thus, it is meaningless in the longitudinal Doppler effect to distinguish between the cases "stationary source and moving observer" and "stationary observer and moving source". The classical Doppler effect for light gives two values [$v_0\left(1+\frac{u}{c}\right), v_0/\left(1-\frac{u}{c}\right)$] for the approaching observer and source. In (RDE), the observed Doppler shift is the same for the two formulas because the (TDE) exactly offsets the differences. This means,

$$v_0\frac{1+\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}}, \textit{ i.e., } v_0\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}.$$

Similarly,

$$\frac{v_0\sqrt{1-\frac{u^2}{c^2}}}{1-\frac{u}{c}}, \textit{ i.e., } v_0\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}.$$

From these relations we have the identity

$$\frac{1+\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}}=\frac{\sqrt{1-\frac{u^2}{c^2}}}{1-\frac{u}{c}}$$

In the other case we have also an identity

$$\frac{1-\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}}=\frac{\sqrt{1-\frac{u^2}{c^2}}}{1+\frac{u}{c}}$$

Due to the identity, the (RDE) formula for a moving observer can be also written in the form used for a moving source.

Finally, we can describe the aberration of light by using both Eqs. (9a, 9b) and (16).

The velocity components for the emitted photon in frame S are

$$V_x=\frac{c^2p_x}{\varepsilon}=c\cos\theta \tag{21a}$$

$$V_y=\frac{c^2p_y}{\varepsilon}=c\sin\theta \tag{21b}$$

According to Eqs. (15), the velocity components in frame S' are

$$V'_x = \frac{c^2 p'_x}{\varepsilon'} = c\cos\theta' \tag{22a}$$

$$V'_y = \frac{c^2 p'_y}{\varepsilon'} = c\sin\theta' \tag{22b}$$

From Eq. (9a) and (9b) we obtain

$$\cos\theta' = \frac{\cos\theta - \frac{u}{c}}{1 - \frac{u}{c}\cos\theta} \tag{23a}$$

$$\sin\theta' = \frac{\sin\theta}{\gamma\left(1 - \frac{u}{c}\cos\theta\right)} \tag{23b}$$

Angle θ' is related to the angle θ via the well known expression, Eq. (23a). However, Eq. (23a) is usually quoted as equation (23b), both of which are derived from the (LT) in (SRT's) formalism. Eq. (23a) describes the aberration of light.

(SRT) explains the Doppler shift for light as being caused by the motion of the light source relative to the observer: the blue/red shift is caused by a change in space/ time due to that motion. So the Doppler principle in (SRT) is intrinsically kinematic, described through Maxwell's theory and (LT), but the lacking the intrinsic structure of light.

(SRT) left out the important fact that the frequency shift through motion is caused directly by the variation in energy $(\varepsilon \approx \nu)$ between light and the observer. Therefore we derived the Doppler principle starting with Eq. (8b), which means that energy, and consequently the frequency is relative. The relative velocity u is a vector, meaning that results are dependent on direction [Eqs. (19)], and also dependent on the speed u, which direction is disregarded. Therefore in normal motion, the radial component is zero, $u_r = 0$, and since

$$u^2 = u_t^2 + u_r^2 = u_t^2$$

We also obtain a change in frequency as in Equation (20). The modifications of relativity are considered symmetrical between source and observer. Therefore we have two formulas of (TDE), Eqs. (20) and not one [Eq.(4)] related to time dilation only.

Conclusion

The result of Michelson-Morley's experiment raised two questions:
1 - Does the ether exist?
2 - If not, then to what is the speed of light relative?

(SRT) provided a dubious answer:

Regardless of the nature of light and the existence of ether; all physical laws of nature should conform to LT.

Einstein's approach may have eliminated doubts about the invariance of light speed, but the following question remains:

If the space–time dependence of photons is:

$$c = \frac{dx}{dt} = \frac{dx'}{dt'} \tag{24}$$

Then the modification of space–time with relative motion is required to maintain a constant light speed as Eq. (24) states. In our proposal, (LT) and preferred reference frames are not required. The intrinsic properties of light (v, k) may be the possible cause of the Doppler shift which satisfies the relation,

$$c = \lambda v = \lambda' v'$$

This means that frequency varies to maintain a constant speed of c.

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