Comparison of Predictions of Planetary Quantization and Implications of the Sedna Finding

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In this article we compare some existing methods to predict quantization of planetary orbits, including a recent Cantorian Superfluid Vortex hypothesis by this author. It is concluded that there exists some plausible linkage between these methods within the framework of Quantum Cosmology hypothesis, which in turn may be due to gravitation-related phenomena from boson condensation.

Keywords: quantization of planetary orbits, Quantum Cosmology, vortices, boson condensation, gravitation

Introduction

As we know, in recent years there have been some methods proposed in order to predict the planetary orbits using quantum-like approach, instead of classical dynamics approach. These new approaches have similarity, that they extend the Bohr-Sommerfeld hypothesis of quantization of angular momentum to planetary systems. This application of wave mechanics to large-scale structures [1] has led to several impressive results in terms of prediction of planetary semimajor axes, particularly to predict orbits of exoplanets [2][3][4][5]. However, a question arises as to how to describe the physical origin of wave mechanics of such large-scale structures.

An interesting approach to explain this is by considering the known fact of scale-invariant spectrum [6], which is sometimes called as Harrison-Zel'dovich spectrum. For instance, Clayton & Moffat recently argued using variable light speed argument, that the Cosmic-Microwave Background Radiation (CMBR) anisotropy may be explained in terms of this kind of spectrum [7]. This notion of scale-invariant spectrum may also be related to noncommutative geometry representation of cosmology [8]. What is interesting here is that perhaps this scale-invariant spectrum may correspond to the fact mentioned before by G. Burbidge, i.e. if we supposed that if ρ is the density of visible matter in the universe and that He/H ratio by mass in it is 0.244, then the thermalized energy which has been released in producing He leads to blackbody temperature of $T=2.76\ ^{o}K$. This value is astonishingly near to the value of $2.73\ ^{o}K$ observed by COBE [9]. And because the CMBR's observed low temperature may be related to Bose-Einstein condensate, of course an interesting question is whether the universe resembles a large Bose-Einstein condensate in its entirety [10][11][12][13].

While at first glance this proposition appears quite fantastic, this can be regarded as no more than an observational implication of the notion of Quantum Cosmology hypothesis as proposed by some authors, including Vilenkin [14][15]. Provided this relationship corresponds to the facts, then it seems reasonable to hypothesize further that all predictions of planetary orbits using quantum-like approach shall somehow comprise the same theoretical implication, i.e. they correspond to the Quantum Cosmology hypothesis. Therefore it seems worth to compare these predictions here, which to this author's knowledge has not been made before, though a comparison of Titius-Bode law and a random stable solar system hypothesis is available elsewhere [16][19][20].

In this article we would compare the following approaches available in the literatures:

a. Nottale's Scale Relativity theory [4];
b. Chechelnitsky's Wave Universe theory [17];
c. Ilyanok's Macroquantum Condensate theory [12];
d. Neto *et al.*'s Schrödinger-type diffusion equation [18];
e. Cantorian Superfluid Vortices hypothesis.

We begin with a short description of each approach considered. It is worth noting here that this article does not attempt to examine validity of each of these theories, but instead we merely present what these authors intend to say as is. Therefore the original notations by these authors are kept intact.

Scale Relativity

Nottale [4] argued that equation of motion for celestial bodies could be expressed in terms of a scale-relativistic Euler-Newton equation, by separating the real and imaginary part of Schrödinger-like equation. Then he obtained a generalized Euler-Newton equation of (Ref .[4] p. 384):

$$m.(\partial/\partial t+V.\nabla)V=-V(f+Q) \tag{1}$$

$$\partial r/\partial t+div(rV)=0 \tag{2}$$

$$\Delta f=-4pGr \tag{3}$$

Using these set of equations, Nottale came up with the generalised Schrödinger equation, by giving up the notion of *differentiability* of spacetime. For a Kepler potential and in the time-independent case, this equation reads (Ref [4] p. 380):

$$2D^2\Delta\Psi + (E/m + GM/r).\Psi = 0 \tag{4}$$

Solving this equation, he obtained that planetary orbits are quantized according to the law:

$$a_n = GMn^2 / v_o{}^2 \tag{5}$$

where a_n,G,M,n,v_o each represents orbit radius for given n, Newton gravitation constant, mass of the Sun, quantum number, and specific velocity (v_o=144 km/sec for Solar system and also exoplanet systems), respectively. Furthermore, according to Nottale, the ratio

$$\boldsymbol{a}_g = v_o / c \tag{6}$$

actually corresponds to gravitational coupling constant, similar to fine coupling constant in quantum electrodynamics. These equations form the basis of Nottale's Scale Relativity prediction of planetary orbits both in Solar system and also in exoplanet systems. The result of this equation (5) for the solar system is presented in Table 1.

Wave Universe

Chechelnitsky's Wave Universe hypothesis began with a fundamental wave equation, which reads as follows [17]:

$$\nabla\Psi + 2/d^2.[e - U]\Psi = 0 \tag{7}$$

where for the solar system, U=-K/a; and K=1.327x10^{11} km^3/sec^{-2}, as the gravitational parameter of the Sun. The result of this equation is also presented in Table 1.

What is interesting here is that Chechelnitsky does not invoke argument of non-differentiability of spacetime, as Notale did. Furthermore, he also arrived at some Jovian planetary orbits beyond

Pluto, which obviously recommend an observation for verification or refutation.

Macroquantum condensate

Ilyanok & Timoshenko [12] took a bold step further by hypothesizing that the universe resembles a large Bose-Einstein condensate, therefore the distribution of all celestial bodies must also be quantized. This conjecture may be originated from the fact that according to BCS theory, superconductivity could exhibit macroquantum phenomena [21]. Therefore it seems also reasonable to argue that the universe resembles such macroquantum phenomena, at least in the context of Quantum Cosmology hypothesis [14][15].

According to Ilyanok and Timoshenko, the quantization of planetary orbits in solar system follows a formula of orbit radii and orbital velocity represented by [12]:

$$R_n = \left(n/3 + 2/3.(2m+1)\right)^2 .R_1 \tag{8}$$

$$v_n = 3v_1 / \left[n + 2(2m+1)\right] \tag{9}$$

where n,m are integers and v_1 and R_1 represents orbital velocity and orbit radius of Mercury, as follows:

$$n = 1,2,3,4,5,6,7,8,9$$

$$m = 0,0,0,0,1,2,3,4,5$$

$$v_1 = 3a^2 c = 47.89307 km/\sec \tag{10}$$

$$R_1 = h/\left(a^{12} m_p c\right) = 5.796x10^{10} m \tag{11}$$

where α, m_p, c each represents fine structure constant (1/137), proton mass, and the speed of light, respectively. The result of this method is presented in Table 1.

It seems worth noting here that at first glimpse this method appears similar to Nottale's quantization approach [4]. However, Ilyanok & Timoshenko attempt to build a direct linkage between fine structure constant and the quantization of planetary orbits, while Nottale puts forth a conjecture of gravitational coupling constant (6). It is perhaps also interesting to remark that Ilyanok & Timoshenko do not invoke argument of *nondifferentiability* of spacetime, which notion is essential in Notale's derivation. In a macroquantum condensate context, this approach seems reasonable, considering the fact that Bose-Einstein condensate with Hausdorff dimension D_H~2 could exhibit fractality [22], implying a conjecture of nondifferentiability of spacetime perhaps is not required. The same fractality property has been observed in astrophysics [23][24][25], which in turn may bring us back to an explanation of the origin of multifractal spectrum as described by Gorski [6].

Neto *et al.*'s Schrodinger-type diffusion

In a recent article, Neto *et al.* considered an *axisymmetrical flat* analytical solution of Schrödinger-type equation involving an attractive central field, which is given by [18]:

$$-g^2/2m.\left(\partial^2\Psi/\partial r + r^{-1}.\partial\Psi/\partial r + r^{-2}.\partial^2\Psi/\partial j^{\,2}\right) + V(r)\Psi = E\Psi \quad (12)$$

where g is a constant and μ is reduced mass. Then they derived a solution using separation of variables:

$$\Psi(r,j\,) = f(r).\Phi(j\,) \quad (13)$$

After a rescaling and defining $n = mGMm/g^2 b$, and by using $V(r) = -GMm/r$, they obtained:

$$u'' + [-1/4 + n/r - (\ell^2 - 1/4)/r^2]u(r) = 0 \quad (14)$$

which is a confluent hypergeometric equation, referred as Whittaker's equation. This equation has a regular solution given by a hypergeometric series which converges if and only if,

$$n = |\ell| + 1/2 + k \quad \text{k=0,1,2,3} \tag{15}$$

from which condition they obtained the solution for f(r) in (13):

$$f(r) = c_1.(2br - 1).\exp(-br) \tag{16}$$

It is obvious therefore that in order to find the appropriate asymptotic expression of Schrödinger-type equation they invoke some arbitrary assumptions. Furthermore their result is based on *averaging* planetary masses, and also their equation (16) leads to prediction of planetary orbits which is equivalent the observed planetary data in Solar system except for Earth and Venus. Therefore, in order to reconcile with observed data, they have to invoke a second quantum number.

The result of their method is also presented in Table 1.

Cantorian superfluid vortex hypothesis

In principle the Cantorian superfluid vortex hypothesis as proposed by this author suggests that distribution of planetary systems can be modeled using superfluid vortices [26]. For a planar cylindrical case of solar system, this hypothesis leads to a known Bohr-Sommerfeld-type quantization of planetary orbits [27].

This hypothesis starts with observation that in quantum fluid systems like superfluidity, it is known that such vortices are subject to quantization condition of integer multiples of 2π, or $\oint v_s.dl = 2\pi . n\hbar / m_4$. Furthermore, such quantized vortices are distributed in equal distance, which phenomenon is known as vorticity. In large superfluid system, usually we use Landau two-fluid model, with normal and superfluid component. The normal fluid component always possesses some nonvanishing amount of viscosity and mutual friction. Similar approach with this proposed model has been considered in the context of neutron stars [28], and this proposed quantized vortice model may also be related to Wolter's vortex [29].

To obtain planetary orbit prediction from this hypothesis we could begin with the Bohr-Sommerfeld's conjecture of quantization of angular momentum. As we know, for the wave function to be well defined and unique, the momenta must satisfy Bohr-Sommerfeld's quantization condition [30]:

$$\oint_{\Gamma} p.dx = 2\mathbf{p}.n\hbar \tag{17}$$

for any closed classical orbit Γ. For the free particle of unit mass on the unit sphere the left-hand side is

$$\int_0^T v^2.dt = \mathbf{w}^2.T = 2\mathbf{p}.\mathbf{w} \tag{18}$$

where T=2π/ω is the period of the orbit. Hence the quantization rule amounts to quantization of the rotation frequency (the angular momentum): $\mathbf{w} = n\hbar$.

Then we can write the force balance relation of Newton's equation of motion:

$$GMm/r^2 = mv^2/r \tag{19}$$

Using Bohr-Sommerfeld's hypothesis of quantization of angular momentum (18), a new constant g was introduced:

$$mvr = ng/2\mathbf{p} \tag{20}$$

Just like in the elementary Bohr theory (before Schrödinger), this pair of equations yields a known simple solution for the orbit radius for any quantum number of the form [26]:

$$r = n^2.g^2/(4p^2.GM.m^2) \tag{21}$$

or

$$r = n^2.GM / v_o^2 \tag{22}$$

where r, n, G, M, v_o represents orbit radii (semimajor axes), quantum number (n=1,2,3,…), Newton gravitation constant, and mass of the nucleus of orbit, and specific velocity, respectively. In this equation (22), we denote

$$v_o = (2p / g).GMm \tag{23}$$

This result (23) is the same with Nottale's equation for predicting semimajor axes of planetary-like systems (5).The value of m is an adjustable parameter (similar to g). The result of this equation (22) is also presented in Table 1. While this method results in the same prediction with Nottale's equation (5) for inner orbits, this author uses a different approach for Jovian orbits. It is known that Nottale has to invoke a second quantum number for Jovian planets, while the Solar system is actually a *planar* cylindrical system [18], therefore a second quantum number seems to be superfluous. Therefore, instead of a second quantum number, in CSV hypothesis we describe outer Jovian planet orbits using a conjecture of reduced mass, μ [26].

Perhaps it would be more interesting if we note here that the same Bohr-Sommerfeld's quantization of orbits could also be treated using the viewpoint of quantum Hall liquid in the context of Chern-Simons theory [31][32]. According to L. Susskind [31] we could assume that the particles making up the fluid are electrically charged and move in a background magnetic field B. Furthermore he showed that the conservation law requires the "magnetic field" at each point y, to be time independent, and the analog of a vortex is a δ function magnetic field [31]:

$$\nabla x A = 2p . r_o q d^2(y) \tag{24}$$

where q measures the strength of the vortex. The solution of this equation is unique up to a gauge transformation. In the Coulomb gauge,

$$\nabla . A = 0 \tag{25}$$

it is given by

$$A_i = q r_o \in_{ij} y_j / y^2 \tag{26}$$

To further understand the quasiparticle we must quantize the fluid. Assume the fluid is composed of particles of charge e. Then the momentum of each particle is [31]:

$$p_a = eB \in_a x_b / 2 \tag{27}$$

The standard Bohr-Sommerfeld quantization condition is

$$\oint_\Gamma p_a . dx_a = 2p . n \tag{28}$$

Inserting equation (27) into (28), then the quantization condition becomes [31]:

$$eB \oint_\Gamma (\in_a x_b / 2) . dx_a = 2p . n \tag{29}$$

Using equation (26) then gives:

$$eBq = 2pn \tag{30}$$

Therefore an elementary quasiparticle (n=1) has electric charge:

$$e_{pq} = 2p r_o / B \tag{31}$$

which result agree with the quasiparticle charge from Laughlin's theory [31]. This expression could be extended to include a source. What interests us here from these relationships as described by Susskind is that it was understood recently that Bose-Einstein condensate in dilute atomic gases could be used to describe the physics of vortex matter when they undergo rotation [33]. Furthermore, there is a possibility that at larger angular velocity (ω) the vortex lattice melts and is replaced by a *quantum Hall liquid*. Exactly at this point, it seems we could find a plausible linkage between a quantum Hall liquid and quantization of planetary motion. And the electron fluid representation in quantum Hall liquid may correspond to the *'sea of electron'* terms of Dirac. In this regards, it is worth noting here that universality of quantum Hall liquid has been around in the literature for more than a decade [34], and and it has also been argued that Hall effect could also have some roles in star formation [35].

It may also be worth to remark here, that according to Obukhov [36] it is possible to explain the CMBR anisotropy from the viewpoint of rotating universe [37], which seems to support our conjecture that the universe in its entirety resembles a large rotating Bose-Einstein condensate. While of course this conjecture is not conclusive yet, it seems that CMBR anisotropy could become a test problem; i.e. to observe whether the proposed Bose-Einstein condensate vortices cosmology model could explain this phenomenon.

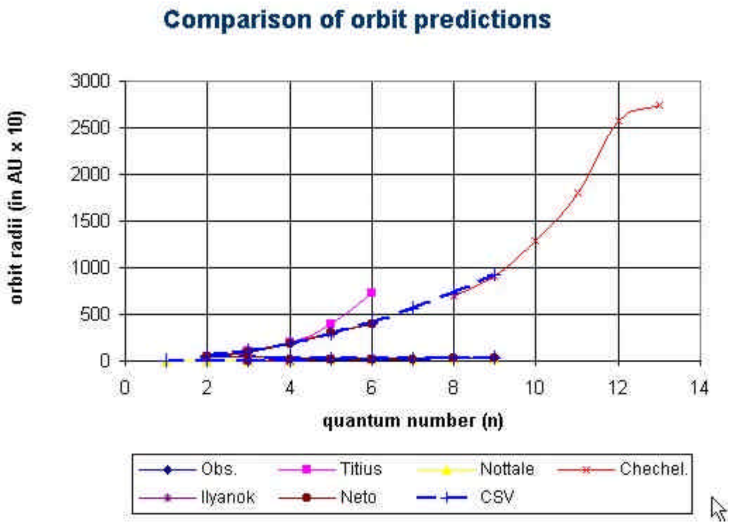
Comparison of predictions and implications of Sedna finding

Based on predicting methods as described above, a comparison table is presented in Table 1.

Astroobject	COMPARISON (in AU x 10' unit)								
	n	Obs.	Titius	Nottale	Chechel.	Ilyanok	Neto	CSV	
	1			0.4				0.43	
	2			1.7				1.71	
Mercury	3	3.87	4	3.9	3.9	3.9	3.9	3.85	
Venus	4	7.32	7	6.8	7.2	6.9	8.3	6.84	
Earth	5	10.00	10	10.7	10.0	10.8	10.0	10.69	
Mars	6	15.24	16	15.4	15.2	15.5	15.4	15.40	
Hungarias	7	20.99		21.0			19.4	20.96	
Asteroid	8	27.00		27.4	27.7		28.8	27.38	
Camilla	9	31.50		34.7			34.8	34.65	
Jupiter	2	52.03	52		52.0	52.1	50.3	45.52	
Saturn	3	95.39	100		95.2	110.2	93.4	102.43	
Uranus	4	191.90	196		191.6	189.8	183.0	182.09	
Neptune	5	301.00	388		300.7	291.0	302.0	284.52	
Pluto	6	395.00	722		393.7	413.7	399.0	409.70	
Π_1	7							557.65	
Π_2	8				700.0			728.36	
Π_3 (Sedna)	9	860			904.4			921.83	(March'04)
Π_4	10				1295				
Π_5	11				1810				
	14				4070				

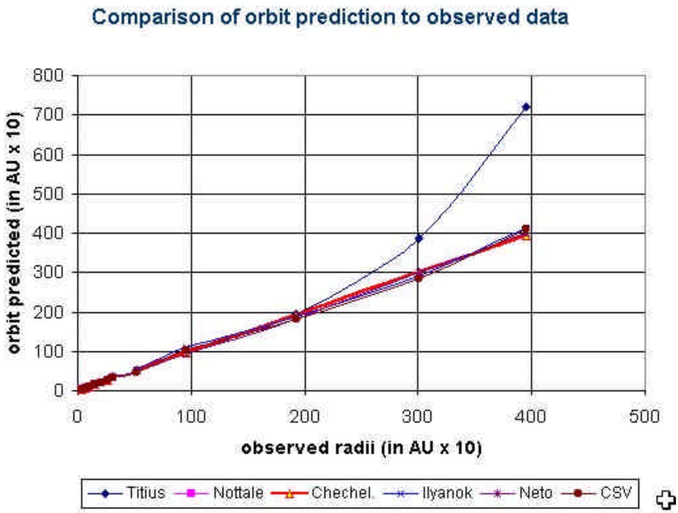
Table 1. Comparison of several methods of orbit prediction

It also seems interesting here to make graph plots for these data in Table 1. The two graphs presented below clearly show how prediction varies against quantum number (n), and against the observed data (Obs). Of course, for an exactly corresponding prediction values to observed data, we will get a gradient =1, corresponding to y=x+0.



Graph 1. Comparison of orbit predictions to quantum number

From Table 1 and its graphplots we observe that all methods compared are very near to the observed data, which seems to support our argument above of the similarity of wave mechanics approach for planetary quantization. We also note that Titius-Bode law overpredicts large orbits, at least for Pluto. Furthermore, there are only two methods which predict planetary orbits beyond Pluto, i.e. Chechelnitsky's Wave Universe hypothesis and the CSV hypothesis suggested by this author. Therefore it seems further observational data is required to verify or refute these predicted orbits beyond Pluto.



Graph 2. Comparison of orbit predictions to observed data

In this regard, it seems worth to put a recent observation of Sedna in this context of planetary quantization, corresponding to n=9 of Jovian planets in Table 1, though it does not mean that Sedna could not be explained in other ways than planetary quantization. As we know, Sedna has found by M. Brown *et al*. from Caltech [38] [39], having around 1770 km in diameter. This Sedna finding obviously leads to some interesting implications. First of all, in numerical terms this finding is very near to a quantum number n=9 as presented in Table 1, within error range of 6.7% as compared with CSV prediction of 92.2AU. Another recent article has also post-predicted this finding, though it was based on Jeans instability [40]. Other interesting aspect of this Sedna includes its very elliptical orbit.

In this article we compared and discussed some methods to predict planetary orbits based on wave-mechanics-type arguments. If the proposition described in this article corresponds to the facts, i.e. the wave mechanics description of celestial bodies correspond to a kind of Quantum Cosmology hypothesis, then it seems further theoretical development could be expected, for instance to extend noncommutative representation of Dirac equation to large scale structure of the universe [41]. Furthermore, a vortex interpretation of Schrödinger equation has also been suggested elsewhere [42][43]. While these are of course not the only plausible approaches, these seem quite interesting in order to find more precise cosmological theories, considering some recent remarkable observation of exoplanets as predicted by such a wave mechanics approach.

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