On the Relativistic Transformation of Electromagnetic Fields

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By investigating the motion of a point charge in an electrostatic and in a magnetostatic field, it is shown that the relativistic transformation of electromagnetic fields leads to ambiguous results. The necessity for developing an 'electrodynamics for moving matter' is emphasized.

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I Introduction

Classical electrodynamics, as it is taught today [1], is based on Hertz's formulation [2] of Maxwell's field equations for matter at rest, and on the Lorentz force which describes the action of the fields on electric particles. It appears unnecessary to formulate an electrodynamics for moving matter, as Hertz attempted in his second paper of 1890 [3], since Einstein's concept [4] of transforming the electromagnetic field into a moving system is supposed to cover the electric phenomena connected with the motion of matter.

This view is not entirely shared by Feynman [5]. He emphasizes that there are two quite distinct laws responsible for the creation of electric fields in a moving conductor in which Ohm's law $\vec{E} = \eta \vec{j}$ holds. There is a contribution to the electric field due to induction by a changing magnetic flux, and a second one due to the motion of the conductor in a magnetic field. Feynman writes that "we know of no place in physics where such a simple and accurate general principle requires for its real understanding an analysis in terms of *two different phenomena*."

Einstein was similarly puzzled by the asymmetry inherent to classical electrodynamics. In the introduction to his famous paper of 1905 [4] he expressed his dissatisfaction about the twofold approach in classical electrodynamics: When a current is produced in a conductor loop due to the relative motion of a magnet, one has to distinguish between whether the conductor is at rest and the magnet moves, or whether the magnet is at rest and the conductor moves. In the first case Faraday's induction law applies, and in the second case Maxwell's electromotive force must be adopted. Einstein sought to unify the two laws which, apparently, lead to the same physical effect. The field $\vec{v} \times \vec{B}$ should turn out to be a 'pseudo-force', similarly like the Coriolis force in an accelerated coordinate system. The Lorentz transformation, which Einstein re-derived from his relativity principle, appeared suitable to achieve the unification. Once a law is known in a system at rest, the same law can be formulated in a moving system by imposing 'Lorentz invariance'. Although the Lorentz transformation has been derived (by Voigt) for the *special* case of constant velocity, Einstein assumed that his formulae for the transformed fields would also hold when the velocity varies in space and time [6].

In the present paper the concept of special relativity, namely to substitute an 'electrodynamics for moving bodies' by an 'electrodynamics for matter at rest' combined with a prescription for transforming the fields, is scrutinized. In Sections III and IV the motion of a charged particle in an electrostatic and in a magnetostatic field, respectively, is calculated in two frames moving at a constant velocity relatively to each other. Adopting the relativistic expressions for the transformed fields, we obtain ambiguous results. It turns out that Einstein's concept is only viable in very singular cases. It is apparently necessary to develop a true electrodynamics for moving matter, in general.

II Basic equations of classical electrodynamics

Hertz [2] gave Maxwell's equations a compact formulation:

$$\epsilon_0 \, div \, \vec{E} = \rho \tag{1}$$

$$rot \, \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{2}$$

$$div \, \vec{B} = 0 \tag{3}$$

$$rot \, \vec{B} = \mu_0 \, \vec{j} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \tag{4}$$

which is valid in vacuo, when the bodies carrying charges and currents are at rest. The mechanical force density on the electrified bodies is given by the divergence of Maxwell's stress tensor:

$$\vec{f} = \rho \vec{E} + \vec{j} \times \vec{B} \tag{5}$$

In Maxwell's Treatise [7] equation (2) is not contained. Instead, Maxwell gave an explicit expression for the 'electromotive force':

$$\vec{E}^{*} = \vec{v} \times \vec{B} - \frac{\partial \vec{A}}{\partial t} - \nabla \phi \tag{6}$$

where $\vec{A}$ is the vector potential in Coulomb gauge ($div\,\vec{A} = 0$) from which the magnetic field is derived: $\vec{B} = rot\,\vec{A}$, and ϕ is the scalar potential satisfying: $\Delta\phi = -\rho/\epsilon_0$. For matter at rest ($\vec{v} = 0$), Maxwell's electromotive force $\vec{E}^{*}$ is identical with the electric field $\vec{E}$, as given by (1 - 4) for given charge and current distributions. In case of a moving conductor, in which Ohm's law $\vec{E} = \eta\,\vec{j}$ holds, the electromotive force (6) has to be inserted for $\vec{E}$, as pointed out in the Introduction.

Lorentz has multiplied (6) with the electric charge of a particle to obtain the Lorentz force [8]:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \tag{7}$$

which is sometimes called the 'fifth postulate', in addition to equations (1 - 4). Since the force density (5) may be derived from (7) by assuming smeared out charge and current distributions, textbooks, such as [1], give frequently the impression that all electromagnetic problems can be solved, in principle, with equations (1 - 4) and (7). This is, however, not entirely true, as the meaning of the velocity in (7) is not quite the same as in (6). Furthermore, it is not perfectly clear what the fields are, when the sources in (1) and (4) are moving.

The velocity in Maxwell's electromotive force (6) does not pertain to the velocity of individual electric particles as in (7), but to the volume element of a moving body. The $\vec{v}\times\vec{B}$ term acts like an electric field to create a current in a moving conductor, as already mentioned, or to produce 'motional Stark effect' in a neutral atom, for example. Hence, one cannot abandon (6), since the $\vec{v}\times\vec{B}$ term is not available as an electric field from (1 - 4).

Special relativity is supposed to extend classical electrodynamics for matter at rest to all situations where matter moves. The five classical postulates of electrodynamics are, therefore, complemented by a further postulate, the Lorentz transformation, which yields the transformed fields acting on a charge in a moving system [4]:

$$\begin{aligned} E'_x &= E_x & B'_x &= B_x & & \\ E'_y &= \gamma\left(E_y - v\,B_z\right), & B'_y &= \gamma\left(B_y + \frac{v}{c^2}\,E_z\right), & \gamma &= 1/\sqrt{1-\beta^2} \\ E'_z &= \gamma\left(E_z + v\,B_y\right), & B'_z &= \gamma\left(B_z - \frac{v}{c^2}\,E_y\right), & \beta &= \frac{v}{c} \end{aligned} \quad (8)$$

Here it is assumed that the fields are given in a system (x, y, z) at rest, and transform into new fields in a system (x', y', z'), which moves with velocity v parallel to the x-axis.

The Lorentz force must be contained in (8) for the following reason: The force on a charge, which is at rest relative to the sources in (1) and (4), is known to be $q\,\vec{E}$. When the charge moves, the electric field in the rest-frame of the charge can be obtained by transforming the electric field according to (8), which should yield the force (7). This is, indeed, the case for $\beta^2 \ll 1$. For $\beta \sim 1$, however, the force $q\,\vec{E}'$, perpendicular to the velocity, is larger than $q\,\vec{E}$ by the γ-factor. It would follow then that (7) cannot be an exact law.

On the other hand, it is found experimentally that for particles moving with velocity $v \sim c$ equation (7) does apply, as long as radiation damping can be neglected. The way out of the impasse is to assume (claim) that all forces perpendicular to the velocity of a moving system, when 'seen' from a system at rest, are increased by the γ-factor. This assumption is necessary, since a charge subjected to an electric field, but balanced by another force, for example gravitation, would loose its equilibrium when observed from a moving system, if the gravitational force would not transform like the electric field. This is a far reaching consequence following from (7) and (8). In the following Section we check, whether the transformation law (8) is compatible with the known transformation law of the inertial force, by calculating the accelerated motion of a charged particle in an electrostatic field.

III Motion of a charged particle in an electrostatic field

Let us assume that a uniform electric field is produced by a large plate condenser. At time $t = 0$ an electric particle moves between the plates with velocity v in negative x-direction as shown in Figure 1. Inserting (7) into the relativistic equation of

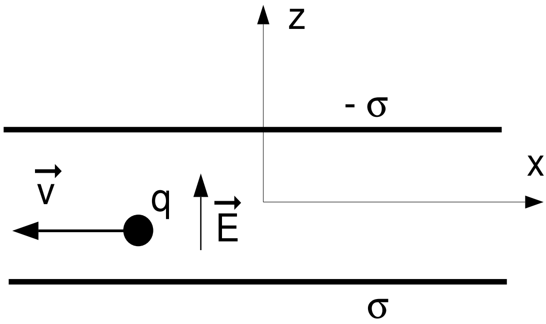

Figure 1: Charged particle moving in an electric field

motion of the particle we have:

$$\frac{d}{dt}\,(m\,v_x) = 0\;, \quad \frac{d}{dt}\,(m\,v_z) = q\,E_z\;, \quad m = \frac{m_0\,c}{\sqrt{c^2 - v_x^2 - v_z^2}} \tag{9}$$

Adopting the initial conditions $v_x\,(0) = -v$, $v_z\,(0) = 0$ one obtains for the velocity components:

$$v_x = -\frac{v}{\sqrt{1+\tau^2}}\;, \quad v_z = \frac{c\,\tau}{\sqrt{1+\tau^2}}\;, \quad \tau = \frac{q\,E_z}{\gamma\,m_0\,c}\,t \tag{10}$$

From $dx/dt = v_x$, $dz/dt = v_z$ the trajectory of the particle can be calculated by further integration of (10).

In the inertial system where the particle is at rest initially, the equation of motion becomes with (7):

$$\frac{d}{dt'}\,(m'\,v'_x) = -q\,v'_z\,B'_y\;, \quad \frac{d}{dt'}\,(m'\,v'_z) = q\,\left(E'_z + v'_x\,B'_y\right)$$
$$m' = \frac{m_0\,c}{\sqrt{c^2 - v'^2_x - v'^2_z}} \tag{11}$$

Substituting the field transformation law (8) and integrating over t' yields for the momentum components of the particle:

$$m'\,v'_x = \frac{\gamma\,q\,v}{c^2}\,E_z\,\left(z' - z'_0\right)\quad,\quad m'\,v'_z = \gamma\,q\,E_z\,\left(t' - \frac{v\,x'}{c^2}\right) \tag{12}$$

where the initial conditions $v'_x\,(0) = v'_z\,(0) = 0\,,\ z'\,(0) = z'_0\,,$ $x'\,(0) = 0$ were chosen. Since we have $t = \gamma\,(t' - v\,x'/c^2)$ according to the Lorentz transformation, the particle gains in both systems the same amount of momentum in z-direction. The momentum gain in x-direction is, however, different: It vanishes in the unprimed system according to (9), but it is finite in the primed system according to the first equation of (12). This is only possible, when there is a reaction force on the plate condenser acting in negative x-direction.

The force density exerted by the particle on the plates is according to (5):

$$\vec{f} = \rho'\,\vec{E}_p + \rho'\,\left(\vec{v} \times \vec{B}_p\right) \tag{13}$$

where the fields produced by the moving particle are given by the expressions:

$$\vec{E}_p = \frac{q}{4\pi\,\epsilon_0}\,\frac{\vec{x}\,' - \vec{x}_0\,'}{|\vec{x}\,' - \vec{x}_0\,'|^3}\quad,\quad \vec{B}_p = \frac{1}{c^2}\,\left(\vec{v}_p \times \vec{E}_p\right) \tag{14}$$

The total force in z-direction integrated over the volume of the plates becomes:

$$\begin{aligned} F_z &= \frac{q\,\sigma'}{4\,\pi\,\epsilon_0}\int_0^\infty \left[\left(1 - \frac{v\,v'_x}{c^2}\right)\frac{z' - z'_0}{\left(r^2 + (z' - z'_0)^2\right)^{\frac{3}{2}}}\right]_h^{-h} 2\pi\,r\,dr \\ &= -\frac{q\,\sigma'}{\epsilon_0}\left(1 - \frac{v\,v'_x}{c^2}\right) = -q\,\gamma\,E_z\,\left(1 - \frac{v\,v'_x}{c^2}\right) \end{aligned} \tag{15}$$

where $2\,h$ is the distance between the plates and equation (19) below was used. Integration over t' yields exactly the same negative momentum in z-direction as given by the second equation of (12), so that Newton's third law is satisfied for the z-component of the force.

In x-direction the force density is according to (13):

$$f_x = \frac{q\,\rho'}{4\,\pi\,\epsilon_0}\,\frac{x' - x'_0}{|\vec{x} - \vec{x}\,'|^3} \tag{16}$$

Integrated over the volume of the plates, this expression vanishes. Hence, the momentum gain as described by the first equation of (12) remains unbalanced. We must conclude then that the momentum of the total system: particle plus condenser is not conserved, when it is calculated in the primed system by adopting the transformation law (8).

There is a further problem, when Maxwell's equations are transformed into a moving system. In addition to the transformation rules (8), one must postulate that the charge density transforms according to the rule:

$$\rho' = \gamma\,\rho \tag{17}$$

in order to ensure that Maxwell's equations are Lorentz-invariant in the moving system. In case of a large condenser as in Figure 1, the electric field is related to the surface charge density by the simple formula following from (1):

$$E_z = \sigma/\epsilon_0 \quad , \quad \sigma = \int \rho\,dz \tag{18}$$

This is also so in the rest-system of the charge:

$$E'_z = \sigma'/\epsilon_0 = \gamma\,\sigma/\epsilon_0 \tag{19}$$

in agreement with (8) and (17). For a condenser with finite dimensions, however, one finds a different electric field depending on whether it is calculated from the transformation rules (8), or directly from Maxwell's equation (1) using (17).

Let us assume that the plates of the condenser in Figure 1 are of circular shape with radius a. The potential produced by the lower plate is then in polar coordinates:

$$\begin{aligned} \phi &= \frac{1}{4\,\pi\,\epsilon_0}\int\limits_0^{2\pi}\!\!\int\limits_0^{a} \frac{\sigma\, r_i\, dr_i\, d\varphi_i}{R} \\ R^2 &= r^2 + r_i^2 - 2\,r\,r_i\,\cos\,(\varphi-\varphi_i) + (z+h)^2 \end{aligned} \tag{20}$$

The electric field in z-direction is $E_z = -\partial\phi/\partial z$ and becomes in the rest-frame of the particle:

$$E'_z = \frac{\gamma}{4\,\pi\,\epsilon_0}\int\limits_0^{2\pi}\!\!\int\limits_0^{a} \frac{\sigma\;\,(z+h)\;\,r_i\,dr_i\,d\varphi_i}{R^3} \tag{21}$$

according to the field transformation (8).

The potential calculated in the rest-frame of the charge is in Cartesian coordinates, because of (17):

$$\begin{aligned} \phi &= \frac{1}{4\,\pi\,\epsilon_0}\iint \frac{\sigma\,\gamma\,dx'_i\,dy'_i}{\left((x'-x'_i)^2 + (y'-y'_i)^2 + (z'-z'_i)^2\right)^{\frac{1}{2}}} \\ &= \frac{1}{4\,\pi\,\epsilon_0}\iint \frac{\sigma\,dx_i\,dy_i}{\left((x-x_i)^2\;(1-\beta^2) + (y-y_i)^2 + (z-z_i)^2\right)^{\frac{1}{2}}} \end{aligned} \tag{22}$$

where Lorentz-contraction in x-direction was taken into account.

Employing polar coordinates the electric field becomes:

$$E'_z = \frac{1}{4\,\pi\,\epsilon_0} \int_0^{2\pi}\!\!\int_0^{a} \frac{\sigma\;\,(z+h)\;\,r_i\,dr_i\,d\varphi_i}{\left(R^2 - \beta^2\;\,(r\cos\varphi - r_i\cos\varphi_i)^2\right)^{\frac{3}{2}}} \tag{23}$$

Comparison with (21) shows that the denominator in (23) is different so that the two expressions yield different fields. This is already obvious by noting that (21) is axially symmetric, but (23) is not.

IV Motion of a charged particle in a magnetostatic field

In the previous Section it was shown that Einstein's method of transforming the electric field leads to ambiguous results. In the following it will be shown that it fails also to replace a magnetostatic field by an electric field.

As long as a particle moves with constant velocity, one can always define a coordinate system which moves with the particle, so that the magnetic force in (7) vanishes. Since the force acting on the particle cannot depend on the choice of the coordinate system up to a γ-factor, the $\vec{v} \times \vec{B}$ term must be replaced by an electric field in the framework of the Lorentz force. If the magnetic field is produced by a neutral current, the particle must 'see' an apparent charge density on the conductor, which produces an electrostatic field acting on the particle, instead of the magnetic field. In the relativistic formalism the charge density appearing on a neutral conductor for a moving observer is:

$$\rho = \frac{\gamma\,\vec{j}\cdot\vec{v}}{c^2} \tag{24}$$

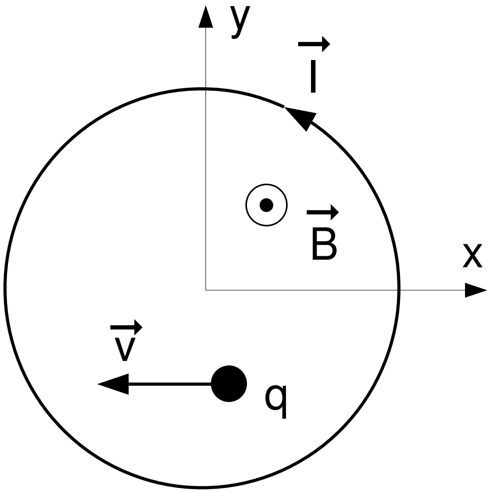

Figure 2: Charged particle moving in a magnetic field

Feynman [9] demonstrates for a straight wire, which carries a constant current, that the charge density (24) yields indeed an electric field which is the same as that which can be obtained by transforming the magnetic field into an electric field with (8). A general proof for the validity of the method is, however, not given.

Let us assume that a magnetic field is produced by a circular current loop of very small cross section as shown in Figure 2. An electric particle moves with constant velocity v in negative x-direction. The force components on the particle are according to (7):

$$F_x = 0 \quad , \quad F_y = q\,v\,B_z \quad , \quad F_z = -q\,v\,B_y \tag{25}$$

The magnetic field may be derived from the vector potential of the current loop with radius a:

$$A_\varphi = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{a \cos\alpha \, d\alpha}{(r^2 + a^2 - 2\, r\, a \cos\alpha + z^2)^{\frac{1}{2}}} \tag{26}$$

which yields the field components from $\vec{B} = rot\,\vec{A}$:

$$B_y = -\frac{y}{r}\frac{\partial A_\varphi}{\partial z} \quad , \quad B_z = \frac{1}{r}\frac{\partial\,(r\, A_\varphi)}{\partial r} \tag{27}$$

In the rest-frame of the moving particle a charge density arises according to (24):

$$\rho = \frac{\gamma\, v}{c^2}\, j_x = -\frac{\gamma\, v}{c^2}\, j_\varphi \sin\varphi \tag{28}$$

which produces an electrostatic potential:

$$\begin{aligned} \phi &= \frac{v}{4\pi\,\epsilon_0\, c^2} \iiint \frac{j_x\, dx'\, dy'\, dz'}{\left((x-x')^2 + (y-y')^2 + (z-z')^2\right)^{\frac{1}{2}}} \\ &= -\frac{\mu_0\, v\, I}{4\pi} \int_0^{2\pi} \frac{a \sin\varphi' \, d\varphi'}{(r^2 + a^2 - 2\, r\, a \cos(\varphi - \varphi') + z^2)^{\frac{1}{2}}} \\ &= -\frac{\mu_0\, v\, I}{4\pi} \int_0^{2\pi} \frac{a\,(\sin\varphi \cos\alpha + \cos\varphi \sin\alpha)\; d\alpha}{(r^2 + a^2 - 2\, r\, a \cos\alpha + z^2)^{\frac{1}{2}}} \end{aligned} \tag{29}$$

Here it was assumed that Lorentz-contraction does not play a role ($\beta^2 \ll 1$), in order to facilitate the calculation. For $\beta \sim 1$ one encounters a similar discrepancy as between equations

(21) and (23) in the previous Section, because of the elliptical deformation of the current ring. Since the uneven term in (29) vanishes upon integration over α, the integrals in (26) and (29) are the same and may be denoted by S:

$$S\,(x,\,y,\,z) = \int\limits_0^{2\pi} \frac{a\,\cos\alpha\;d\alpha}{(r^2 + a^2 - 2\,r\,a\,\cos\alpha + z^2)^{\frac{1}{2}}}\;\;,\quad r^2 = x^2 + y^2 \tag{30}$$

The force on the particle in the moving system is $\vec{F} = -q\,\nabla\phi$. Comparing now the force components as given by (25 - 27) with the gradient force derived from (29) one obtains with (30):

$$\begin{aligned} 0 \;&= C\,\frac{\partial}{\partial x}\left(\frac{y\,S}{r}\right)\;, \qquad C\,\frac{1}{r}\,\frac{\partial\,(r\,S)}{\partial r} = C\,\frac{\partial}{\partial y}\left(\frac{y\,S}{r}\right) \\ C\,\frac{y}{r}\,\frac{\partial\,S}{\partial z} &= C\,\frac{\partial}{\partial z}\left(\frac{y\,S}{r}\right)\;, \qquad C = \frac{q\,v\,\mu_0\,I}{4\,\pi} \end{aligned} \tag{31}$$

Only the z-components of the magnetic force and the electric force in (31) are equal, but neither the x- nor the y-components agree. It turns out that the cross product in (7) cannot be replaced by a gradient, in general.

This result is quite understandable from the structure of the $\vec{v}\times\vec{B}$ term. When a particle moves in a magnetic field, its kinetic energy is not changed, since the magnetic force is perpendicular to the velocity. Replacing the magnetic force by an electric gradient-force means, that the energy of the particle is now a function of its position in the scalar potential field, which is produced by the apparent charge. Hence, the initial energy will change, when the particle moves under the influence of the electric force. This is not the case, when the particle is only subjected to a magnetic field.

In the above analysis only an electric gradient field was considered to replace the magnetic force. The reason was that the rotational part of the electric field vanishes, when we assume that the current in the ring is kept constant. One could argue that a particle travelling in a vector potential, which is constant in time, but non-uniform in space, experiences a time variation of the vector potential due to the motion. With $\vec{E} = -\partial\vec{A}/\partial t$ a force on the particle should then arise. This is, however, not the case: If a particle travels outside an infinitely long solenoid in the region where the magnetic field vanishes, but the vector potential is finite, the particle is not deflected, unless the magnetic field inside the solenoid changes in time. For reasons of symmetry one would not expect that the particle experiences a force, in case it is at rest and the solenoid moves. This is why the $\partial\vec{A}/\partial t$ term was ignored when comparing the forces in equation (31).

If one adopts, nevertheless, the full expression $\vec{E} = -\nabla\phi - \partial\vec{A}/\partial t$ for the electric field in the case of Figure 2, when the particle is at rest, but the current ring moves, one obtains for the force components:

$$q\,E_x = -C\,\sin\varphi\,\cos\varphi\,\frac{S}{r}\quad,\quad q\,E_y = C\,\left(\frac{\partial S}{\partial r} + \cos^2\varphi\,\frac{S}{r}\right) \tag{32}$$

This is still not the same as the magnetic force components given by the cross product $q\,\left(\vec{v}\times\vec{B}\right)$.

V Discussion and Conclusion

From the analysis in Sections III and IV it became obvious that a solution of 'Maxwell's equations for matter at rest' cannot be made into a solution for moving matter by applying the Lorentz transformation, in order to obtain the fields in a moving system. It is questionable anyway, whether this method would work when the velocity varies in space and time, since the Lorentz transformation is restricted to constant motion. Einstein thought [6], nevertheless, that the field transformation rules (8) have general validity, but this was just a speculation which was not based on experiments. In a recent paper [10] by the present author, it was shown that the Lorentz transformation applied to electromagnetic waves predicts certain optical phenomena, which are not supported by experiments[1]. It is, therefore, not surprising that it fails also, when applied to Maxwell's first order equations.

The result in Section IV points to a serious problem which arises in classical electrodynamics, independent of the relativistic formalism. The Lorentz force requires to find an electric force which replaces the magnetic force in a system moving with the particle. It was shown that the required electric field cannot be obtained, in general, from 'Maxwell's equations for matter at rest', at least not when the 'apparent' charge density (24) is used. From energetic considerations we even concluded that it is

[1] In a recent experiment [11] it was found that the time dilation factor is, in fact, absent, when microwaves are received by an antenna which moves perpendicular to the wave vector. This is in agreement with equation (21) in Reference [10], but in disagreement with the prediction of the Lorentz transformation.

not possible, in principle, to substitute the cross-product $\vec{v} \times \vec{B}$ by a gradient field derived from a potential. Thus, either the Lorentz force, or the field equations, or both must be suitably modified to account for the force on a particle in its rest-frame. It is, of course, well known that the Lorentz force must be modified anyway to include the effect of radiation damping, when a charge produces electromagnetic waves due to strong acceleration. Whether a modification of the Lorentz force alone leaves equations (1 - 4) intact, is an open question. In 1890 Hertz [2] was aware of the fact that *the final forms of the forces are not yet found*. In case the open problems could not be solved, he was not even certain that Faraday's and Maxwell's field concept is viable at all.

Having shown that the transformation of the electromagnetic fields, as proposed by special relativity, is not a feasible concept to establish an 'electrodynamics for moving matter', it is obvious that the work started by Lorentz [8] and Hertz [3] should be taken up again, both theoretically and experimentally. It remains to be seen to what extent classical electrodynamics will require a basic revision.

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