A Cantorian Superfluid Vortex and the Quantization of Planetary Motion

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This article suggests a preliminary version of a Cantorian superfluid vortex hypothesis as a plausible model of non-linear cosmology. Though some parts of the proposed theory resemble several elements of what have been proposed by Consoli (2000, 2002), Gibson (1999), Nottale (1996, 1997, 2001, 2002a), and Winterberg (2002b), it seems such a Cantorian superfluid vortex model instead of superfluid or vortex theory alone has *never* been proposed before. Implications of the proposed theory will be discussed subsequently, including prediction of some new outer planets in solar system beyond Pluto orbit. Therefore further observational data is recommended to falsify or verify these predictions. If the proposed hypothesis corresponds to the observed facts, then it could be used to solve certain unsolved problems, such as gravitation instability, clustering, vorticity and void formation in galaxies, and the distribution of planet orbits both in solar system and also exoplanets.

Keywords: multiple vortices, superfluid aether, nonlinear cosmology, gravitation instability, Bose-Einstein condensate, Cantorian spacetime, fluid dynamics.

Introduction

In recent years, there has been a growing interest in the quantum-like approach to describe orbits of celestial bodies. While this approach has not been widely accepted, motivating idea of this approach was originated from Bohr-Sommerfeld's hypothesis of quantization of angular momentum, and therefore it has some resemblance with Schrödinger's wave equation (Chavanis 1999, Nottale 1996, Neto *et al*. 2002). This application of wave mechanics to large-scale structures (Coles 2002) has led to several impressive results in terms of the prediction of planetary semimajor axes, particularly to predict orbits of exoplanets (Armitage *et al*. 2002, Lineweaver *et al.* 2003, Nottale *et al*. 1997, 2000, Weldrake 2002). However, a question arises as how to describe the physical origin of wave mechanics of such large-scale structures. This leads us to hypothesis by Volovik-Winterberg of superfluid phonon-roton as quantum vacuum aether (Volovik 2001, Winterberg 2002a, 2002b).

In this context, gravitation could be considered as result of diffusion process of such Schrödinger-like wave equation in the context of Euler-Newton equations of motion (Kobelev 2001, Neto *et al*. 2002, Rosu 1994, Zakir 1999, Zurek 1995). And large-scale structures emerge as condensed objects within such a quantum vacuum aether.

In the mean time, despite rapid advancement in theoretical cosmology development, there are certain issues that remain unexplainable in the presently available theories; one of these issues concern the origin and nature of gravitation instability (Coles 2002, Gibson 1999). Recent studies that have incorporated condensation, and void formation occurring on the non-acoustic density nuclei produced by turbulent mixing, appear to indicate that the universe is inherently *nonlinear* nature. Thus a very different nonlinear cosmology is emerging to replace the presently accepted linear cosmology model.

For instance, recently Gibson (1999) suggested that the theory of gravitational structure formation in astrophysics and cosmology should be revised based on *real* fluid behavior and turbulent mixing[i] theory, which leads us to nonlinear fluid model. His reasoning of this suggestion is based on the following argument: "*The Jeans theory of gravitational instability fails to describe this highly nonlinear phenomenon because it is based on a linear perturbation stability analysis of an inadequate set of conservation equations excluding turbulence, turbulent mixing, viscous forces, and molecular and* gravitational *diffusivity.*" This is because Jeans' theory neglects viscous and nonlinear terms in Navier-Stokes *momentum equations*, thus reducing the problem of gravitational instability in a nearly uniform gas to one of linear acoustics.[ii]

In related work, Nottale (1996, 1997) argued that equation of motion for celestial bodies could be expressed in terms of a scale-relativistic Euler-Newton equation.[iii] By separating the real and imaginary part of Schrödinger-like equation, he obtained a generalized Euler-Newton equation and the continuity-equation (which is therefore now part of the dynamics), so the system becomes (Nottale 1997, Nottale *et al*. 2000 p. 384):

$$m.(\partial/\partial t + V.\nabla)V = -V(f + Q) \tag{1a}$$

$$\partial r/\partial t + div(rV) = 0 \tag{1b}$$

$$\Delta f = -4pGr \tag{1c}$$

It is clear therefore Nottale's basic Euler-Newton equations above, while including the inertial vortex force, neglect viscous terms (– **ν ΔV)** in Navier-Stokes momentum equations,[iv] so his equations will obviously lead us to certain reduction of gravitational instability phenomena similar to Jeans' theory. Though Nottale's expression could offer a plausible explanation on the origin of *dark energy* (Ginzburg 2002, Nottale 2002a p. 20-22, Nottale 2002b p. 13-14), his expression appears to be not complete enough to explain other phenomena in a nonlinear cosmology, such as clustering, gravitation condensation and void formation.

Therefore the subsequent arguments will be based on a more complete form of Navier-Stokes equations including inertial-vortex force (Gibson 1999). Furthermore in the present article, two basic conjectures are proposed, i.e.

(i) in accordance with Thouless *et al*. (2001), it is proposed here: Instead of using the Euler-Lagrange equation, '*the nonlinear Navier-Stokes equations are applicable to represent the superfluid equations of motion'*. By doing so we can expect to obtain an extended expression of Nottale's Euler-Schrödinger equations (Nottale 1996, 1997, 2000, 2001, 2002a).

(ii) by taking into consideration recent developments in Cantorian spacetime physics, particularly by Castro *et al.* (2000, 2001) and Celerier & Nottale (2002), we propose that *modeling the universe using superfluid aether is compatible (at least in principle) with Nottale's scale relativity framework*. This is the second basic conjecture in this article.[v]

Accordingly, this article suggests that the nonlinear dynamics of Cantorian vortices in superfluid aether can serve as the basis of a nonlinear cosmological model. The term 'Cantorian' here represents the notion of 'transfinite set' introduced by Georg Cantor.[vi] Recently this term has been reintroduced for instance by Castro *et al.* (2000) and Castro & Granik (2001) to describe the exact dimension of the universe. As we know, a transfinite set is associated with the mapping of a set onto itself, producing a '*self-similar*' pattern. This pattern is observed in various natural phenomena, including turbulence and tropical hurricane phenomena.

Turbulence usually occurs when conditions of low viscosity and high-speed gradients are present. A turbulent fluid can be visually identified by the presence of *vortices*. As we know, a flow pattern, whose streamlines are concentric circles, is known as circular vortex (vortice). If the fluid particle rotates around its own axis, the vortex is called rotational. Such vortices continually form and evolve over time, giving rise to highly complex motions. In this context, vortices are defined as the curl of the velocity ($\nabla \times V$) in Navier-Stokes equations.[vii] Landau describes turbulence as a superposition of an *infinite* number of vortices, with sizes varying over all scales (this '*all scales*' notion leads us to Cantorian term). From the large scale vortices, energy is transmitted down to smaller ones without loss. The energy of the fluid is finally dissipated to the environment when it reaches the smallest vortices in the range of scales. The solutions to the velocity field are unique when the helicity $= v\ .\ \mathrm{curl}\ v = 0$; otherwise the solutions are not unique.

As we know, real fluid flow is never irrotational, though the mean pattern of turbulent flow outside the boundary layer resembles the pattern of irrotational flow. In rotational flow of real fluids, vorticity can develop as an effect of viscosity. Provided other factors remain the same, vortices can neither be created nor destroyed in a non-viscous fluid. Since the vortex moves with the fluid, vortex tube retain the same fluid elements and these elements retain their vorticity. The term 'vorticity' here is defined as the number of circulations in a certain area, and it equals to the circulation around an elemental surface divided by the area of the surface (supposing such vortex lattice exists within equal distance).[viii]

In quantum fluid systems like superfluidity, such vortices are subject to quantization condition of integer multiples,[ix] i.e. they are present in certain N number of atoms, as experimentally established in the superfluid phase of ^{4}He,

$$\oint v_s . dl = 2p . n\hbar / m_4 = n.k_o \qquad (2)$$

where m_4 is the helium particle mass, and κ_o is the quantum of circulation (Nozieres & Pines 1990, Thouless *et al*. 2001). Furthermore, quantized vortices is a topological excited state, which takes form of circulation with equidistance distribution known as vorticity (Carter 1999, Kiehn 2001). Usually the Landau two-fluid model is used, with a normal and superfluid component. The normal fluid component always possesses some nonvanishing amount of viscosity and mutual friction; therefore it could exhibit quantum vorticity as observed in Ketterle's experiments.

A 'Cantorian vortice' can be defined in simple terms as tendency of the dynamics of both fluids and superfluids to produce multiple regions of vortex and circulation structures at various *scales* (Barge & Sommeria 1995, Castro *et al*. 2002, Chavanis 1999, Kobelev 2001, Nozieres & Pines 1990, Volovik 2000b, 2000c). In principle, the notion of Cantorian Superfluid Vortex suggests that there is a tendency in nature, particularly at the astronomical level scale, to produce mini vortices within the bigger vortices *ad infinitum*. Though some parts of the proposed theory resemble several elements of what have been proposed by Consoli (2000, 2002), Gibson (1999), Nottale (1996, 1997, 2001, 2002a), Volovik (2000a, 2000b, 2001), and also Winterberg (2002a, 2002b), to the author's present knowledge the idea of using a Cantorian superfluid vortex model instead of (ordinary) superfluid model or vortex theory alone has *never* been proposed before. The Cantorian term here implies that such a superfluid vortice is—in accordance with Landau's definition of turbulence—supposed to exist both as quantum vacuum aether background (micro phenomena) and as representation of various condensed objects such as neutron stars (macro phenomena). The proposed hypothesis results in a non-homogenous isotropic Euclidean flat-spacetime expanding universe at *all scales*, but *without* a cosmological constant. This cosmology constant nullity is somewhat in accordance with some recent articles, for instance by Guendelman *et al*. (2002), Volovik (2001), and Winterberg (2002a, 2002b).

Implications of the proposed model will be discussed subsequently, where first results of the method yield improved prediction of three new planets in outer planet orbits of the solar system beyond Pluto. If the predictions of the proposed hypothesis correspond to the observed facts, it is intuitively conjectured that the proposed theory could offer an improved explanation for several unexplainable things (at least not yet in a quantifiable form) in regards to the origin of gravitation instability, void formation, and unifying gravity and quantum theory.

A review of recent developments

Throughout the last century of theoretical physics since Planck era, physicists have investigated almost every conceivable idea of how geometry can be used or modified to describe physical phenomena. For instance, Minkowski refined his 4D spacetime-geometry to explain Einstein's STR. Others have come up with 5D (Kaluza-Klein), 6D, and then ten D, eleven D, and recently 26D (bosonic string theory as a dual resonance model in 26D; see Winterberg 2002a). It seems like the number of geometrical dimensions simply grow with time. We could also note a considerable amount of study has been devoted to geometry with infinite-dimension or Hilbert space.

However, recently it seems there is also a reverse drift of simplifying these high dimensional (integer) numbers, for instance by use of the replacement of the dual resonance model in 26D with QCD in 4D to describe nuclear forces; and by using of the aforementioned analogies between Yang-Mills theories and *vortex dynamics*, there is a suggestion that string theory should perhaps be reinstated by some kind of vortex dynamics at the Planck scale (Winterberg 2002a). Furthermore, Castro *et al*. (2000, 2001) have proposed that the *exact* dimension of the universe is only a bit higher than Minkowskian 4D (less than 5D). They arrived at this conclusion after reconciling Cantorian spacetime geometry with the so-called Golden Section. Therefore instead of proposing a trivial argument over which geometry is superior, this article proposing accepting the hypothesis that the Cantorian fractal spacetime dimension as proposed by Castro *et al*. (2000) can be the *real* geometric dimension of the universe. This fractal dimension will be called the Cantorian-Minkowski dimension. This conjecture is somewhat in accordance with a recent suggestion made by Kobelev (2001) that Newton equation is a diffusion equation of multifractal universe.

In the mean time, despite the fact that most theoretical physics efforts are devoted toward the proper expressions of fields, fields are not the only objects which one can think as occupying spacetime, there are also fluids. When there is no equation of state specified they are *more general* than fields (Roberts 2001).[x] In this regards quantum fluids, which are usually understood as a limited class of objects used to describe low-temperature physics phenomena, have in recent years been used to model various cosmological phenomena, for instance neutron stars (Andersson & Comer 2001, Elgaroy & DeBlassio 2001, Sedrakian & Cordes 1997, Yakovlev 2000). It is not surprising therefore that there is increasing research in using superfluid model to represent cosmology dynamics (Liu 2002, Roberts 2001, Volovik 2000a, 2000b, 2000c, 2001, Zurek 1995).

In this context, it is worth noting here some recent development in superfluidity research. This direction of research includes application of NLSE (Nonlinear Schrödinger equation) as a model of the Bose-Einstein condensate under various conditions (Quist 2002). There are also NLSE proposals representing Cantorian fractal spacetime phenomena (Castro *et al*. 2002). Experiments on Bose-Einstein condensates have now *begun to address vortex systems*. Superfluid turbulence issues and its explanation in terms of quantum *vortex dynamics* have become one of the most interesting physics research these days (Volovik 2000a, 2002b, Zurek 1995). For instance, recent experiments in the past few years showed that some turbulent flows of the superfluid phase of ^{4}He (helium II) are similar to analogous turbulent flow in a classical fluid (Thouless *et al.* 2001). In theoretical realm, there is also new interest in the relationship between the topology (broken by reconnections, hence release of energy) and the geometry of structure—sometimes known as *topological defects* in cosmology (Yates 1996, Zurek 1995)—which cannot be changed arbitrarily as done traditionally by topologists but changes according to the dynamics (NLSE or Navier-Stokes equation[xi]).

Winterberg (2002a) has suggested that the universe can actually be considered an Euclidean flat-spacetime provided we include superfluid aether quantum vacuum into the model. Winterberg's aether is a densely filled substance with an equal number of positive and negative Planck masses $m_P = \sqrt{(hc/G)}$ which interact locally through contact-type delta-function potentials. In the framework of this approach Winterberg (2002a, 2002b) has shown that quantum mechanics can be derived as an approximate solution of the Boltzmann equation for the Planck aether masses. The particle in his model is a formation appeared as result of the interaction between the positive and negative Planck masses similar to the phonon in a solid. This suggestion is seemingly in a good agreement with other study of gravity phenomena as long wave-length excitation of Bose-Einstein condensate by Consoli (2000, 2002). Consoli (2000) noted that the basic idea that gravity can be a long-wavelength effect induced by the peculiar ground state of an underlying quantum field theory leads to considering the implications of spontaneous symmetry breaking through an elementary scalar field. He pointed out that Bose-Einstein condensation implies the existence of long-range order and of a gap-less mode of the Higgs-field. This gives rise to a 1/r potential and couplings with infinitesimal strength to the inertial mass of known particles. If this is interpreted as the origin of Newtonian gravity one finds a natural solution of the hierarchy problem. In the spirit of Landau, Consoli (2000, 2002) has also considered similarity between his condensate model and superfluid aether hypothesis. Furthermore, he also suggested: "*all classical experimental tests of general relativity would be fulfilled in any theory incorporating the Equivalence Principle*."

Furthermore, recently Celerier & Nottale (2002) have shown that the Dirac equation can be derived from the scale relativity theory. Since the Dirac equation implies the existence of aether, this derivation can be interpreted as: modeling superfluid aether in the universe is compatible (at least in principle) with Nottale's scale relativity framework.[xii] Nottale's conjecture on the applicability of the Schrödinger equation to describe macroscopic phenomena (up to astronomic scale) seems also to imply the presence of a certain form of fluid (aether) as the medium of vacuum quantum fluctuation or a zero point field (Roberts 2001). And because the only type of matter capable of resembling such quantum phenomena macroscopically is Bose-Einstein condensate or its special case superfluid (Consoli 2000, 2002), then this leads us to a conjecture that the *aether medium is very likely a quantum fluid*.

Combining the character of these selected recent developments, this article suggests that the nonlinear wave dynamics of Cantorian vortices of superfluid aether can serve as the basis of a nonlinear cosmological model, which will be capable of describing various phenomena including a plausible mechanism of continuous particle generation in the universe. The preceding work (albeit somewhat controversial from the present accepted view) suggests that this alternative and nonlinear cosmological model shall include: (a) an aether, (b) Euclidean flat spacetime[xiii], (c) vortex dynamics, (d) superfluid (Bose-Einstein condensate), and (e) fractal phenomena—as the basis of real physical model and also the theoretical analysis of nonlinear cosmology. It is the opinion of this author that a proper combination will lead us to a consistent real model.

Therefore, in theoretical terms this article argues in favor of combining *Cantorian-Minkowski geometry with Nottale-Gibson-Winterberg's vortex of superfluid aether.* The proposed model results in a Euclidean flat spacetime with some fluctuations induced by fractal phenomena (expressed as a non-integer dimension in Cantorian universe) arising from multiple vortices. A real physically-observed model is chosen here instead of geometrical construct, because it will directly lead us to a set of experimental tests which can be used to determine if the model is not valid. With regards to superfluidity research, perhaps the conjectures of this article can be considered as extending Volovik's (2000a, 2000b, 2001) superfluid theory to Cantorian spacetime case.

A derivation of the basic vortex model and quantization of semimajor axes

The Schrödinger equation of wave mechanics can be interpreted as a description for the tendency of micro aggregates of matter to make structures. In this regards, Nottale (1993, 1996, 1997) put forth a conjecture that spacetime is *non-differentiable,*[xiv] which led to a fractal version of the Schrödinger-like equation capable of predicting the semimajor axes of both planetary-like systems as well as micro orbits at molecular level. This reasoning could be considered as an alternative interpretation of Ehrenfest Theorem.

However, such a quantum-like approach in a large-scale structure has not been widely accepted (Coles 2002), for the quantization of macroscopic systems is something outside the scope of known physics (Neto *et al*. 2002). Nevertheless, some possible origins for such effects have been outlined. For instance Bohr-Sommerfeld's hypothesis of quantization of angular momentum, appears to be more direct than the Schrödinger-like equation, at least for (planar case of) planetary orbits in the solar system. For a spherical case (for some exoplanet systems) we should derive solution of the Schrödinger-like equation.

As we know, for the wave function to be well defined and single-valued, the momenta must satisfy Bohr-Sommerfeld's quantization conditions (Van Holten 2001):

$$\oint_\Gamma p.dx = 2\mathbf{p}\,.n\hbar \tag{3}$$

for any closed classical orbit Γ. For the free particle of unit mass on the unit sphere the left-hand side is

$$\int_0^T v^2.dt = w^2.T = 2p.w \tag{3a}$$

where T = 2π/ω is the period of the orbit. Hence the quantization rule amounts to quantization of the rotation frequency (the angular momentum): $w = n\hbar$.

Then the force balance relation of Newton's equation of motion:

$$GMm/r^2 = mv^2/r \tag{3b}$$

Using Bohr-Sommerfeld's hypothesis of quantization of angular momentum (3a), a new constant g was introduced (which plays the role of a gravitational analog of the Planck constant):

$$mvr = ng/2p$$

Just like in the elementary Bohr theory (before Schrödinger), this pair of equations yields a known simple solution for the orbit radius for any quantum number of the form:

$$r = n^2.g^2/(4p^2.GM.m^2) \tag{5}$$

or

$$r = n^2.GM/v_o^2 \tag{6}$$

where r, n, G, M, v_o represents semimajor axes, quantum number (n = 1,2,3,…), Newton gravitation constant, and mass of the nucleus of orbit, and specific velocity, respectively. In this equation (6), we denote

$$v_o = (2p/g).GMm \tag{6a}$$

This result (6) is the same as Nottale's basic equation for predicting semimajor axes of planetary-like systems (Nottale 1996, Nottale *et al.* 1997, 2000). It can be shown that equation (6) could be derived directly from the Schrödinger equation for planar case (Christianto

2001), therefore it represents the solution of the Schrödinger equation for *planar axisymmetric cylindrical case*. The value of m is an adjustable parameter (similar to g). For a planetary system including exoplanets Nottale *et al*. (1997, 2000) has found the specific velocity v_o is $\pm$ 144 km/s. Therefore this equation (6) implies the semimajor axes distribution can be predicted from a sequence of quantum numbers. This equation (5) is also comparable with Neto *et al*.'s (2002) approach, where they propose $m = 2.1 \times 10^{26}$ kg (the average mass of the planets in solar system).

It is worth noting here Nottale *et al*. (1997, 2000) reported this equation (6) agrees very well with observed data including those for exoplanets, and particularly for *inner planet orbits* in the solar system. Indeed the number of exoplanets found has increased fivefold since their first study (Nottale *et al*. 2000). However, a question arises when we compare this prediction with *outer planet orbits* in the solar system, since this results in very low predictions compared with observed data, i.e. 52.6% for Jupiter, 36.3% for Saturn, 22.3% for Uranus, 17.2% for Neptune, and 15.6% for Pluto. Therefore, Nottale (1996) proposed to use a different value for v_o to get the distribution of outer planets (the so-called Jovian planets).

Nottale (1996) proposed a plausible explanation for this discrepancy by suggesting outer planets from Jupiter to Pluto are part of different systems since they apparently consist of *different* physical and chemical planetary compositions, so we can expect two different diffusion coefficients for them. Therefore he proposed the following relation to predict orbits of inner planets and outer planets (Nottale 1996, p. 51) $a = n.(n + ½).a_o$. Nottale then suggested the proper values are $a_{o.inner} = 0.038025$AU for inner orbits and $a_{o.outer} = 1.028196$AU for outer orbits, and based on these values the discrepancy in predicting outer planet distribution can be reconciled.

While Nottale's (1996, p. 53) description on these different chemical and physical compositions, distribution of mass, and distribution of angular momentum seem to be at least near to right, he did not offer any explanation of *why* there are different chemical and physical compositions if these outer planets were generated by the same Sun in the past. Nottale's proposed equation was based on the second quantum number l, derived from Schrödinger-type equation for *spherical case.* However, it should be noted that while the second quantum number could plausibly explain the different orbits for outer planets, it cannot provide any explanation for their different chemical and physical compositions. Therefore, this leads us to a conjecture, i.e. these *differences of planetary distribution and different chemical and physical compositions of the outer planets in the solar system are the consequences of the interaction of a negative mass (star) with the Sun*.[xv] From this author's opinion, it seems only through using this conjecture we could explain why the outer planets are physico-chemically different from the inner planets. From this conjecture, then we reinterpreted Nottale's conjecture that Jupiter should be the second planet ($n = 2$) in the outer orbit system, to obtain predicted values of semimajor axes of those Jovian planets, based on the notion of reduced mass μ. The result of this approach will be described subsequently.

Another plausible explanation of the outer planets distribution has been suggested by Chavanis (1999) based on *two-fluids model*. However, while this suggestion is in good agreement with observation of outer planet orbits, in the author opinion it also does not offer a convincing argument for the difference of chemical and physical composition *if* those inner and Jovian planets were generated by the same Sun.

Now let's turn our attention to the implications of equation (6) in regards to the basic vortex model. If T is the orbit period of the above planet around the Sun, then by Kepler's third law,

$$r^3 \approx T^2 \approx (2pr/v)^2 \tag{7}$$

Or

$$v^2 r \approx 4p^2 = k_{spring}$$

where r, T, v, k_{spring} represents semimajor axes, orbit period, orbit velocity, and 'spring constant' of the dynamics system, respectively.[xvi] For gravity case, one obtains $k_{spring} = G.M$. We remark here this constant k_{spring} could be comparable with Nottale's (Nottale *et al.* 2000) notion of parameter $D = G.M/2\omega$; thus $k_{spring} = D.2\omega = D.2\alpha_g c$. This alternative expression comes from the definition of gravitation coupling constant $\alpha_g = \omega/c$, where $\alpha_g^{-1} = 2072 \pm 7$ (Nottale *et al*. 2000).

By observing the above expressions, we conclude that equation (8) has the same basic form of Nottale's equation (6). We also note here Nozieres & Pines (1990) suggested that a vortex structure exists in a superfluid if its velocity is *radius-dependent* (v = f(1/r)). Since from equation (8) the quadratic of velocity is radius-dependent $v^2 = (k/r)$, we propose here that equation (8) also implies a *special case* of vortex motion. Therefore, we conclude equation (6) also implies a vortex motion. This seems to be in agreement with Nottale *et al.*'s (1997, 2000) assertion that specific velocity v_o = 144 km/s represents a new fundamental constant observed from the planetary up to extragalactic scale.

In order to generalize further equation (6), we proposed using Kobelev's (2001) idea that Newton's equations may be treated as a diffusion process in a multi-fractal universe. Provided such a relationship exists, we could conclude that equation (6) implies a

Cantorian fractality of vortex structure in the universe. But a question arises here as to whether a scaling factor is required to represent equation of motion of celestial bodies at various scales using equation (6). Therefore, by using a fractional derivative method as described by Kolwankar (1998, eq. 2.9), then

$$d^q f(bx)/[dx]^q = b^q.\{d^q f(bx)/[d(bx)]^q\} \tag{9}$$

where it is assumed that for $dx \to 0, d(bx) \approx dx$. Hence this author obtained (Christianto 2002b) a linear scaling factor for equation (6):

$$a_0 = f.n^2.GM / v_o^2 \tag{10}$$

This equation implies :

$$v_1^2 = (v_o^2 / f_o) \tag{11}$$

In other words, for different scaling reference frames, specific velocity v_1 may vary and may be influenced by a scale effect ϕ. To this author's present knowledge, such a scaling factor has never appeared before elsewhere; neither in Nottale's work (1996, 1997, 2001, 2002) nor in Neto *et al*. (2002). A plausible reason for this is that Nottale's and Neto *et al*.'s theory were intended to describe planetary orbits only.

A note on this interpretation is perhaps worth making. While of course this Cantorian fractality of vortex structure in the universe is not the only possible interpretation, we believe this is the nearest interpretation considering the turbulence phenomena.[xvii] It is known that turbulent flows seem to display *self-similar* statistical properties at length scales smaller than the scales at which energy is delivered to the flow (this sometimes referred to as 'multi-fractality' of turbulence). For instance, Kolmogorov argued that at these scales, in three dimensions, the fluids display universal statistical features (Bernard 2000, Foias *et al*. 2001 p. 17, Gibson 1991, Weinan 2000).

Turbulent flow is conventionally visualized as a cascade of large vortices (large scale components of the flow) breaking up into ever smaller sized vortices (fine-scale components of the flow) – the principal cascading entity is the 'enstrophy'.[xviii]

Recent observational data of the similar size of semimajor axes between solar system and exoplanet systems ($a/M = 0.043$ AU/M_o for $n = 1$; and $a/M = 0.17$ AU/M_o for $n = 2$) seems to indicate that those are clusters of celestial objects at the same hierarchy (scale) of quantized vortices (Armitage *et al*. 2002, Lineweaver *et al.* 2003, Neto *et al*. 2002, Nottale *et al*. 1997, 2000, Weldrake 2002). This seems to imply that the proposed Cantorian vortices interpretation is in good agreement with observed data.

Superfluid vortices model

It is worth discussing here the *rationale* for suggesting a Cantorian superfluid aether as a real physical model for nonlinear cosmology. This brings us back in time to where GTR was first introduced (in passing we note in pre-GTR era aether hypothesis was almost entirely abandoned because of the growing acceptance of STR; see Munera 1998).

It is known that in GTR there is no explicit description of the medium of interaction in space (aether), though actually this was considered by Einstein himself in his lecture in Leiden 1921, "*Ether and Relativity*" (Einstein 1921):

> *"..According to the general theory of relativity space without an ether is **unthinkable**; for in such a space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense. But this ether may not*

> *be thought of as endowed with the quality characteristic of ponderable media, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it."*

A perfect fluid in GTR is therefore could be thought of as a liquid medium with no viscosity and no heat induction. Such a perfect fluid is basically a special case of quantum liquid or superfluid (Nozieres & Pines 1990). We note the term 'special case' because the superfluid here should be able to represent non-ponderable (weightless) characteristic of the aether medium, though perhaps it could have motion.

It is clear therefore aether is inherently implied in a GTR geometrical construct (see also Consoli 2002). Furthermore, it is possible to explain the frame dragging phenomena in a GTR geometrical construct as it is actually a fluid vortex—with a massive object in its vortex centre (Prix 2000)—capturing a volume of surrounding fluid and entraining its rotation.

In Maxwell's hypothesis, aether is a frictionless fluid. Based on this conjecture Winterberg (2002a, 2002b) has proposed an aether model, which consists of a quantum fluid made up of Bose particles. This analogy leads to the Planckian aether hypothesis which makes the assumption the vacuum of space is a kind of plasma (see also Roberts 2001). The ultimate building blocks of matter are Planck mass particles obeying the laws of classical Newtonian mechanics, but there are also *negative* Planck mass particles. Furthermore, with the Planck aether having an equal number of positive and negative Planck mass particles, the cosmological constant is zero and the universe is Euclidean flat-spacetime. In its groundstate the Planck aether is a two component positive-negative mass superfluid with a *phonon-roton* energy spectrum for each component.

The theory of superfluid vortices is based upon various versions of the Landau's two-component fluid model (Godfrey *et al*. 2001), and is adequately described by many researchers (Kivshar *et al*. 1998, Quist 2002, Thouless *et al*. 2001, Tornkvist & Schroder 1997, Volovik 2000c, 2001, Zurek 1995). For applications to Cosmology, it is presumed that the "vacuum" is a superfluid-like continuum in which the formation of topological defects as "vortices" generates the stars and galaxies as components of the normal fluid. The diffusive and dissipative Navier-Stokes fluid equations, with constraints that lead to the Complex Ginzburg-Landau equations to describe the superfluid, form the basis of the mathematical model. The topological defects can be homogeneously defined, hence they are self-similar, and scale covariant. Such topological defect domains can support not only fractals but also quantum like integer values for their closed integrals.

The conceptual map (Figure 1) depicts how the various parts of the most recent theories could plausibly be used to form a Cantorian superfluid vortex model for nonlinear cosmology.

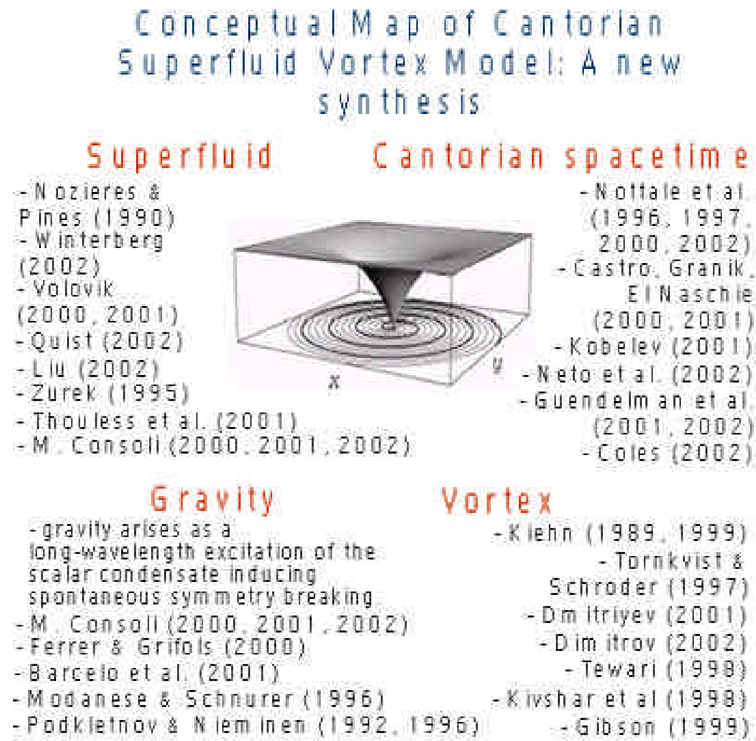

Figure 1. Conceptual map of the plausible synthesis of a Cantorian superfluid vortex model for nonlinear cosmology

Now we are going to illustrate how the equation of motion (6) is compatible with the proposed superfluid vortices model as described above. In other words, we will provide an argument to link the solution of the Schrödinger equation (6) with the solution of Navier-Stokes equations. Theoretically, R. Kiehn (1989, 1999) has shown that there is an exact mapping between the Schrödinger equation and Navier-Stokes equation, though without reference yet to its cosmological implications. Therefore now we extend his conjecture to a cosmological setting. In order to do this, we consider two approaches here:

- Gibson's (1999) Navier-Stokes model for cosmology;
- Godfrey *et al*.'s (2001) model of superfluid vortices.

First, we note here that Gibson (1999) has shown that his Navier-Stokes-Newton model yields the following solution:

$$v_r = m'.Gt / r^2 \tag{12}$$

where r, t, G, m', v_r represents semimajor axes, time elapsed, Newton gravitation constant, mass of the nucleus of orbit, and specific velocity, respectively. It is clear therefore that equation (12) admits mass growth rate as time elapsed, which is permitted by Gibson's Navier-Stokes model. Now we assert $v = 2pr / T$ or $r = vT / 2p = vt$, and substitute this value to one of r in equation (12). We get:

$$r = m'.G / v^2 \tag{13}$$

which is very similar to equation (6), except the expression for quadratic quantum number n^2. A plausible reason for this missing quantum number is that Gibson (1999) assumed a normal fluid in his model instead of quantum liquid. He also argued that equation (12) only governs the formation stage (such as spiral nebulae formation); while equation (13) is also applicable for present time provided we assert a quantum liquid for the system. Therefore we also conclude again that Nottale's equation (6) *actually implies a quantum liquid as medium of interaction.*

For the second method, we note here that according to Godfrey *et al*. (2001) the analytic form of an oscillating plane boundary layer flow of superfluid vortices can be derived from the Navier-Stokes equation, and the velocity u(*z,t*) is given by:

$$u = A.e^{-kz}.\cos(wt - kz) \tag{14}$$

where $k = \sqrt{(w/2v)}$, $w = 2p/T$ is the angular frequency of oscillation, T is the period of oscillation, ν is the kinematic viscosity and A is an arbitrary constant. In the limit that the coupling of the superfluid and normal fluid components through mutual friction is negligible, we may take this oscillating velocity profile for the normal fluid, with the superfluid remaining at rest. Because we can assert velocity u = dz/dt = dΨ/dt, therefore we can obtain Ψ and also its second differentiation $d^2\Psi/dt^2$. Hence we get:

$$d^2\Psi / dt^2 = -A.e^{-kz}.\sin(wt - kz).w \tag{15}$$

or

$$d^2\Psi / dt^2 + w^2.\Psi = 0 \tag{16}$$

which is the most basic form of the Schrödinger equation. In other words, we obtain the Schrödinger equation from a velocity expression derived from the Navier-Stokes equation for superfluid vortices (Godfrey *et al*. 2001). These two methods confirm Kiehn's (1989, 1999) conjecture that there is exact mapping between the Schrödinger equation and Navier-Stokes equation *regardless* of the scale of the system considered. This conclusion, which was based on a two-fluid model of superfluid vortices, is the *main result* of this article; and to this author's present knowledge this conclusion has never been made before for the astronomical domain (neither in Chavanis 1999, Neto *et al.* 2002, nor Nottale 1996, 1997, 2001, 2002). In this author opinion, Chavanis' article (1999) is the nearest to this approach, because he already considered two-fluid model for the Schrödinger equation (though without reference to superfluidity), though he did not mention the role of Navier-Stokes equations like Gibson (1999).

A distinctive feature of this proposed superfluid vortices approach is that we could directly compare our model with laboratory observation (Volovik 2001, Zurek 1995). For instance, using this model Godfrey *et al*. (2001) argued that the fluid at the edge of the disk moves a distance $4\phi_c R$ in a time T (with angular velocity $\omega = 2\pi/T$), thus having a critical dimensional linear velocity of

$$v_{disk} = 2w.f_c R / p \tag{17}$$

In this equation, ϕ_c represents critical amplitude where damping of the oscillations reduce to a value, which was interpreted as the damping due only to viscosity of the normal fluid component. In this regards, interpretation of the experiment is that superfluid boundary layer vortices are the cause of critical amplitude of oscillations observed. Therefore it seems we could expect to observe such critical amplitude for the motion of celestial objects. Of course for spherical orbit systems the equation of critical dimensional linear velocity is somewhat different from equation (17) above (Godfrey *et al*. 2001). To this author's present knowledge such theoretical linkage between critical amplitude of superfluid vortices and astronomical orbital motions has also never been made before; neither in Chavanis (1999), Nottale (1996, 1997, 2001, 2002), Volovik (2000a, 2000b, 2000c, 2001), nor Zurek (1995).

New planets prediction in solar system

Based on equation (6) and using Nottale's conjecture of Jupiter should be the second planet (n = 2) in the outer orbit system, we derive predicted and observed values of semimajor axes of those outer planets. Then by using Nottale's (1996, p. 53) conjecture for quantization of galaxy pairs, and minimizing the standard deviation (s) between these observed and predicted values, we can solve equation (6) for the reduced mass μ to get the most probable distribution for outer planet orbits:

$$m = (m_1.m_2)/(m_1 + m_2) \tag{18}$$

It is worth noting here, that a somewhat similar approach using reduced mass μ to derive planetary orbits has also been used by Neto *et al.* (2002), as follows:

$$-g^2/2m.\left(\partial^2\Psi/\partial r+\partial\Psi/r\partial r+r^{-2}.\partial^2\Psi/\partial j^{\,2}\right)+V\Psi=E\Psi \quad (18a)$$

though Neto *et al*. (2002) did not come to the same conclusion as presented here. Result of this method (18) is presented in Table 1 below.

Msun = 0.198951 10^34 g k = **26.60** k.Msun = 5.29 10^34 g

Astroobject	Orbit Size	Observed r_{actual} (AU x 10)[1]	r_{actual} (10^6 km)	Prediction based on Nottale-Ord n	r_{pred} (10^6 km)	n	r_{pred}/r_{actual} (%)	$(r_{pred}-r_{act})^2$
				1	6.40			
				2	25.60	2		
Mercury	4	3.87	57.89	3	57.60	3	99.484	0.1
Venus	7	7.32	109.51	4	102.39	4	93.504	50.6
Earth	10	10.00	149.60	5	159.99	5	106.945	107.9
Mars	16	15.24	227.99	6	230.38	6	101.050	5.7
Hungarias[3]			314.00	7	313.58	7	99.865	0.2
Asteroid[4]		27.00	403.91	8	409.57	8	101.400	32.0
		31.50	471.23	9	518.36	9	110.001	2220.9
Jupiter	52	52.03	778.36	2	681.01	2	87.493	9476.8
Saturn	100	95.39	1427.01	3	1532.27	3	107.376	11078.9
Uranus	196	191.90	2870.78	4	2724.04	4	94.888	21534.7
Neptune	388	301.00	4502.90	5	4256.31	5	94.524	60806.4
Pluto	722	395.00	5909.12	6	6129.08	6	103.722	48384.8
Π_1				7	8342.36	7		
Π_2[5]				8	10896.14	8		
Π_3[6]				9	13790.43	9		

Table 1. Predicted orbit values of inner and outer planets in Solar system

From Table 1 above we obtain $\mu = 26.604.m_1$, for the minimum standard deviation s = 0.76AU.[xix] Inserting this μ value into equation (18) and solving it, we get the most likely companion mass of $m_2 = -(26.604/25.604).m_1$. Therefore we conclude it is *very likely* there is a negative-mass star (NMS) interacting with the Sun. This NMS has a mass value of very near to the Sun but with a negative sign, so this can be considered as the dim twin-companion star of the Sun. This is somewhat comparable to what some astronomers suggest of the hypothetical '*dark star*' (Damgov *et al*. 2002), though to this author's present knowledge none of the existing astronomic literatures has considered a negative-mass star as plausible candidate of the twin-companion of the Sun. Therefore thus far, this conclusion of the plausible presence of a large negative-mass object in the solar system could only be explained using *superfluid/superconducting model* (DeAquino 2002). [xx]

On the basis of this value of $\mu = 26.604.m_1$, we obtained a set of predicted orbit values for both inner planets and Jovian planets. For inner planets, our prediction values are very similar to Nottale's (1996) values, starting from n = 3 for Mercury; for n = 7 Nottale reported minor object called Hungarias; for Jovian planets from n = 2 for Jupiter up to n = 6 for Pluto our prediction values are also somewhat similar with Nottale's (1996) values. It is worth noting here, we don't have to invoke an *ad hoc* quantum number to predict orbits of Venus and Earth as Neto *et al*. (2002) did. We also note here that the proposed method results in prediction of orbit values, which are within a 7% error range compared to observed values, except for Jupiter which is within a 12.6% error range.

The departure of our predicted values compared to Nottale's predicted values (1996, 1997, 2001) appear in outer planet orbits starting from n = 7. We proposed some new predictions of the possible presence of three outer planets beyond Pluto (for n = 7, n = 8, n = 9) to be called here as Π_1, Π_2, Π_3 at orbits around 55.77 ± 1.24AU, 72.84 ± 1.24AU, and 92.18 ± 1.24AU, respectively. This prediction of most likely semimajor axes has taken into consideration standard deviation found above s = 0.76AU (Table 1). Two of these predicted orbits of outer planets are somewhat in agreement with previous predictions by some astronomers on the possible presence of outer planets beyond Pluto around ~50AU and around ~100AU (Horner *et al*. 2001). However, it is worth noting here, the predicted planet (for n = 8) at orbit 72.84 ± 1.24AU is purely based on equation of quantization of orbit (6) for Jovian planets. It is also worth noting here, that these proposed planets beyond Pluto are different from what is predicted by Matese *et al*. (1999), since Matese's planet is supposed to be somewhere around the outer Oort cloud.

Further remarks are worth considering here concerning predicted orbits at n = 8 and n = 9. We consider first for the case of inner orbits. It was suggested by Olber and also recently by Van Flandern in 1993 (Damgov *et al*. 2002) of a planet (or *planets*) existed until relatively recently between Mars and Jupiter, at the location where a missing planet is expected by the well-known Titius-Bode law (see Table 1 under column 'Orbit size'). As we know, Titius-Bode law was based on series of numbers 0,3,6,12,24,48,96… which then translated by factor 4. Thus we have series of 4,7,10,16,28,52,… which are supposed to be able to predict the orbit size of planets in solar system. This argument was subsequently supported by Nottale's equation except for orbits at n = 7 and n = 9, between Mars and Jupiter, which can be regarded as departure from the Titius-Bode law. However, while Nottale (1996, p. 51) has reported planets (or at least, recognizable objects) at n = 8 and n = 9 for *inner* orbit in solar system were observed, to our present knowledge no similar prediction has been made for n = 8 and n = 9 for outer orbits. Therefore new observational data is highly recommended to find the real semimajor axes of the proposed new outer planets beyond Pluto.

If these new outer planets correspond to the observational data, it is conjectured intuitively that the proposed Cantorian superfluid vortices model could offer an improved explanation for several things unexplainable (at least not yet in a observable and quantifiable form) thus far with regards to the origin of continuous particle generation, gravitation instability, and unifying gravity and quantum theory.

Notes on the superfluid experiments for cosmology: fractal superfluid

Zurek (1995) and Volovik (2000b) have proposed some aspects of superfluid analogies to describe various cosmological phenomena. However, extending this view towards Cantorian Superfluid Vortex hypothesis implies we should be able to observe fractal phenomena of superfluid and also Bose-Einstein condensate systems. While this has not become the accepted view, recent articles indicate such phenomena were *already* observed (Kivotides *et al.* 2001, 2001b, Ktitorov 2002).

In this regards, some recent observations have shown that the number of galaxies N(r) within a sphere of radius r, centered on any galaxy, is not proportional to r^3 as would be expected of a homogeneous distribution. Instead N(r) is proportional to r^D, where D is approximately equal to 2, which is symptomatic of distribution with fractal dimension D. It is interesting to note, that for $D = 2$, the cosmological gravitational redshift gives the linear distance-redshift relation and becomes an observable phenomenon (Mittal & Lohiya 2001). This non-integer dimension is known as Hausdorff dimension d_H, which can be computed to be within the range of $1.6 \sim 2.0$ up to the scale $1 \sim 200$ Mpc (Baryshev 1994, 1999). Furthermore, transition to homogeneity distribution has not been found yet. In this regards Anderson *et al.*[xxi] also admitted: "*These findings (of clustering and void formation) have become increasingly difficult to reconcile with standard cosmological theories, in which the approach to homogeneity at large-scales is central element.*" What more interests us here is that an extended version of Gross-Pitaevskii equation admits self-similar solutions and also it corresponds to Hausdorff dimension $d_H \sim 2$, which seems to substantiate our hypothesis that there is exact correspondence between cosmological phenomena and condensed matter physics.[xxii]

In principle, the proposed Cantorian Superfluid Vortex theory leads us to a fractal superfluid description of Euclidean flat-spacetime universe, which is scale-invariant and expanding at all scales, but without a cosmological constant (this was also suggested by Guendelman *et al.* 2002, Winterberg 2002a, 2002b). This Cantorian Superfluid Vortex model is inhomogeneous though it is perhaps isotropic (in accordance with Einstein-Mandelbrot Cosmological Principle; Mittal & Lohiya 2001). Gibson (1999) has also described how the nonlinear cosmology model based on Navier-Stokes equations could explain the hidden-universe problem. Furthermore, it seems that the superfluid vortice model could explain why the inner cylindrical core of earth rotates independently of the rest of the planet.[xxiii]

It seems therefore we could expect that further research will divulge more interesting fractal phenomena of Bose-Einstein condensate and superfluid systems (somewhat related to superfluid turbulence and its damping phenomena; Godfrey *et al*. 2001), which could lead us to further generalization of the proposed Cantorian Superfluid Vortex model.

A new method to predict quantization of planetary orbits has been proposed based on a Cantorian superfluid vortex hypothesis. It could be expected that in the near future there will be more precise nonlinear cosmology models based on real fluid theory.

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Endnotes

[i] Term 'turbulent mixing' here has been used in accord with Gibson's original terminology. Turbulence is defined as "an eddy-like state of fluid motion where the inertial-vortex forces of the eddies are larger than the viscous, buoyancy, electromagnetic or any other forces which tend to damp the eddies." Furthermore, natural flows at very high Reynolds, Froude, Rossby numbers in the ocean, atmosphere, stars and interstellar medium develop highly intermittent turbulent and mixing (Gibson 1991, also Foias et al. 2001).

[ii] For other publications of C. Gibson related to this issue, see arXiv.org: astro-ph/9904230, astro-ph/9904237, astro-ph/9904260, astro-ph/9904284, astro-ph/9904283, astro-ph/9904317, astro-ph/9911264, astro-ph/9904362, astro-ph/0003147, astro-ph/0002381, astro-ph/9810456, astro-ph/0003352, astro-ph/9904366, astro-ph/9908335.

[iii] See also Castro, Mahecha, Rodriguez (2002) for further discussion on this approach from the fractal diffusion viewpoint.

[iv] As we know $\rho(V.\nabla)V$ is the only nonlinear term in the Navier-Stokes equations; this term is also called the inertial (vortex) term. The Navier-Stokes equations are among the very few equations of mathematical physics for which the nonlinearity arises not from the physical attributes of the system but rather from the mathematical (kinematical) aspects of the system. In divergence free condition div $\mathbf{u} = 0$, the Navier-Stokes equations for a viscous, incompressible, homogenous flow are usually expressed as:

$$\partial u / \partial t - v.\Delta u + (u.\nabla)u + \nabla p = f,$$

$$\nabla.u = 0$$

where for notational simplicity, we represent the divergence of u by $\nabla.\mathbf{u}$, and for all practical purposes the density has been normalized to unity, $\rho=1$ (C. Foias et al., 2001). It shall be worthnoting, however, the origin of viscosity imposes a limit on the domain of validity of the Navier-Stokes equations. We should learn of some natural lengths characterizing the length scale region in which flow energy dissipation is dominated by viscous phenomena.

Therefore we find the significance of the Reynolds number emerges by comparing the inertial and dissipation terms of the Navier-Stokes equations. The inertial term dominates when:

$$\mathrm{Re} = L_* U_* / \nu >> 1$$

By setting the Re = +∞ (i.e. ν = 0), we obtain the case of inviscid flows. In this case, the divergence-free condition is retained but the momentum equation changes, resulting in the Euler equations for inviscid perfect fluids:

$$\partial u / \partial t + (u.\nabla)u + \nabla p = f,$$
$$\nabla.u = 0$$

Note here, some of the difficulties encountered in studying turbulent behavior, a largely inviscid regime, arise because of transition from Euler's equations to the Navier-Stokes equations necessitates a change from a first-order system to a second-order one in space (∇ to Δ) (C. Foias et al. 2001).

[v] We admit here the accepted viewpoint is superfluidity implies no dissipation (no turbulence is possible); the condensations –as long-lived states perhaps far from equilibrium – are indeed related to superfluidity, where the solutions are harmonic, so dissipative effects do not appear. Hence chaos can appear in the superfluid but not irreversible turbulence. However, recent research have begun to embrace this 'superfluid turbulence' issue (see Proceedings of the Isaac Newton Institute Workshop on Quantized Vortex Dynamics and Superfluid Turbulence, Cambridge, UK, Aug. 2000). They discussed for instance: hydrodynamic description of superfluid helium turbulence with quantum vortices; valuable comparison between the physics of Navier-Stokes and helium II turbulence; and a realistic possibility of experimental study of quantum turbulence in superfluid ^{3}He.

Other researchers have considered the possibility of superfluid turbulence phenomena, particularly for superfluid ^{3}He and He^4. Zurek (1995, 16) considered turbulent tangle of vortex lines. Volovik (2000b) considered ^{3}He-A effects to represent turbulent cosmic plasmas, though he admits these effects are less dramatic. Some experiments showing unusual properties damping and viscosity properties of helium II, indicating turbulence phenomenon, have also been reported by (Godfrey *et al*. 2001). Therefore we could expect under certain condition superfluid (helium) could exhibit such turbulence phenomena.

[vi] See also for instance *arXiv:math-ph/9909033*.

[vii] Inspired by Landau two-fluid theory, a number of researchers share a viewpoint that a **vortex** can be a singularity in a "background" fluid. The background fluid is the superconductor (or superfluid) which can admit circulation, but without vorticity and without dissipation. The defect "vortex" regions are then topological defects (Yates 1996), which, if not empty holes, are bounded regions of real vorticity, with a vorticity discontinuity on the boundary of the defect domain. The discontinuity implies a lack of differentiability. In the limit, these regions are taken to be "vortex" threads or strings, but this is only part of the story for there are other types of topologically bounded regions of "vorticity" which in many cases can have persistent lifetimes, and therefore represent "objects" in the background fluid (see Kiehn 2001). In this regards, an active community sponsored by ESF in Europe, COSLAB-VORTEX-BEC2000+ groups have combined to give a workshop in Bilbao this summer (2003), see http://tp.lc.ehu.es/ILE/bilbaocoslab.htm. It appears that the objective of COSLAB is to see how these objects in a laboratory superfluid may be considered as models of a cosmology (Zurek 1995, Volovik 2000b). In effect, the background is the "vacuum aether superfluid" and the stars and galaxies are the "condensed objects" within it.

[viii] Vorticity in cosmology has been considered in a recent article, C. Schmid, arXiv:gr-qc/0201095 (2002); while the idea of condensation may correspond to article by G. Chapline, arXiv:hep-th/9812129 (1998).

[ix] Such vortices sometimes are known as 'circulatory wave' or Wolter's vortex, see H. Rosu, arXiv:quant-ph/9506015 (1997).

[x] This argument can be considered as based on the simple observation, i.e. one can represent natural objects like gas or water as (kinematic) dynamics of

fluids, but not as fields. Therefore we could conclude the domains of application of fields are *less than* those of fluids.

[xi] It is known there exist exact solutions to the Navier-Stokes equations that – at constant vorticity- create bounded regions of fluid bubbles of isolated vorticity which are formed as the mean translational flow increases. It seems this could be an example of particle generation in dissipative media. It is perhaps also worth noting here, i.e. there does exist one-to-one correspondence between the Schroedinger equation and the Navier-Stokes equation for viscous compresible fluids, not just Madelung-Eulerian fluids (Kiehn 1989, 1999). The square of the wavefunction is the enstrophy of these fluids.

[xii] At this point, it is worthnoting here this previous works by Cartan have shown that Dirac equation can be generalized without any recourse to non-differentiability nor to an aether. Therefore, such aether interpretation could be considered merely as plausible alternative interpretation, somewhat in accordance with the previous works of Prokhovik, Rothwarf (1998), Consoli *arXiv:hep-ph/0109215* etc.

[xiii] Similar suggestion of *flat spacetime* universe has also been argued recently for instance by Moniz (*arXiv:gr-qc/0011098*) and K. Akama (*arXiv:hep-th/0007001, hep-th/0001113*).

[xiv] *Non-differentiable function* is defined here in simple term as function, which has a derivative nowhere. It is known there are such functions, which are continuous but nowhere differentiable. Some mathematicians propose Weierstrass function belongs to this group.

[xv] Alternatively, we could consider negative mass is inherent in the structure of the core of the Sun (*arXiv:physics/0205040*). This possibility has been discussed by DeAquino for the case of neutron stars. Otherwise, perhaps this negative mass could be considered as effects related to (ultra-cold superfluid neutron) boson stars as theorised by several authors.

[xvi] There is also known transformation (Kustaanheimo-Steifel) from the Kepler problem to the harmonic oscillator problem. An alternative expression was given by Tewari (1998).

[xvii] See also *Apeiron* Vol. 9 No. 2 (2002), though this article discusses atmospheric flows instead of the motion of celestial bodies.

[xviii] Mandelbrot also suggested turbulent velocity fields may have fractal structure with a non-integer Hausdorff dimension: a pattern of spiral with smaller spirals on them—and so on to increasingly smaller scales. This is in

accordance with Landau's (1963) turbulence definition as "superposition of an *infinite* number of vortices, or eddies, with sizes varying *over all scales*." For discussion on possible limitations of such *scale symmetry* assumption, we refer to E.I. Guendelman, *arXiv:gr-qc/0004011, arXiv:gr-qc/9901067*.

[xix] This method uses *Ordinary Least Square* (OLS) theorem, or known as 'least square error' principle. However it shall be kept in mind, this OLS method has seven well-known premises known as "Gauss-Markov assumptions."

[xx] For discussion on the plausibility of the proposed Negative-Mass Star (NMS), see for instance F. De Aquino, *arXiv:physics/0205040* (2002a). In principle, he conjectures there is negative mass inside the vortex core of neutron stars. Therefore either we could observe a distant negative mass star as companion of the Sun, or perhaps the negative mass with mass approximately equivalent with the mass of the Sun is located inside the core of the Sun, as part of its inner structure. Alternatively, we could think such a negative mass as extension to Cantorian space of negative electron mass in Hall effect theory: $-eEm_h / m_e = +eE$ which can only hold if m_h=-m_e. See H. Myers, *Introductory solid state physics*, Taylor & Francis, 2nd ed. (1997), p. 266-267.

[xxi] Anderson, P.W., *et al., Europhys. Lett.* (), arXiv:astro-ph/0002054 (2000).

[xxii] Kolomeisky, E., *et al*., arXiv:cond-mat/0002282 (2000).

[xxiii] X. Song and P. Richards of Columbia University's Lamont-Doherty, http://www.ldeo.columbia.edu/song/pr/html