The Twin Paradox in Special Relativity and in Lorentz Ether Theory

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The present paper analyses the twin paradox as presented by Van Flandern [1] and confirms the results obtained by standard relativistic calculations. A variety of the twin paradox with symmetrical causal chains of events is considered, where the motional trajectories of two twins can be realized with some probability. It seems that such a probabilistic presentation of the twin paradox destroys a conception about equivalence of all inertial observers in SRT. The twin problem has been considered within Lorentz ether theory (LET). For better understanding of the observations of both twins, a mathematical apparatus of LET has been developed. It has been proved that LET postulates represent a direct consequence of the Galilean transformations in physical space-time under limited speed of light.

Keywords: twin paradox, special theory of relativity, Lorentz ether theory

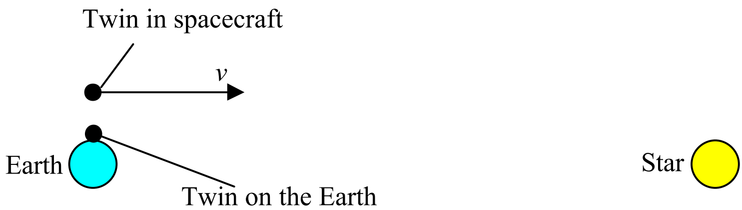


Fig. 1. Initial time instant: the journey of spacecraft twin is starting.

1. Introduction

Since the appearance of the special relativity theory (SRT), the twin paradox has been discussed in many papers and textbooks. One of the most interesting papers on the subject was recently published in Apeiron [1]. This publication prompts me to continue a discussion about this paradox. Section 2 analyses the problems to be found in [1] in more detail. Section 3 presents a variety of twin paradox, where the motional trajectories of two twins are not definite, but may be realized with some probability. Finally, Section 4 explains the twin paradox in LET, as well as in any other possible ether theory, which adopts a Galilean metric of absolute space. The subsection 4.1 develops a mathematical apparatus for LET.

2. The Twin paradox as presented by Tom Van Flandern

In ref. [1] the twin paradox was considered with an imaginary Global Positioning System filling space between Earth and a star. The results obtained in [1] can also be presented at a schematic level, as will be done in the present Section.

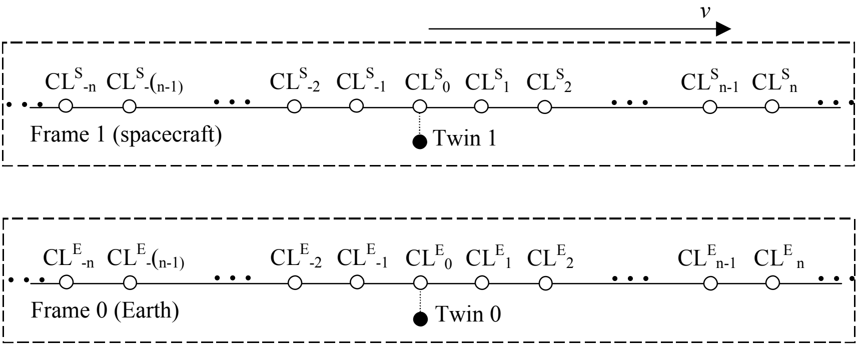


Fig. 2. The inertial reference frames of twin 0 and twin 1. $CL^E{}_0$ is in the hands of twin 0, and $CL^S{}_0$ is in the hands of twin 1.

At the initial time instant let there be two twins. One of them stays on the Earth, while the another one moves at constant velocity v along the axis x toward the star S (Fig. 1).

In the stationary reference frame (Earth frame 0), there is an almost infinite set of clocks $\{CL^E{}_n\}$ to be placed at spatial points x_n, and, for simplicity, $x_n – x_{n-1}$ is a constant value for any n. The same set of clocks $\{CL^S{}_n\}$ is in the frame of moving twin (spacecraft frame 1), where x'_n-x'_{n-1} is also a constant value for any n (Fig. 2). All clocks in their own frame are Einstein-synchronized. Then under travel of twin 1, the twin 0 sees that the clock $CL^S{}_0$ in the hands of twin 1 ticks slower by g times in comparison with $CL^E{}_0$, held in his own hands. (Hereinafter $g = 1\big/\sqrt{1 - v^2/c^2}$). Ref. [1] called attention to an interesting effect: if twin 0 looks for a rate of succession of clocks $\{CL^S{}_i\}$, passing across his laboratory with the speed v, he finds that time in the succession goes by g times faster than for the Earth clock. This is a consequence of the Lorentz transformation, or the “time slippage effect,” defined in [1]. There is no mystery in this effect, if we remember that the remote clocks, being synchronized in frame 1, are not synchronized in frame 0. In particular, one can show that the momentary readings of the clock $CL^S{}_0$ and $CL^S{}_{-i}$ in the Earth frame 0 differ by the term $\Delta t = g\, x_i v/c^2$, if these clocks are Einstein-synchronized in frame 1. In its turn , in the Earth frame $x_i = vt$, where t is the travel time of the spacecraft to point x_i. Hence, $\Delta t = g\, t\, v^2/c^2$. At the instant t the Earth twin sees that $CL^S{}_0$ shows $t_0^s = t/g$. Thus, the reading of $CL^S{}_{-i}$, passing across the Earth at the this instant, is

$$t_i^S = t_0^S + \Delta t = t/g + g\, t v^2/c^2 = g\, t\,. \tag{1}$$

This explains the observation stated above: judged in the Earth frame all individual traveling clocks go slower, but in the succession of clocks passing a fixed Earth point time goes faster.

A symmetrical situation is realized in the spacecraft frame 1. The traveling twin sees that $CL^E{}_0$ ticks slower by a factor g than his own clock $CL^S{}_0$, while in succession of passing clocks $\{CL^E{}_i\}$ time goes g times faster than for $CL^S{}_0$. The latter is due to the fact that two Einstein-synchronized remote clocks in the Earth frame are not synchronized with each other in the spacecraft frame 1. This symmetrical situation emerges when the spacecraft reaches star S.

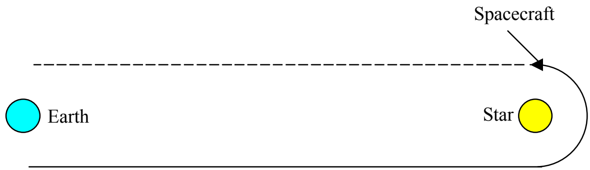
Thus, the travelling twin arrives at the star S, which is remote from the Earth at the distance x_n in the Earth frame 0. At this instant twin 1 sees the clock $CL^E{}_n$ in the window of his spacecraft, while the Earth twin sees clocks $CL^S{}_{-n'}$, passing across his laboratory. (Here $n \neq n'$ due to the scale contraction effect.) The Earth twin fixes the reading of his clock $CL^E{}_0$ $t_0^E = x_n / v$, and the reading of $CL^S{}_{-n'}$, due to Eq. (1),

$t^S_{-n'} = g\, x_n / v$ *. In addition, he concludes that the reading of the clock on the board of spacecraft is $t^S_0 = t^E_0 / g$. The spacecraft twin sees the reading of his clock CL^S_0 $t'^S_0 = x'_n / v = x_n / g\, v$, and the reading of CL^E_n $t'^E_n = g\, t'_0 = x_n / v$. In addition, he concludes that due to the time dilation effect, the Earth clock CL^E_0 indicates $t'^E_0 = t'^S_0 / g$. Assuming the numerical values adopted in [1] ($t_0 = 49$ months ≈ 4 years, $v = 0.99c$, and $g = 7$), we obtain $t^S_{-n'} = 7 t^E_0 = 343$ months ≈ 28 years, $t^S_0 = t^E_0 / 7 = 7$ months, $t'^S_0 = 7$ months, $t'^E_0 = t'^S_0 / 7 = x_n / g^2 v = 1$ month. We therefore have the following table:

What Earth twin sees:	**What Spacecraft twin sees:**
t^E_0 = 49 months (x_n / v)	t'^S_0 = 7 months ($x_n / g\, v$)
$t^S_{-n'}$ = 343 months ($g\, x_n / v$)	t'^E_n = 49 months (x_n / v)
t^S_0 = 7 months ($x_n / g\, v$)	t'^E_0 = 1 month ($x_n / g^2 v$)

Further, let the twin 1 turn back. In ref. [1] Van Flandern supposed a rotation about the star S (Fig. 3). Then, according to [1], at the beginning of rotation the spacecraft twin infers that only one month has elapsed back on Earth since his journey began (t'^E_0 = 1

* Hereinafter we adopt the following designations for space-time coordinates in both frames: the subscript signifies a point of location $\{x_i\}$, the superscript indicates an affiliation with a reference frame (E- Earth, S – spacecraft); primed four-vectors belong to the spacecraft frame 1, while unprimed four-vectors belong to the Earth frame 0. For example, t'^E_n indicates time for twin 1 on the Earth clock at the point x_n.



month). When the spacecraft turns around, then, according to [1], the Earth time for the twin 1 is 8 years: four years into the future

Fig. 3. In order to return home, the twin 1 orbits around the star S.

instead of the past. This is a consequence of the "time slippage effect". Thus, for the traveling twin Earth time changed suddenly, if his rotation time is negligible in comparison with his total journey time. This paradox becomes especially drastic if twin 1 orbits S several times [1]. Then each time the traveler heads away from Earth in that orbit, Earth time drops back to 1 month; and each time the traveler heads toward Earth, inferred Earth time becomes 8 years. This is a quite paradoxical result. At the same time, in terms of "time slippage effect" it is difficult to understand its physics. Now let us explain this observation, considering the process of acceleration of twin 1. To simplify consideration further, we substitute for rotational motion a progressive motion at the constant (in relativistic meaning) negative acceleration a along the axis x. Let us divide this process into two stages: deceleration from the initial velocity v to $v = 0$ (which can be associated with rotation during the first quarter of the rotational period); and further acceleration from $v = 0$ to $-v$ (rotation during the second quarter). Here we assume that the acceleration a is so large that the time of acceleration is negligible in comparison with travel time. During this rotation the spacecraft twin infers the presence of a constant gravitation field, increasing with increase of x. Then according to a well-known result of relativity, he observes a different rate of clocks for different points x: clocks tick faster with decrease of x. Hence, the clock $CL^E{}_0$ (located in the point $-x'_n$) ticks faster, than $CL^E{}_n$ (located in the point $x' = 0$), during acceleration of the spacecraft frame. As a result, an additional time difference between readings of these clocks appears due to deceleration. The calculations are very complex, because we have to take into account time-dependent scale contraction and time dilation effects. However, the final result can be easily predicted from physical reasoning: at the time instant when the momentary velocity of the spacecraft is equal to zero, the Earth frame 0 becomes an instantaneously co-moving inertial frame. Hence, all clocks, being synchronized in frame 0, are synchronized in frame 1, too. This means that at $v(t) = 0$ the clocks $CL^E{}_0$ and $CL^E{}_n$ show equal time in frame 1. Since $CL^E{}_n$ (t'^E_n) shows 4 years, then $CL^E{}_0$ also shows 4 years. Before the deceleration $CL^E{}_0$ (t'^E_0) indicated 1 month (see table). Therefore, we get a sudden change of Earth time for the spacecraft twin. This change is found as a difference of t'^E_n and t'^E_0 before the deceleration process:

$$\Delta t_a = t'^E_n - t'^E_0 = x_n/v - x_n/g^2 v = x_n v/c^2 . \quad (2)$$

It is interesting to note that a short-time deceleration process "causes" a large time change Δt_a, which does not depend on deceleration time. This result can be explained at a qualitative level for $v << c$ and $ax/c^2 << 1$, for which the calculations are greatly simplified. In this approximation a dependence of time on the space coordinate in a uniformly accelerated frame is written as [2]

$$t(x) \approx t(0)\left(1 - \frac{ax}{c^2}\right) \tag{3}$$

for any admissible definition of an accelerated frame. Then for the point $x = -x_n$ (in the adopted approximation x' can be taken to be equal to x in Eq. (3)), we derive

$$t(x) \approx t(0)\left(1 + \frac{ax_n}{c^2}\right) = t(0) + \frac{t(0)ax_n}{c^2}. \tag{4}$$

Taking $t(0)$ to be equal to the deceleration time, we get $at(0) = v$, and

$$t(x) = t(0) + \frac{vx_n}{c^2},\ \Delta t_a = t(x) - t(0) = \frac{vx_n}{c^2}, \tag{5}$$

which coincides with (2).

One can add that for the second stage of non-inertial motion of twin 1 (acceleration from $v = 0$ to $+v$), the value Δt_a increases two times, and at the end of this stage the traveler sees the Earth time 8 years.

Thus, in full agreement with [1] we conclude that there is no essential change of local time under acceleration of a twin; sudden changes occur only with remote clocks. This is a cornerstone of the relativistic explanation of the known forms of twin paradox, and many supporters of relativity theory are quite satisfied with this explanation. However, there is a very difficult logical point for relativity theory in this explanation. It is clear that there is no causal relation between acceleration of a spacecraft and sudden changes of time in remote clocks. Therefore, we have to conclude that such changes of time are "*apparent*" phenomena for the spacecraft twin. This is especially clearly seen for rotation of spacecraft around the star: the Earth time cannot physically change by 8 years and oscillate within this range during continuous orbiting of the spacecraft.

However, special relativity does not admit "apparent" phenomena in space-time: all that we measure is supposed to be real. This conclusion follows from Einstein's postulates. Indeed, an equivalence of all inertial reference frames and constancy of light velocity mean that the geometry of empty space-time in all inertial reference frames is pseudo-Euclidean with Galilean metrics in Cartesian coordinates. (The Galilean metric tensor $\mathbf{g_G}$ has the form: $g_{00} = 1$, $g_{11} = g_{22} = g_{33} = -1$, all other $g_{ik} = 0$). This and only this geometry has an exclusive property: the measured space and time coordinates give their physical magnitudes directly. Hence, according to special relativity, there are no "apparent" phenomena in an empty space-time, and the oscillation of Earth time within 8 years during orbit of spacecraft around a star is a real effect. We may certainly state that this is not physically correct. At the same time, the lack of causal relationship between the Earth's and star's events does not allow us to derive mathematically inconsistent results in consideration of the twin paradox.

The problem becomes even more complicated when we postulate a probabilistic nature of the behavior of the twins. This variety of twin paradox is considered in the next Section.

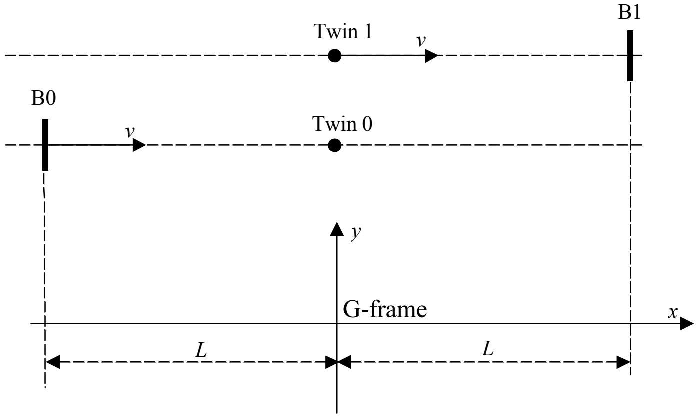


Fig. 4. The twin 1 moves toward the barrier B1, and the barrier B0 moves toward twin 0 in the "global" inertial frame G.

3. Butterflies paradox

Consider the following *gedanken* experiment. Let the twin 0 be at rest in some "global" inertial frame G, and the twin 1 move at the constant velocity v along the x axis. Both twins are inside their own identical spacecrafts. At $t = 0$ both of them have the space coordinate $x = 0$. At the points $x = \pm L$ of frame G (which initially coincides with the frame of twin 0) let there be two identical special barriers (Fig. 4), which have the following property: with the probability ½ the spacecraft passes across a barrier without interaction, and with the probability ½ the spacecraft collides with the barrier and suddenly loses its velocity. It is always a pleasure to conduct *gedanken* experiments, where we may dispense with technical details. Nevertheless, in our case we can imagine the barriers in Fig. 5 as two rotating very large half-circles with mass to be equal to the mass of spacecraft. The right barrier B1 is at rest in G, while the left barrier moves at the same constant

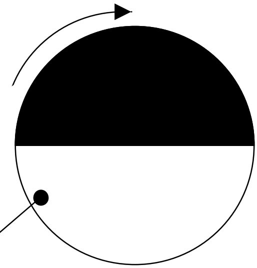


Fig. 5. Imaginary construction of barrier. We assume that a collision of spacecraft with the barrier is fully elastic.

velocity v along the axis x toward twin 0. We again assume that $v = 0.99c$, and $g = 7$. Let L/v be equal to 7 hours, and let there be a butterfly in each spacecraft that lives 20 hours. Our problem is to determine the state of each butterfly (either it is alive, or has died of old age) at the instant when each spacecraft meets its own barrier. The problem can be solved for all available observers: twin 0, twin 1, and observer in G-frame. We also assume that each twin reports to the other participants of this story about the state of his butterfly.

The problem is easily solved for the G-frame. Each spacecraft meets its own barrier at the moment $t_G = L/v = 7$ hours; the same moment is fixed by twin 0. Thus, after meeting with the barrier B0, twin 0 sends a report to the other observers that his butterfly is alive. This result does not depend on whether he collides or passes across B0: even if he collides, nothing changes in the local time of twin 0. The elapsed time in the spacecraft of twin 1 at the moment of his meeting with B1 is $L/gv = 1$ hour, and this twin also reports that his butterfly is living, regardless of a result of his interaction with B1.

Let us consider the same problem in frame 0. Until a meeting with B0, the frame 0 coincides with G-frame, and both twins interact with their barriers at $t_G = L/v = 7$ hours. The elapsed time in the spacecraft of twin 1 at the moment of interaction with B1 is $L/gv = 1$ hour. Further, let us imagine that both twins passed across their barriers without interaction. (The probability of this event is ¼). Then each twin reports that his butterfly flies after passing the barriers. Now let us consider another situation, which is also realized with the probability ¼: twin 1 passes across B1 without collision, while twin 0 collides with B0 and suddenly acquires a constant velocity v in G-frame. The acceleration process changes nothing in his local time, and twin 0 reports to both other observers that his butterfly is living. At the same time, he reveals that his frame 0 had acceleration, while the frame 1 was always inertial. Hence, in accordance with a well known result of relativity, he concludes that his own clock ticked slower than the clock in spacecraft 1. Since his clock indicates 7 hours, the clock of twin 1 should indicate g times more, *i.e.*, 49 hours. One can show that this result can be directly derived from Eqs. (2)-(5). This time essentially exceeds the lifetime of the butterfly (20 hours). Hence, twin 0 concludes that the butterfly in spacecraft 1 has died.

Thus, if twin 0 does not collide with B0, he expects that the butterfly in spacecraft 1 is living; if he collides with B0, he infers that butterfly 1 has died many hours ago. The most interesting point is that just before the collision, twin 0 does not know what to think: in fact, he rolls the dice in order to decide whether the butterfly 1 is alive or dead. We notice that the second conclusion contradicts the factual observation that was made in the G-frame, where both butterflies live after collision. Here we meet a contradiction with causality.

Now we omit a consideration of this problem for other motional trajectories of twins 0 and 1, depending on a result of their interaction with B0 and B1, as well as an analysis of the butterfly paradox in frame 1: again there appear situations that contradict one another. (It should be borne in mind that in frame 1 the interactions with B0 and B1 are not simultaneous; in addition, the distance to B0 at the initial moment is L/g). So, it seems that we have to ignore all these solutions, adopting the optimistic (for butterflies) observation in G-frame: both butterflies are alive after the spacecraft interact with the barriers. A reason to accept this resolution of the problem is an absence of any absolute events in the G-frame. Hence that frame can be taken as preferred for the problem considered (and "G" can be transcribed as God). However, it contradicts relativity. We again turn in circles.

4. Twin paradox in the Lorentz ether theory

Strictly speaking, the title of this Section is not correct: Lorentz ether theory does not know any paradoxes of relative motion. It postulates the presence of a preferred (absolute) reference frame, wherein all observations are true by definition. The author of [1] states that the local gravity field serves as the "preferred frame" of LET. This assumption, without change in the mathematical structure of the theory, constitutes "Lorentzian relativity".

Now we will show that the twin paradox can be also resolved in an empty space, under direct application of the LET postulates without specification of a physical model of ether†.

† Let us recall the postulates of LET in its modern form:

1) There is an "absolute" reference frame K_0, wherein light velocity is isotropic and equal to c. **2)** In an arbitrary reference frame K, moving at constant velocity $\vec{v}$ in K_0, the velocity of light is equal to $\vec{c}\,'=\vec{c}-\vec{v}$. **3)** In this

Let us imagine that both twins use so-called light clocks. We recall that the latter is composed of two faced mirrors, and a light ray reflects infinitely between the mirrors along their common normal. Time of light propagation from one mirror to another and back is taken as the unit time.

In the absolute reference frame ?$_0$ let two identical light clocks be at rest. The distance from one mirror (M1) to another (M2) is equal to L. The axes of clocks (normal to M1, M2) are parallel to the axis y. Time of light propagation from one mirror to another and back is $t_0 = 2L/c$. Further, let the first clock (Cl_0) remain at rest together with twin 0, while the second clock Cl_1 together with twin 1 acquires a constant velocity $-v$ along the x axis. Then one can see that the time of light propagation from $M1_0$ to $M2_0$ and back (in Cl_0) in the frame K_0 is equal to $2L/c$ (Fig. 6,a, left side), while the time of light propagation from $M1_1$ to $M2_1$ and back (in Cl_1) becomes equal to

$$t_1 = 2L\Big/c\sqrt{1-v^2/c^2}\,, \tag{6}$$

(Fig. 6,a, right side), *i.e.*, the rate of the moving clock slows down by g. Now let us go to the inertial frame K_1 of twin 1, which is attached to the moving clock Cl_1. In this frame a physical light velocity c_{ph} is determined by the Galilean law of speed composition, in accordance with the second LET postulate, Hence, the light ray, propagating from $M1_0$ to $M2_0$ (in clock Cl_0), has the following projections upon the coordinate axes of K_1: $\left(c_{\text{ph}}\right)_x = v$; $\left(c_{\text{ph}}\right)_y = c$. A modulus of light velocity is $c_{\text{ph}} = \sqrt{c^2 + v^2}$ (see Fig. 6,b, right side). Then the propagation time of light from $M1_0$ to $M2_0$ and back is

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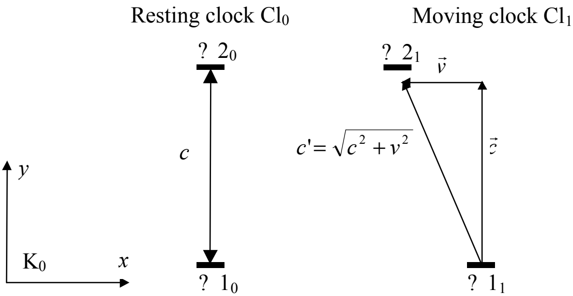
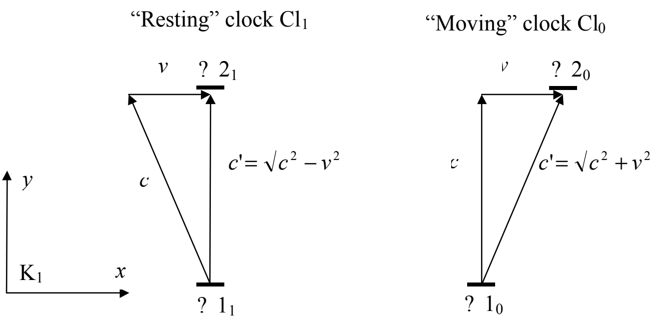
reference frame K a linear scale is contracted by $\sqrt{1-v^2/c^2}$ along the vector $\vec{v}$.

4) In this reference frame K time is dilated by $\sqrt{1-v^2/c^2}$.



a: what the twin 0 does see



b: what the twin 1 does see

Fig. 6. Diagram of light propagation in light clocks Cl_0 and Cl_1 for twin 0 (a) and twin 1 (b). Due to Galilean law of speed composition, both twins fix the identical true rates of Cl_0 and Cl_1 in their reference frames: dilation of time is absolute.

$2L/\left(c_{\mathrm{ph}}\right)_y = t_0$, *i.e.*, the true (physical) rate of Cl_0 for twin 1 does not depend on the velocity of K_1 in K_0.

To determine the rate of clock Cl_1, twin 1 should also apply the Galilean law of speed composition. The vectors $\vec{c}$ and $\vec{v}$ are added so as to give a resultant vector $\vec{c}\,'$ along the y axis (Fig. 6,b, left side). Since $\vec{c}\,'$ and $\vec{v}$ are mutually perpendicular, it follows that $c' = \sqrt{c^2 - v^2}$. From this the time of light propagation from $M1_1$ to $M2_1$ and back in the frame K_1 is

$$t'_1 = 2L\big/c\sqrt{1 - v^2/c^2}\,, \tag{7}$$

i.e., the clock Cl_1 slows down its true rate in both K_0 and K_1 inertial frames.

As a result, both twins get an identical true rate of each light clock. We can add that for twin 0 the corresponding time intervals, measured experimentally, are directly described by Eqs. (6), (7). But what will twin 1 measure in his moving frame? We assert that the results of his measurements differ from Eqs. (6), (7). In order to explain this assertion, let us, first of all, define a model of inertial reference frame wherein a measurement of space and time intervals is carried out. Let us take the most popular Einstein model: the use of a standard scale for measurement of lengths, a standard clock for measurement of time and exchange of light signals between distant clocks for their synchronization (Einstein method). We further assume that both inertial frames K_0 and K_1 are equipped with the same measuring instruments, and we look for the results of measurements in moving frame K_1. Due to the third LET postulate, the unit scales of twin 1 are contracted g times in comparison with their values for $v = 0$; the time of standard clocks is dilated by g times, and neither effect is detectable in K_1. In addition, the Einstein synchronization of two distant clocks in K_1 does not assure their instantaneous true readings will be equal: due to the light velocity anisotropy, two clocks Cl_0 and Cl_1, lying along the x axis at separation L/g show a difference of their instantaneous readings

$$\Delta t_s = \frac{1}{2g}\left(\frac{L}{c-v} - \frac{L}{c+v}\right) = \frac{Lv}{g\left(c^2 - v^2\right)}. \tag{8}$$

Keeping in mind Eq. (8), let us determine a rate of light clock Cl_0 for twin 1. At the instant $t_{ph} = 0$ let a light signal in Cl_0 be emitted from $M1_0$ to $M2_0$. We call x_{ph} the coordinate of $M1_0$ along the axis at $t_{ph} = 0$. A standard clock, recording this instant, is also at the point x_{ph}. At the time moment Δt_{ph} the light signal returns to $M1_0$, which has moved through the distance $\Delta x_{ph} = v\Delta t_{ph}$. This instant is recorded by a second standard clock placed at the point $x_{ph}+\Delta x_{ph}$. True (physical) time of light propagation from $M1_0$ to $M2_0$ and back, Δt_{ph}, as shown above, equals to $2L/c$ in the frame K_1. However, now we should take into account an error of standard clock synchronization according to eq. (8), where L/g should be replaced by $\Delta x_{ph} = v\Delta t_{ph} = 2vL/c$. Hence, we obtain

$$\Delta t_s = \frac{2L}{c}\frac{v^2}{c^2 - v^2}.$$

That is why a difference of indications of light clocks (fixing the instants of departure and arrival of light pulse at $M1_0$) at the points x_{ph} and $x_{ph}+\Delta x_{ph}$ is

$$\Delta t = \Delta t_{ph} + \Delta t_s = \left(\frac{2L}{c} + \frac{2L}{c}\frac{v^2}{c^2 - v^2}\right) = \frac{2L}{c\left(1 - v^2/c^2\right)}.$$

This time interval is measured by a standard clock, which is slowed down by g times. Therefore, a result of measurement of Δt with such a standard clock is g times less:

$$\Delta t_{\text{ex}} = \Delta t \sqrt{1 - \frac{v^2}{c^2}} = 2L/c\sqrt{1 - \left(v^2/c^2\right)} \tag{9}$$

Such is the rate of CL_0 for twin 1, obtained by means of his instruments. We see that twin 1 records a dilation of time in the "moving" frame K_0, and that Δt_{ex} is exactly equal to the value derived from the Lorentz transformations.

Now let us determine the result of twin 1's time measurement by his own clock Cl_1. According to Eq. (7), Cl_1 slows down by g times in the frame K_1. Here we remember that the rate of this clock is compared with the rate of a standard clock in K_1. However, the standard clock time is dilated g times, in accordance with the fourth LET postulate. Therefore, a time interval measured by this standard clock should be g times smaller, *i.e.*, $2L/c$. As a result, the absolute dilation of proper time in the frame K_1 is experimentally not observable, and twin 1 detects no dependence of clock rate on the absolute velocity of his frame.

Thus, when he measures the rates of clocks Cl_0 and Cl_1, twin 1 concludes that his own clock ticks faster than Cl_0, although the physical rate of Cl_1 is g times slower than Cl_0. This explains the illusory relativity of the time dilation effect in LET.

It is quite interesting to notice that such an illusory relativity of the time dilation effect, described by the Lorentz transformations, is not an exclusive property of LET. The same result is obtained in a wide class of ether theories, which adopt a pseudo-Euclidean geometry of ether with Galilean metrics. This problem is considered in the next subsection.

4.1. Mathematization of LET

We now emphasize an important result obtained above: in a moving inertial reference frame, given acceptance of LET postulates, we have to distinguish the physical (true) time intervals and the time intervals obtained *via* measurements. (In fact, this means rejecting the relativity postulates: see the end of Section 2). We can generalize this conclusion, and introduce the physical (true) four-vectors x_{ph}^{i} $(i = 0\ldots3)$ and "measured" four-vectors x_{ex}^{i} in an arbitrary inertial reference frame. This can be done not only for LET, but for any other ether theory that adopts a dependence of physical space and time intervals on absolute velocity. Next, in ether theories, assuming a Galilean metric of absolute space, we formally introduce the Minkowskian four-vectors x_{L} (without determination of their physical meaning), which are subject to the Lorentz transformation **L**:

$$\left(x_{\mathrm{L}}\right)_i = L_{ij}\left(x'_{\mathrm{L}}\right)^j, \tag{10}$$

Since motion of an arbitrary inertial frame does not change the geometry of absolute space, it continues to be pseudo-Euclidean for any moving inertial observer. However, due to possible dependence of space and time intervals on the absolute velocity, the metric tensor **g** in a moving frame is no longer Galilean. This means that physical four-vectors in arbitrary inertial frames should be linear functions of Minkowskian four-vectors:

$$\left(x_{\mathrm{ph}}\right)_i = B_{ij}\left(x_{\mathrm{L}}\right)^j, \tag{11}$$

where the coefficients B_{ij} do not depend on space-time coordinates of a moving inertial frame; they only depend on its absolute velocity $\vec{v}$. (This kind of pseudo-Euclidean geometry has so-called oblique-angled metrics). It is essential that for $v = 0$, the matrix **B** reduces to the unit matrix, and

$$\left(x_{\mathrm{ph}}\right)_i (v = 0) = \left(x_{\mathrm{L}}\right)_i. \tag{12}$$

Eq. (11) can then be rewritten in the form

$$\left(x_{\mathrm{ph}}\right)_i(\vec{v}) = B_{ij}(\vec{v})\left(x_{\mathrm{ph}}\right)^j(v=0), \tag{13}$$

which clearly indicates that the matrix **B** describes a dependence of physical space and time intervals in a moving inertial frame on absolute velocity $\vec{v}$.

Transformation (11) allows one to write a relationship between time components of the four-vectors x_{ph} and x_{L}:

$$\left(x_{\mathrm{ph}}\right)_0 = B_{00}\left(x_{\mathrm{L}}\right)^0 + B_{0a}\left(x_{\mathrm{L}}\right)^a \tag{14}$$

($a = 1..3$). For two events at a fixed spatial point ($\left(x_{\mathrm{ph}}\right)^a = 0$)

$$\left(x_{\mathrm{ph}}\right)_0(\vec{v}) = B_{00}\left(x_{\mathrm{L}}\right)^0 = B_{00}\left(x_{\mathrm{ph}}\right)_i(v=0). \tag{15}$$

Hence, the coefficient B_{00} describes the change of clock rate at a fixed spatial point in motion at the constant absolute velocity $\vec{v}$. The change takes place for both standard and physical time intervals. Therefore, the measured time interval at a fixed spatial point is

$$\left(x_{\mathrm{ex}}\right)_0 = \frac{1}{B_{00}}\left(x_{\mathrm{ph}}\right)^0. \tag{16}$$

For time intervals at two different spatial points, separated by the distance $\left(x_{\mathrm{ph}}\right)_a$, we write

$$\left(x_{\mathrm{ex}}\right)_0 = \frac{1}{B_{00}}\left[\left(x_{\mathrm{ph}}\right)_0 + \Delta\left(x_{\mathrm{ph}}\right)_0\right]. \tag{17}$$

where $\Delta\left(x_{\mathrm{ph}}\right)_0$ is the error of synchronization of clocks separated by the distance $\left(x_{\mathrm{ph}}\right)_a$ in oblique-angled space-time. The value of $\Delta\left(x_{\mathrm{ph}}\right)_0$ can be found from the equality

$$\left(x_{\mathrm{ph}}\right)_{02}=\left(x_{\mathrm{ph}}\right)_{01}/2 \tag{18}$$

(Einstein's method of clock synchronization), where $\left(x_{\mathrm{ph}}\right)_{01}$ stands for the time for light propagation from the first clock Cl_1 (at the origin of coordinates) to the second clock Cl_2 (at the point $\left(x_{\mathrm{ph}}\right)_a$) and back according to Cl_1, while $\left(x_{\mathrm{ph}}\right)_{02}$ is the reading of Cl_2 at the moment of arrival of the light pulse. For oblique-angled space-time the propagation time of light from Cl_1 to Cl_2 $\left(x_{\mathrm{ph}}\right)_{0+}$ is not equal, in general, to the propagation time in the reverse direction $\left(x_{\mathrm{ph}}\right)_{0-}$. Hence, implementation of the equality (18) is possible only in the case where the readings of both clocks at the initial moment of time differ by the value $\Delta\left(x_{\mathrm{ph}}\right)_0$, and

$$\left(x_{\mathrm{ph}}\right)_{01}=\frac{1}{2}\left[\left(x_{\mathrm{ph}}\right)_{0+}+\left(x_{\mathrm{ph}}\right)_{0-}\right],$$

$$\left(x_{\mathrm{ph}}\right)_{02}=\frac{1}{2}\left[\left(x_{\mathrm{ph}}\right)_{0+}+\Delta\left(x_{\mathrm{ph}}\right)_{0}\right]. \tag{19}$$

Therefore, using Eq. (18), we obtain:

$$\Delta\left(x_{\mathrm{ph}}\right)_0=\frac{1}{2}\left[\left(x_{\mathrm{ph}}\right)_{0-}+\left(x_{\mathrm{ph}}\right)_{0+}\right]. \tag{20}$$

Expressions for $(x_{\mathrm{ph}})_{0+}$ and $(x_{\mathrm{ph}})_{0-}$ can be found from Eq. (11):

$$\left(x_{\mathrm{ph}}\right)_{0+} = B_{00}\left(x_{\mathrm{L}}\right)^{0} + B_{0a}\left(x_{\mathrm{L}}\right)^{a},$$
$$\left(x_{\mathrm{ph}}\right)_{0-} = B_{00}\left(x_{\mathrm{L}}\right)^{0} - B_{0a}\left(x_{\mathrm{L}}\right)^{a}. \tag{21}$$

Substituting Eq. (21) into Eq. (20), we get:

$$\Delta\left(x_{\mathrm{ph}}\right)_{0} = -B_{0a}\left(x_{\mathrm{L}}\right)^{a}. \tag{22}$$

(This equation represents a generalization of Eq. (8)). Further substitution of Eqs. (22) and (14) into Eq. (17) gives:

$$\left(x_{\mathrm{ex}}\right)_{0} = \left(x_{\mathrm{L}}\right)_{0}. \tag{23}$$

Thus, we have derived a remarkable result: for any ether theory adopting Galilean metrics of absolute space, the measured time intervals always obey the Lorentz transformations. Consequently, the twin paradox now loses all physical meaning: the time dilation effect is absolute, but due to a strange property of Nature, each twin ascribes a relative dilation of time to the other's clock. And this conclusion is true not only for LET, but for an infinite number of ether space-time theories!

Looking at Eq. (23), we may ask the following interesting question: is this equality valid for space intervals, too? In other words, do we get the equality

$$\left(x_{\mathrm{ex}}\right)^{a} = \left(x_{\mathrm{L}}\right)^{a} \tag{24}$$

for an arbitrary matrix **B** in Eq. (11)? In general, this is not the case. We can show (see, *e.g.* [3-5]) that Eq. (24) is realized in the case where the coefficients $B_{a0} = 0$. Then Eqs. (23), (24) can be written simultaneously as

$$\left(x_{\mathrm{ex}}\right)^{i}=\left(x_{\mathrm{L}}\right)^{i}, \tag{25}$$

($i=0\ldots3$), which means that departures of oblique-angled metrics in moving inertial frames from Galilean metrics are not experimentally observable. In other words, an observer in any inertial frame moving in the absolute space sees the world as in SRT, for an infinite set of ether space-time theories. However, this does not yet mean that SRT and all ether theories cannot be distinguished experimentally (see, *e.g.* [6]). This problem falls outside the scope of the present paper.

Now let us determine a physical meaning of ether theories with $B_{a0}=0$ in Eq. (11). For such theories, due to the equality (25), a transformation for measured space-time coordinates can be written as

$$\left(x_{\mathrm{ex}}\right)_{i}=L_{ij}\left(x'_{\mathrm{ex}}\right)^{j}. \tag{26}$$

(In this subsection the primed four-vectors belong to the absolute frame K_0). Further, let us assume that true (physical) four-vectors in the absolute frame K_0 and in an arbitrary inertial frame K are related by some admissible linear transformation **A**:

$$\left(x_{\mathrm{ph}}\right)_{i}=A_{ij}(\vec{v})\left(x'_{\mathrm{ph}}\right)^{j}. \tag{27}$$

Here we exclude the trivial rotations and translations of space, so that the matrix **A** depends only on the absolute velocity $\vec{v}$ of the frame K. Taking into account that in the absolute frame K_0

$$x'_{\mathrm{ph}}=x'_{\mathrm{ex}}=x'_{\mathrm{L}}, \tag{28}$$

we obtain from Eqs. (26)-(28) a relationship between four-vectors x_{ph} and x_{L} in an arbitrary inertial frame K:

$$\left(x_{\mathrm{ph}}\right)_{i}=A_{i}^{k}(\vec{v})L_{kj}^{-1}(\vec{v})\left(x_{\mathrm{L}}\right)^{j}. \tag{29}$$

Comparing Eq. (29) with Eq. (11), we find

$$B_{ij}(\vec{v}) = A_i^k(\vec{v}) L_{kj}^{-1}(\vec{v}). \tag{30}$$

Further, assuming validity of a reciprocity principle:

$$\mathbf{A}^{-1}(-\vec{v}\,) = \mathbf{A}(\vec{v}\,), \tag{31}$$

and substituting into Eq. (30) the known form of the matrix **L** (see, *e.g*. [2]), we get:

$$B_{00} = g / A_{00}\,, \tag{32a}$$

$$B_{a0} = 0\,, \tag{32b}$$

$$B_{0a} = A_{0a} + A_{00} \times [\frac{v_a}{c^2} g + (\frac{1}{A_{00}^2} - 1) \frac{v_a}{v^2} (g-1)]\,, \tag{32c}$$

$$B_{ab} = A_{ab} + A_{a0} \frac{v_b}{v^2} (1 - \frac{1}{g})\,. \tag{32d}$$

We see that the coefficient B_{a0} is equal to zero for any matrix **A**. We can show that adoption of the reciprocity principle (31) is essential for vanishing B_{a0}.

Thus, we conclude that the observable world for each inertially moving observer looks as in SRT (Eq. (25)) in any ether theory that postulates a Galilean metric of an absolute space and the reciprocity principle. These two postulates, being essentially more general than SRT postulates, do not allow determination of the matrix of the physical space-time transformation **A** in closed form. We may only investigate the properties of physical space-time for this or that particular choice of the matrix **A**. For example, let us make the simplest choice:

$$\mathbf{A} = \mathbf{G},$$

where **G** is the Galilean matrix: $G_{ii} = 1$, $G_{a0} = -v_a$, and all other $G_{ij} = 0$. Substituting matrix **G** in place of matrix **A** in Eqs. (32), we find the following coefficients of matrix **B**:

$$B_{00} = g\,, B_{a\,0} = 0, B_{0a} = \frac{v_a}{c^2} g\,, B_{ab} = d_{ab} \frac{v_a v_b}{v^2}\left(1 - \frac{1}{g}\right), \tag{33}$$

where d_{ab} is the Kronecker symbol. Further substitution of Eq. (33) into Eq. (13) allows us to determine a dependence of physical space-time four-vectors on the absolute velocity $\vec{v}$ of an arbitrary inertial reference frame K:

$$\vec{r}_{\text{ph}}(\vec{v}) = \vec{r}_{\text{ph}}(v=0) + \frac{\vec{v}\left(\vec{r}_{\text{ph}}(v=0), \vec{v}\right)}{v^2}\left[\sqrt{1-(v^2/c^2)} - 1\right], \tag{34}$$

$$t_{\text{ph}}(\vec{v}) = \frac{t_{\text{ph}}(v=0)}{\sqrt{1-(v^2/c^2)}} + \frac{\vec{r}_{\text{ph}}(v=0)\vec{v}}{c^2\sqrt{1-(v^2/c^2)}}\,. \tag{35}$$

For the time intervals at a fixed spatial point of the frame K ($r_{\text{ph}} = 0$), we obtain the dependence of t_{ph} on $\vec{v}$ [see, Eq. (35)]:

$$t_{\text{ph}}(\vec{v}) = t_{\text{ph}}(v=0) / \sqrt{1-(v^2/c^2)}, \tag{36}$$

which represents a mathematical expression of the fourth postulate of LET: an absolute dilation of physical time by the factor *g*. Furthermore, we obtain from Eq. (34):

$$\begin{gathered}\left(\vec{r}_{\text{ph}}(\vec{v}), \vec{v}\right) = \left(\vec{r}_{\text{ph}}(v=0), \vec{v}\right)\sqrt{1-v^2/c^2}, \\ \left[\vec{r}_{\text{ph}}(\vec{v}) \times \vec{v}\right] = \left[\vec{r}_{\text{ph}}(v=0) \times \vec{v}\right],\end{gathered} \tag{37}$$

which represents a mathematical expression of the third postulate of LET: an absolute contraction of moving physical scale along the absolute velocity vector (Fitzgerald-Lorentz hypothesis). Finally, transformation (27) under **A** = **G** leads to the second LET postulate: the Galilean law of speed addition for physical light velocity c_{ph}. Thus, we have verified the full set of the Lorentz ether postulates in physical space-time for **A** = **G**.

In fact, we reveal the physical meaning of LET: it responds to the Galilean transformation in physical space-time‡. The postulates of LET, which appeared artificial to many physicists for a century, now represent a direct consequence of the Galilean transforms under a natural assumption about Galilean metrics in an absolute space. The simplicity of the Galilean transformations assigns an exclusive place to LET among other admissible ether theories and leads us to believe that this theory alone describes nature.

5. Conclusions

1. Application of the standard formalism of special relativity to the twin paradox in the presentation by Van Flandern repeats his results. There are no rapid changes in local time of an accelerated clock; sudden changes happen with remote clocks. This creates paradoxical situations upon detailed analysis of the twin paradox.
2. A variety of twin paradox, where the motional trajectories of the twins have a probabilistic nature ("butterflies paradox"), shows that it is impossible to explain under equivalence of all inertial reference frames.
3. Application of the LET postulates to analysis of the rate of a light clock in empty space indicates that in ether theories one

‡ An essential difference of LET from classical Newtonian physics is the limited velocity of light, which forbids "non-admissible" metric coefficients in oblique-angled space-time, whereas LET admits the velocities higher than c [4].

should distinguish between true (physical) values and the values obtained *via* measurements. This approach reveals that a dilation of true time is absolute; dilation of "measured" ("illusory") time is relative. This fully explains all possible observations in the twin paradox.

4. In any ether theory that adopts the Galilean metrics for absolute space and the reciprocity principle, the observable world looks as in SRT: the "measured" space and time intervals obey the Lorentz transformations. The exclusive place of LET among an infinite number of such ether theories is defined by a choice of the Galilean transformations for physical space-time four-vectors, the simplest kind of non-trivial transformations in Nature. The Lorentz ether postulates are not independent artificial assumptions, they are derived from the Galilean transformations, keeping Galilean metrics of the absolute space. In this theory the twin problem loses its paradoxical nature.

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