Remarks on the Causality Principle
(comment on a previous paper)

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This paper returns to the problem [1] about two light pulses propagating across an accelerated infinite chain of re-emitters (point absorber-re-emitters) of light, where both non-inertial and inertial observers look for possible intersection of these light pulses. It is shown that the author's previous conclusion about violation of the causality principle in this problem contained a mistake. In this connection the present paper analyses a simplified version of the problem, where two reflectors of light substitute for an infinite chain of reflectors. The compatibility of causality and relativity principles is derived.

Keywords: causality principle, emission/absorption of light, transformations between inertial and non-inertial reference frames

Introduction

The paper [1] of the author, claiming a finding of violation of causality in relativity theory, was published two years ago. Later, due to activity of one of the Editors of Apeiron, a widespread discussion was carried out concerning the physical problem to be found in the paper: propagation of two short light pulses across an infinite chain of uniformly accelerated re-emitters of light. This discussion showed that the problem was correctly solved by the author in the accelerated frame (so that the comment by Vladimir Onoochin [2] was erroneous), while the author's consideration of the problem in an inertial reference frame contained a mistake. The obligation of the author is to recognize the mistake and to express gratitude to Prof. Dvoeglazov for organization of the discussion, as well as to the reviewer who found the mistake. Section 2 reproduces a description of the physical problem from [1] and indicates the mistake in its analysis. Simultaneously it shows that the revealed error still does not resolve a problem about violation of causality in relativity theory, as the author sees this problem: namely, possible violation of causality in the processes of emission/absorption of light, where the corresponding world lines have fracture points (discontinuous slopes). The implemented analysis just indicates that the particular example chosen by the author for demonstration of his idea was wrong. Section 3 considers a simplified version of the problem [1], where only two re-emitters of light replace the infinite chain of re-emitters. This allows a complete quantitative analysis. The result of calculations shows the validity of causality in relativity theory in the processes of emission/absorption of light.

Propagation of two light pulse across an infinite chain of re-emitters of light in a rigid non-inertial reference frame and the causality principle

Let us recall the problem considered in [1].

Imagine a rigid non-inertial reference frame, moving at the constant (in relativistic meaning) acceleration a along the axis x. The rigid frame is defined by the relationships [3]

$$x^a = x'^a; t' = 0; t = t, \tag{1}$$

where primed space and time coordinates belong to successive instantaneously co-moving inertial reference frames, while t stands for the proper time at the origin of coordinates.

Let a short light pulse be emitted from the point $x = 0$ along the axis x of this frame. Let a number of point re-emitters of light RL_m be located along the x-axis at points x_m (RL_0 is located at the point $x = 0$ and, for simplicity, all $\Delta x_m = x_{m+1} - x_m$ are equal to each other). When a light pulse arrives at each re-emitter, it is absorbed by it, and after a fixed interval of its proper time Δt_0 is emitted by RL along the x-axis again.

Further, let the second light pulse be emitted from the point $x = 0$ at a moment of time (taken as $t = 0$), when the first light pulse has a coordinate

$$0 < \Delta x \le x_1. \tag{2}$$

One requires to find the times t_1 and t_2, where t_1 is the moment of time when the first (right) light pulse is emitted by RL_n, while t_2 is the moment of time when the second (left) pulse is reaching RL_n, and n is some number.

It has been shown in the paper [1] that for a spatial point x_n, defined by the equation

$$x_n = -\frac{\Delta t c^2}{a(\Delta t_0 + \Delta t)} \tag{3}$$

the values t_1 and t_2 are equal to each other. (Here $\Delta t = t'_{x_1-0} - t_{x_1-\Delta x}$, where t'_{x_1-0} is the propagation time of the second (left) light pulse from the point $x = 0$ to point x_1, while $t_{x_1-\Delta x}$ is the propagation time of the first (right) light pulse from the point Δx to point x_1. The sign of the acceleration is negative). The equality $t_1 = t_2$ means that an observer in this accelerated frame detects an absolute event: a meeting of both light pulses considered at the spatial point x_n. As an example, Fig. 1 shows a meeting of the right and left light pulses at the point x_n for $n = 4$. One can see from Fig. 1 that a physical reason, which makes possible an intersection of the light pulses at the point x_n, is the different rate of clocks at different points x.

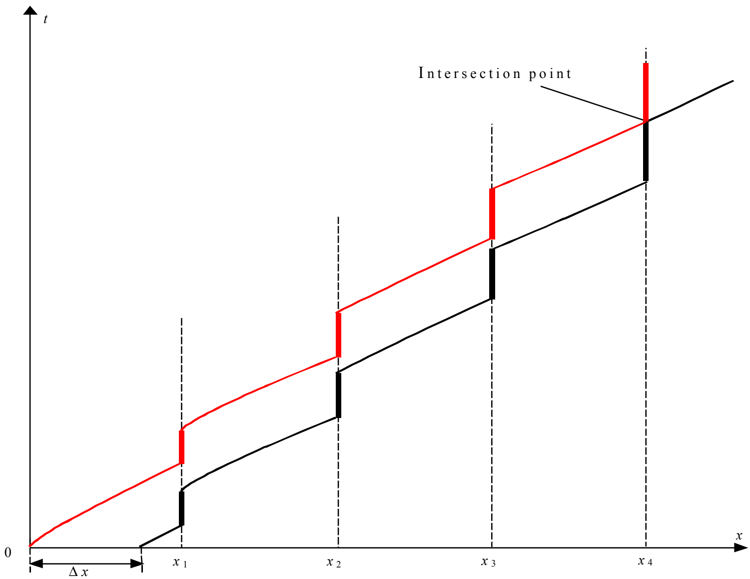

Fig. 1. An intersection of the right and left light pulses at the point x_n for $n = 4$.

On the other hand, the author in ref. [1] claimed that these light pulses would never meet for any external inertial observer. It would mean a violation of causality in relativity theory. In order to prove this assertion in a general form (particular calculations were very complex in an inertial frame), the author assumed that the chain of re-emitters is infinitely long along the axis *x*. Further, let us choose for observing the propagation process of the light pulses an inertial frame K, such that, at the moment when an observer sees the appearance of the left light pulse at the point $x = 0$, he simultaneously sees the right pulse arriving at RL_1. For this time moment we introduced into consideration the second inertial frame K_s shifted along the axis *x* at such a distance (with respect to K) which is equal to the distance between RL_0 and RL_1. (The relative velocity of K and K_s is equal to zero). Due to the space homogeneity in inertial frames, such a shift for an infinitely long chain of re-emitter's seems equivalent to a re-numbering of the re-emitters in K_s: the RL_m (in K) becomes RL_{m-1} (in K_s). Hence, the propagation time from RL_0 to RL_{n-1} for the left pulse seems to be exactly equal to the propagation time from RL_1 to RL_n for the right pulse in both K and K_s frames (since the RL_1, RL_n in K are the RL_0, RL_{n-1} in K_s). Hence, at the moment of time (in K) when RL_n emits the right pulse, the left one is emitted by RL_{n-1} for any n. Therefore, these light pulses will never meet in the inertial frame K.

This problem was discussed for a long time, and finally it was found that the inertial frames K and K_s are not equivalent to each other, if one considers a finite-length chain of re-emitters (an infinitely long chain cannot exist in nature). However, each finite chain has its origin and end, and displacement of an inertial observer along the axis x changes their coordinates. Hence, despite the homogeneity of space in inertial reference frames, two spatially shifted inertial observers are nevertheless not equivalent to each other with respect to the considered physical problem. Thus, we have to analyse this problem (propagation of two light pulses across the chain of accelerated re-emitters) for a single inertial observer. For such an observer, in particular, the momentary velocities of each re-emitter are different due to the time-dependent scale contraction effect for the moving chain. For this reason ($v = v(t)$), the time dilation effect also depends on x. This means that the rate of clocks at different points x is also different. Hence, the two light pulses considered may meet in the inertial frame, too. This makes improvable the statement of the author about possible violation of causality in the problem considered.

Now it is appropriate to explain why the author believed that in such of physical problems, dealing with emission/absorption of light, a violation of causality nevertheless might be expected.

In general, the causality principle (CP) implies two fundamental requirements:

1. The cause-consequence order of events is absolute.
2. Events, which can cause essential inferences (for example, collision of particles), are absolute.

It is known that the finiteness of light velocity provides conformity of the relativity theory with the first requirement of CP. The second requirement of CP is taken into account by a choice of homogeneous admissible space-time transformations for increments of space-time four-vectors. Indeed, the event of collision of two particles (or intersection of two light rays) corresponds to the equality Δt, $\Delta r = 0$, and for homogeneous transformations we get $\Delta t'$, $\Delta r' = 0$, too. Here t, r and t', r' belong to two different reference frames. At the same time, when the space-time coordinates of two particles (or any other point objects) are close to each other, a possibility of intersection (or non-intersection) of their world lines can be expressed locally in terms of relationships between magnitudes and time derivatives of the functions describing these world lines. It is clear that in a physically correct theory the conditions of intersection/non-intersection of the world lines should be the same for observers in any frame of references.

Let us formulate such conditions for a one-dimensional case, using further in this section units with $c = 1$. Let there be two world lines $x^{(1)}(t)$ and $x^{(2)}(t)$ in an empty space time of a reference frame (either inertial or non-inertial). Within some short time interval $\{t_0, t_0+\Delta t\}$ the values $x^{(1)}$ and $x^{(2)}$ are close to each other, and, for example, $x^{(2)}(t_0) > x^{(1)}(t_0)$. Then we may certainly assert that the functions $x^{(1)}(t)$ and $x^{(2)}(t)$ will not have a point of intersection within the time interval Δt, if in this time range

$$x^{(1)}(t_0)+\frac{dx^{(1)}}{dt}\Delta t < x^{(2)}(t_0)+\frac{dx^{(2)}}{dt}\Delta t\text{ , or}$$

$$\frac{dx^{(1)}}{dt}-\frac{dx^{(2)}}{dt}<\frac{\Delta x}{\Delta t}, \tag{4}$$

where we designated $\Delta x = x^{(2)}(t_0)-x^{(1)}(t_0)$. Conversely, a condition of intersection of the world lines considered is

$$\frac{dx^{(1)}}{dt}-\frac{dx^{(2)}}{dt}\geq\frac{\Delta x}{\Delta t}. \tag{5}$$

Due to the homogeneity of space-time transformations, the corresponding space coordinates $X^{(1)}(T)$, $X^{(2)}(T)$ for another observer are also close to each other within a corresponding time interval ΔT. For $X^{(2)}(T_0)>X^{(1)}(T_0)$, two world lines do not have a point of intersection for this observer, if

$$\frac{dX^{(1)}}{dT}-\frac{dX^{(2)}}{dT}<\frac{\Delta X}{\Delta T}. \tag{6}$$

Here we designated $\Delta X = X^{(2)}(T_0)-X^{(1)}(T_0)$. Conversely, the world line intersect, if

$$\frac{dX^{(1)}}{dT}-\frac{dX^{(2)}}{dT}\geq\frac{\Delta X}{\Delta T}. \tag{7}$$

Simultaneous validity of either inequalities (4, 6) or inequalities (5, 7) is a strong requirement of CP.

Thus, we find that the finiteness of light velocity and the homogeneity of space-time transformations are necessary, but still not sufficient, conditions for implementation of the CP in relativity theory. Namely, one requires to realize either inequalities (4, 6) or inequalities (5, 7).

Here we omit a proof that for smooth functions $x(t)$, $X(t)$, the inequalities (4, 6) or inequalities (5, 7) are always implemented simultaneously for all observers, either inertial or non-inertial [4]. It seems that this theorem completely proves a correspondence of relativity theory with CP. However, if at least one of the functions $x^{(1)}(t), x^{(2)}(t), X^{(1)}(T), X^{(2)}(T)$ has a fracture (slope discontinuity) in the considered time range, the proof becomes invalid. Indeed, in such a case at least one of the time derivatives in the inequalities (4)-(7) is infinite at the fracture point t_f, lying inside the considered time interval $\{t_0, t_0+\Delta t\}$. Then for $t_f<t<t_0+\Delta t$, the time derivative may have a value unrelated to its old value. However, from the viewpoint of physics it seems that such fracture points cannot exist: infinite time derivatives dx/dt and dX/dT are impossible, due to the fundamental requirement that the velocity of any entity cannot exceed the velocity of light. This is actually true, with one exception: the cases of absorption (emission) of light. Indeed, let us imagine some absorber/re-emitter of light, which moves in some reference frame. At some instant let it absorb an incident short light pulse, and after a fixed interval of its proper time Δt_r let it re-emit a light pulse in the same direction. Since Δt_r is a definite value, we may imagine that during this time the re-emitter «keeps» information about the absorbed light pulse. Hence, we may join the world lines of absorbed and re-emitted light pulses by the world line of absorber/re-emitter (Fig. 2). In this case the events of absorption/emission of light can be formally considered in macroscopic scale as the “fracture” points of the common full world line in Fig. 2.

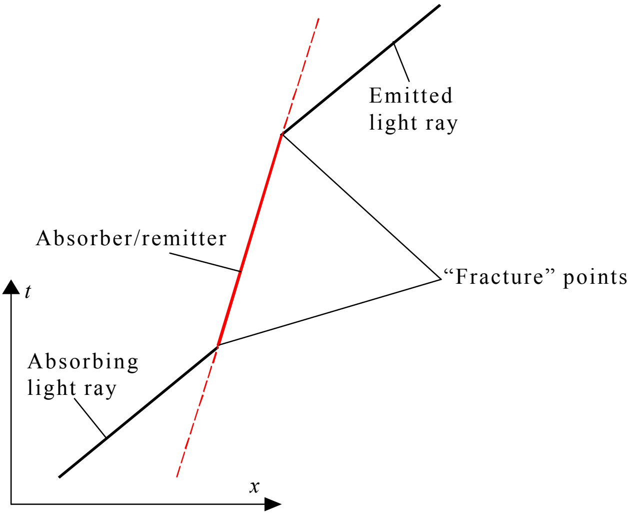

Fig. 2. Common "world line" of "absorbing light ray → absorber/re-emitter → emitted light ray". The events of absorption/re-emission of light can formally be considered as "fracture" points on a macro-physical scale.

Thus, it seems that formal mathematics does not forbid a violation of CP in relativity theory. Ref. [1] represents the first physical example, where the CP in relativity theory can be tested through the processes of emission/absorption of light. However, due to the efforts of many people, it was found that the problem could not answer the question about possible violation of CP: the particular calculations for an inertial observer are stupendously difficult. At the same time, one may greatly simplify this problem, going from an infinite chain of re-emitters of light to only two such re-emitters. This problem, where all calculations can be completed in both non-inertial and inertial reference frames, is analysed in the next Section.

Case of two light pulses and two re-emitters of light in a rigid non-inertial reference frame

In a rigid frame, moving at the constant acceleration a along the axis x and defined by Eqs. (1), let a short light pulse be emitted from the point $x = 0$ along the axis x. Let two re-emitters of light RL_0 and RL_1 be located along the x-axis at the points 0 and x_1. When a light pulse arrives at each re-emitter, it is absorbed by it, and after a fixed interval of its proper time Δt_0 is emitted by RL along the x-axis again.

Further, let the second light pulse be emitted from the point $x = 0$ at a moment of time (taken as $t = 0$) when the first light pulse has a coordinate value satisfying (2). One requires to find the times t_1 and t_2, where t_1 is the moment of time when the first (right) light pulse is emitted by RL_1, while t_2 is the moment of time when the second (left) pulse reaches RL_1.

In order to solve this problem we have to determine the metric coefficients in the accelerated frame. For this purpose let us write a known relationship between space-time coordinates in a fixed inertial reference frame (T, X, Y, Z) and (t, x, y, z) [3]:

$$dT = dt(1+\frac{ax}{c^2})ch\frac{at}{c}+\frac{dx}{c}sh\frac{at}{c}, \tag{8}$$

$$dX = cdt(1+\frac{ax}{c^2})sh\frac{at}{c}+dxch\frac{at}{c}, \quad dY = dy, \quad dZ = dz\,. \tag{9}$$

The space-time metric determined by eqs. (8), (9), is the following:

$$ds^2 = c^2dt^2(1+\frac{ax}{c^2})^2-(dx)^2-(dy)^2-(dz)^2. \tag{10}$$

And the corresponding components of the metric tensor are:

$$g_{00} = (1+\frac{ax}{c^2})^2; g_{0a} = 0; g_{11} = g_{22} = g_{33} = -1, \text{ all others } g_{ab} = 0 .(11)$$

The physical values are related to the coordinate values by the equations

$$dx_{\text{ph}0} = \sqrt{g_{00}}dx^0 + \frac{g_{0a}dx^a}{\sqrt{g_{00}}}, \tag{12a}$$

$$\Sigma dx^2_{\text{ph}a} = \left(-g_{ab} + \frac{g_{0a}g_{0b}}{g_{00}}\right)dx^a dx^b, (\alpha = 1\ldots3). \tag{12b}$$

Substituting the components of the metric tensor from (11) into (12), we obtain

$$dx_{\text{ph}} = dx,\ dy_{\text{ph}} = dy,\ dz_{\text{ph}} = dz; \tag{13}$$

$$dt_{\text{ph}} = dt(1+\frac{ax}{c^2}). \tag{14}$$

Further, let is consider the first (right) light pulse. At the initial time moment it starts moving from the point Δx to point x_1. Along the light signal $dx_{ph}/dt_{ph} = c$, and

$$\frac{dx}{dt} = c_x, \tag{15}$$

where the light velocity in (x, t) coordinates is

$$c_x = c\left(1+ax/c^2\right). \tag{16}$$

(Here we used Eqs. (13), (14)). Further, at the moment

$$t_R = \int_{\Delta x}^{x_1} dx/c_x = \frac{c}{a}\ln\frac{1+ax_1/c}{1+a\Delta x/c}. \tag{17}$$

the light signal reaches RL_1 at the point x_1 and is absorbed there. Here we have a fracture point on the common world line "right light signal + RL_1", and further the world line is described by the equation $x = x_1$ within the time range $\{t_R, t_R+\Delta t_1)$, where Δt_1 can be found from the equation (see, (14)):

$$\Delta \boldsymbol{t}_0 = \int_0^{\Delta t(x)} dt_{ph}(x) = \Delta t_1 (1 + ax/c^2),$$

$$\Delta t_1 = \frac{\Delta \boldsymbol{t}_0}{1 + ax_1/c^2}. \quad (18)$$

Then at the moment

$$t_1 = t_R + \Delta t_1 \quad (19)$$

the right light pulse is re-emitted by RL_1 along the axis x, and we ignore its subsequent history (Fig. 3).

Now let us look for the second (left) light signal. At $t = 0$ it emerges at the point $x = 0$ and is immediately absorbed by RL_0. Then during the time $\Delta \boldsymbol{t}_0$ the common world line "RL_0+left light pulse" is described by the equation $x = 0$. At the moment $\Delta \boldsymbol{t}_0$ the left light pulse is emitted by RL_0 (fracture point on the common world line), and then the world line is described by Eq. (15). The time of passage of the left light signal from the point $x = 0$ to point x_1 is determined as

$$t_L = \int_0^{x_1} dx/c_x, \quad (20)$$

and

$$t_2 = \Delta t_0 + t_L \,. \tag{21}$$

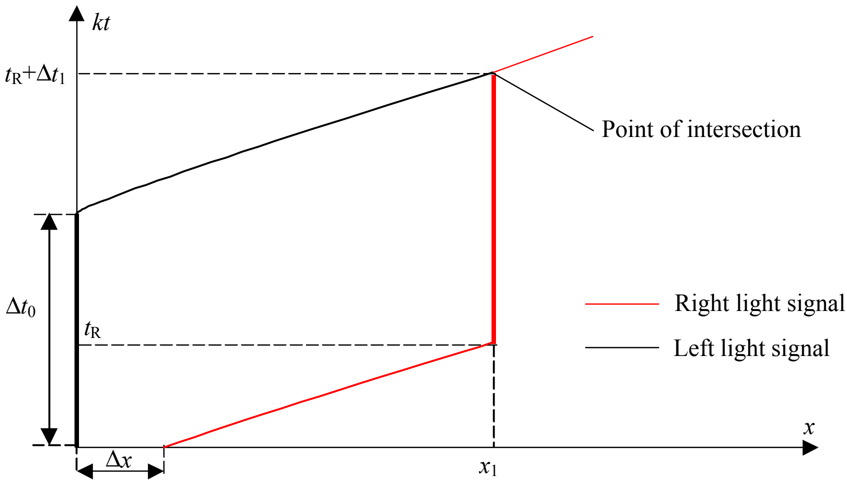

Fig. 3. World lines for the systems “right light pulse + RL_1” and “RL_0 + left light pulse” for a non-inertial observer. Both light pulses meet at the point x_1.

Intersection of the two light pulses occurs for $t_1 = t_2$. Substituting their values from Eqs. (19) and (21), we obtain

$$t_R + \Delta t_1 = \Delta t_0 + t_L \,. \tag{22}$$

Further substitution of Eqs. (17), (18) and (20) gives:

$$\frac{\Delta t_0}{1 + a x_1/c^2} - \Delta t_0 = \int_0^{x_1} dx/c_x - \int_{\Delta x}^{x_1} dx/c_x \text{ , or}$$

$$-\frac{\Delta t_0}{1 + a x_1/c^2}\frac{a x_1}{c^2} = \int_0^{\Delta x} dx/c_x \,.$$

Further manipulations with the help of Eq. (16) lead to

$$\Delta t_0 = -\frac{c^3\left(1+ax_1/c^2\right)\ln\left(1+a\Delta x/c^2\right)}{a^2 x_1}. \tag{23}$$

Thus, if Eq. (23) is satisfied, two light pulses intersect at the point x_1, as depicted in Fig. 3. One can also see from Eq. (23) that such an intersection is possible only for a negative sign of the acceleration a.

Now let us consider the same problem for an inertial observer, assuming that at $t=0$ the momentary relative velocity of the non-inertial and inertial frames is equal to zero. Then the forms of the world lines for the first and second light signals can be obtained from Eqs. (8), (9). First consider the left pulse. The motional equation for RL_0 before the instant of emission can be easily obtained through integration of Eqs. (8), (9) for $x=0$, $dx=0$:

$$dT = dt\,ch\frac{at}{c},\; T_L = \int_0^{\Delta t_0} ch\frac{at}{c}dt = \frac{c}{a}sh\frac{a\Delta t_0}{c}. \tag{24}$$

$$dX = cdt\,sh\frac{at}{c},\; X_L = c\int_0^{\Delta t_0} sh\frac{at}{c}dt = \frac{c^2}{a}\left(ch\frac{a\Delta t_0}{c}-1\right). \tag{25}$$

At the emission instant we have a fracture point, and subsequently the propagation of the left light pulse is described by the equation

$$\Delta X = c\Delta T \tag{26}$$

before its absorption by RL_1.

The right light pulse at the initial time moment has the space coordinate $X(0)=\Delta x$, and before its absorption by RL_1 its motion is described by Eq. (26). Let us designate its coordinate at the absorption instant as T_R. Then the corresponding space coordinate at this instant is

$$X_R = \Delta x + cT_R. \tag{27}$$

At the point (X_R, T_R) the common world line "right light signal + RL_1" has a fracture point (absorption event), and for $T>T_R$ the shape of the world line describes the motion of the re-emitter RL_1. For this motion $x = x_1$ until the moment of re-emission of a light pulse. Substituting the equality $x = x_1$ into Eqs. (8), (9), we get

$$dT = dt\left(1+\frac{ax_1}{c^2}\right)ch\frac{at}{c}, \tag{28}$$

$$dX = cdt\left(1+\frac{ax_1}{c^2}\right)sh\frac{at}{c}. \tag{29}$$

Integration of (28) gives the time interval between absorption and re-emission of the right light pulse:

$$\begin{gathered}\Delta T_R = \left(1+\frac{ax_1}{c^2}\right)\int_{t_R}^{t_R+\Delta t_1} ch\frac{at}{c}dt = \\ \frac{c}{a}\left(1+\frac{ax_1}{c^2}\right)\left(sh\frac{a\left(\Delta t_1+t_R\right)}{c} - sh\frac{at_R}{c}\right)\end{gathered}, \tag{30}$$

and integration of (29) allows us to find the distance of passage by RL_1 between the absorption and emission moments:

$$\begin{gathered}\Delta X_R = c\left(1+\frac{ax_1}{c^2}\right)\int_{t_R}^{t_R+\Delta t_1} sh\frac{at}{c}dt = \\ \frac{c^2}{a}\left(1+\frac{ax_1}{c^2}\right)\left(ch\frac{a\left(\Delta t_1+t_R\right)}{c} - ch\frac{at_R}{c}\right)\end{gathered}. \tag{31}$$

Now let us determine the space coordinate of the left light signal at the moment of emission of the right light pulse ($T_R + \Delta T_R$). It is clear that this coordinate is defined by the equation

$$X_L(T_R+\Delta T_R)=X_L+c\left(T_R+\Delta T_R-T_L\right),$$

where X_L is determined by Eq. (25), and $c\left(T_R+\Delta T_R-T_L\right)$ is the distance covered by this light pulse between the time of its emission T_L and the emission time of the right pulse $T_R+\Delta T_R$. Then the difference of space coordinates of both light pulses at the instant $T_R+\Delta T_R$ should be equal to zero according to the requirement of CP (both light pulses should meet at the point x_1). The expression for this difference for an inertial observer is written as

$$\begin{aligned}&\Delta=X_R+\Delta X_R-X_L(T_R+\Delta T_R)=\\&X_R+\Delta X_R-X_L-c\left(T_R+\Delta T_R-T_L\right)=\\&=\Delta x+\left(\Delta X_R-X_L\right)-c\left(\Delta T_R-T_L\right).\end{aligned} \tag{32}$$

(here we used Eq. (27)).

Substituting corresponding values from Eqs. (31), (30), (25) and (24), one gets:

$$\begin{aligned}&\Delta=\Delta x+\frac{c^2}{a}\left(1+\frac{ax_1}{c^2}\right)\left(ch\frac{a\left(\Delta t_1+t_R\right)}{c}-ch\frac{at_R}{c}\right)-\\&\frac{c^2}{a}\left(ch\frac{a\Delta t_0}{c}-1\right)-\\&-\left[\frac{c^2}{a}\left(1+\frac{ax_1}{c^2}\right)\left(sh\frac{a\left(\Delta t_1+t_R\right)}{c}-sh\frac{at_R}{c}\right)-\frac{c^2}{a}sh\frac{a\Delta t_0}{c}\right]=\\&=\Delta x+\frac{c^2}{a}\left(1+\frac{ax_1}{c^2}\right)\mathrm{e}^{-\frac{at_R}{c}}\left(\mathrm{e}^{-\frac{a\Delta t_1}{c}}-1\right)-\frac{c^2}{a}\left(\mathrm{e}^{-\frac{a\Delta t_0}{c}}-1\right)\end{aligned} \tag{33}$$

(here we have used the identity $chx–shx=\mathrm{e}^{-x}$).

Further substitutions of the values t_R, Δt_1 and Δt_0 from corresponding Eqs. (17), (18) and (23) yield:

$$\Delta = \Delta x + \frac{c^2}{a}\left(1+\frac{ax_1}{c^2}\right)\left(\frac{1+a\Delta x_1/c^2}{1+ax/c^2}\right)\left(\left(1+\frac{a\Delta x}{c^2}\right)^{\frac{c^2}{ax_1}}-1\right)-$$

$$\frac{c^2}{a}\left(\left(1+\frac{a\Delta x}{c^2}\right)^{\frac{c^2}{ax_1}\left(1+\frac{ax_1}{c^2}\right)}-1\right)=$$

$$= \Delta x + \frac{c^2}{a}\left(1+\frac{a\Delta x}{c^2}\right)\left(\left(1+\frac{a\Delta x}{c^2}\right)^{\frac{c^2}{ax_1}}-1\right)-$$

$$\frac{c^2}{a}\left(\left(1+\frac{a\Delta x}{c^2}\right)^{\frac{c^2}{ax_1}}\left(1+\frac{a\Delta x}{c^2}\right)-1\right)=$$

$$= \Delta x + \frac{c^2}{a}\left(1+\frac{a\Delta x}{c^2}\right)\left(1+\frac{a\Delta x}{c^2}\right)^{\frac{c^2}{ax_1}} - \frac{c^2}{a}\left(1+\frac{a\Delta x}{c^2}\right)-$$

$$\frac{c^2}{a}\left(1+\frac{a\Delta x}{c^2}\right)^{\frac{c^2}{ax_1}}\left(1+\frac{a\Delta x}{c^2}\right)+\frac{c^2}{a}=$$

$$= \Delta x - \frac{c^2}{a} - \Delta x + \frac{c^2}{a} = 0 \qquad (34)$$

Thus, we determine that two light pulses intersect for both inertial and non-inertial observer in accordance with CP.

4. Conclusion

We conclude that there is no contradiction between the causality and relativity principles in the processes of emission and absorption of light, when the common world lines "emitter/absorber + light pulse" have fracture points. At such points the time derivatives of the world lines (the velocities of entities) before and after the fracture point are not correlated with each other. However, stepwise changes of time derivatives of world lines are correlated for different observers. That, perhaps, prevents a violation of causality. Nevertheless, the present problem concerning propagation of two light pulses across a chain of accelerated re-emitters of light seems physically interesting in itself.

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