Anomalous Statistics and the Rescaling of Planck's-Constant

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Numerous forms of anomalous particle statistics have been motivated by 2+1 dimensional anayons, the multifractal structure of non-extensive systems, and the possible bosonic-fermionic nature of any fundamental particle. From recent studies in non-commutative geometry and previous studies in the discrete nature of space time we deduce that the unit cell in phase space is changed leading to a rescaling of Planck's constant. The rescaling in turn generates anomalous terms in the Planck distribution and the fermi energy of fermions which can be interpreted in terms of an anomalous statistics parameter.

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Rescaling Planck's Constant

After almost a century of quantum physics and two decades of string theory it has become apparent that space-time might not be a smooth differentiable manifold in which physics can be embedded [1,2,3,4,5]. Quite long ago Caldirola suggested that quantum mechanics should embrace finite differences [6,7] and numerous authors have sought to suggest a primitive origin to space time in terms of graph theory, combinatorics and discreteness [8,9,10,11,12]. In particular we have sought to explore the consequences of discrete space-time in quantum mechanics (Q.M.) by studying electron spin resonance [13], electron spin polarization procession [14], spectral shifts in hydrogen 15] and internal transitions of elementary particles [16]. Recent studies in non-commutative geometry have suggested that time should be discretized [17] leading to an uncertainty principle for space-time forbidding space-time measurements beneath a certain limited scale [18,19]. If, for instance, we write for a modified uncertainty principle

$$\Delta x \Delta P = h + (\Delta x \Delta P)DST$$

(where $(\Delta x \Delta P)_{DST}$ = additional uncertainty due to discrete space-time effects), we have $\Delta x = L_0$ = discrete space interval, and for photons

$$\Delta p = \Delta\left(\frac{h\nu}{C}\right) = \Delta\left(\frac{h}{CT}\right) = \frac{h}{CT^2}\Delta T = \frac{h}{C}\nu^2\tau_0 \left(\Delta T = \tau_0\right)$$

(Here τ_0 = discrete time uncertainty and $\nu = \frac{1}{T}$). Thus

$$\Delta x \Delta p = h + \frac{h\nu^2}{C}\tau_0 L_0 \tag{2.1}$$

In Eq. (2.1) we may think of this as a rescaling of Planck's constant, where $h \rightarrow h + \Delta h, \quad \Delta h = \frac{h\nu^2}{C}\tau_0 L_0$

In a separate branch of physics motivated by the statistics of 2+1 dimensional anayons [20], multifractality [21] and the possible Boson-Fermion nature of any fundamental particle [22], anomalous statistics have developed. To illustrate how the above notions of discrete space-time can describe any of these anomalous corrections to BE and FB statistics we write the energy density of photons as

$$dU(\nu) = \frac{8\pi\nu^3 h \quad d\nu}{C^3 \left(e^{\frac{h\nu}{kT}} - 1 \right)} \tag{2.2}$$

If we let $h \to h + \Delta h$ we find

(2.3)

$$dU(\nu) = \frac{8\pi\nu^2}{C^3} \left(\frac{h\nu d\nu}{e^{\frac{h\nu}{kT}} - 1} \right) + \frac{8\pi\nu^3}{C^3 (K-1)} \left(1 - \frac{h\nu}{kT} \left(\frac{K}{K-1} \right) \right) \Delta h d\nu$$

here $\frac{\Delta h\nu}{kT} < 1$, $\left(K = e^{\frac{k\nu}{kT}} \right)$

From the approach of Ref. (20) we find [23]

$$dU(\nu) = \frac{8\pi h\nu^3}{C^3} \left[\frac{e^{\frac{h\nu}{kT}} - 1 - \alpha}{\left(e^{\frac{h\nu}{kT}} - 1 \right)^2} \right] d\nu \tag{2.4}$$

(α = anomalous statistics parameter). Equating the correction term in Eq. (2.3) to the correction term in Eq. (2.4) we find for $\frac{h\nu}{kT} > 1$,

$$-\frac{\alpha\left(8\pi h\nu^3\right)}{C^3K^2}=-\frac{8\pi\nu^3}{C^3K}\left(\frac{h\nu}{kT}\right)\left(\frac{L_0\tau_0 h\nu^2}{C}\right)$$

leading to

$$\alpha=\frac{KL_0\tau_0 h\nu^3}{(kT)C} \tag{2.5}$$

with $\nu = 10^{14}$ sec^{-1}, T = 100°K, L = 10^{-16} cm, $\tau_0 = 10^{-12}$ sec.

$$\alpha = 10^{-4} \tag{2.6}$$

Eq. (2.6) is within the experimental limits of measurable differences of the CMB [24] from a Planck distribution according to Eq. (2.4) and suggest that both discrete space-time effects and anomalous effects cannot be ruled out and might have a common underlying origin.

Actually the above values for L_0 and τ_0 might be an overestimate of discrete space-time intervals. Roychowdhury and Roy [25] have pointed out that due to stochastic discrete space-time effects the energy levels of hydrogen might suffer a shift and Bracci et. al. [26] have suggested a discrete spatial interval of 10^{-5} fermi based on various experimental situations. In a separate paper, we have suggested a value of $\tau_0 \cong 1.25\times 10^{-18}$ sec by comparing hyperfine corrections in hydrogen spectra with discrete time corrections [27]. These values would give a value of $\alpha = 10^{-12}$ according to Eq. (2.5) which is too small to be measured in the C.M.B. spectra.

As another application of the above ideas in a previous note [28] we have calculated the corrections of the fermi energy of fermions due to anomalous statistics. The corrected value for the fermi energy is (here α = 1-ε, α = 1 for fermi statistics).

$$\varepsilon_F = \frac{1}{2m}\left(\frac{3N}{(1+\varepsilon)8\pi V}\right)^{\frac{2}{3}} h_0^{\ 2}$$

(2.7) or

$$\varepsilon_F \approx \frac{1}{2m}\left(\frac{3N}{8\pi V}\right)^{\frac{2}{3}} h_0^{\ 2}\left(1-\frac{2\varepsilon}{3}\right)$$

If we interpret Eq. (2.7) as a discrete time correction to h as $h \rightarrow h_0 + \Delta h$ so that $\varepsilon_F = \frac{1}{2m}\left(\frac{3N}{8\pi V}\right)^{\frac{2}{3}}\left(h_0^{\ 2} + \partial h_0 \Delta h\right)$ we have $h_0^{\ 2}\left(-\frac{2}{3}\varepsilon\right) = 2h_0\Delta h,$ or $\Delta h = -\frac{1}{3}\varepsilon h_0$ (h_0 = usual Planck constant). Thus for fermions Δh is negative. This would find application to neutron star stability. By balancing the gravitational attraction with the degeneracy pressure for a neutron star we find for the equilibrium radius [29]

(2.8) $$R = \left(\frac{81\pi^2}{16}\right)^{\frac{1}{3}} \frac{N^{-\frac{1}{3}}}{Gm_n^{\ 3}} \frac{1}{4\pi^2} h^2$$

For $h \rightarrow h_0 - \Delta h = h_0 - \frac{mL_0^{\ 2}}{\tau_0}$ ($\Delta h = -(\Delta P \Delta X)_{DST} = -\frac{mL_0^{\ 2}}{\tau_0}$) we find

(2.9) $$\Delta R = -\left(\frac{81\pi^2}{16}\right)^{\frac{1}{3}} \frac{N^{-\frac{1}{3}}}{Gm_n^{\ 3}}\left(\frac{1}{4\pi^2}\right)\left(2h_0\frac{mL_0^{\ 2}}{\tau_0}\right)$$

Thus if τ_0 is small it would diminish R by a factor of $\Delta R = -R\left(\frac{2mL_0{}^2}{\tau_0 h_0}\right)$. For $L_0 \approx 10^{-18}$ sec. $h_0 \cong 6.6 \times 10^{-27}$ (CGS), $m = 1.67 \times 10^{-24}$ cgs $\tau_0 = 10^{-33}$ sec. $\Delta R \cong -R$. Thus for small τ_0 and thus large fluctuations in h we could shrink a neutron star below the corresponding radius for a black hole. Thus discrete space-time corrections to h could lead to the unexpected production of black holes.

The above estimate serves to demonstrate that by combining effects due to anomalous statistics and discrete space-time effects we are lead to rather remarkable astrophysical phenomena in the C.M.B. and the physics of condensed astrophysical objects.

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