A New Proposal Combining Quantum Mechanics and the Special Theory of Relativity

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The technical problem in Feynman's proposal to formulate quantum mechanics in terms of a sum over histories of a particle is that this sum over histories must be performed in imaginary time rather than in the real time that everyone experiences [1]. In this paper, for the derivation of Lorentz transformation from the postulates of Special theory of Relativity, the concept of De Broglie quantum waves is introduced which leads to the conclusions that the derivation of Lorentz transformation does not need the postulate of constancy of light speed in free space in case the postulate of phase coincidence of De Broglie quantum waves is introduced. Consequently, a new proposal is formulated that combines Quantum mechanics and the special theory of relativity and overcomes the technical problem in Feynman's proposal.

Keywords: Feynman's proposal, quantum mechanics, De Broglie waves, Special Theory of Relativity.

Motivation for the new proposal [2]

The special theory of relativity (or the general theory of relativity) deals with the large-scale structure of the universe. Quantum mechanics, on the other hand, deals with phenomena on extremely small scales. Unfortunately, however these two theories are known to be inconsistent with each other—they cannot both be correct. This motivates a search for a new proposal that will incorporate them both.

The Lorentz transformation: [4]

Let us suppose that we are in a frame of reference S and find that a point particle at rest at the origin of S emits at $t=0$ a light photon in the positive X-axis direction. Further, assume that another frame of reference S', moving with velocity v in positive X-axis direction, coincides with frame S at $t=t'=0$ such that the same point particle moving with velocity $(-v)$ along X-axis in S' emits the aforesaid light photon along the positive X'-axis direction at $t'=0$.

This light photon covers distance $x=ct$ along X-axis in S with light speed c in free space in time t Our objective is to find distance x' covered by this light photon along the X-axis after its emission at $t'=0$ in S'.

A reasonable guess as to the nature of the correct relationship between x and x' is

$$x'=k(x-vt) \tag{1}$$

Where k is a factor of proportionality that does not depend upon either x or t but may be a function of v. The choice of equation (1) follows from several considerations:

(a) It is linear in x and x', so that a single event in frame S corresponds to a single event in frame S', as it must.

(b) It is a simple, and simple solutions to a problem must be explored first

(c) It has the possibility of reducing to the equation $(x' = x - vt)$, which we know to be correct in ordinary mechanics.

Because the equations of physics must have the same form in both S and S', we need only change the sign of v (in order to take into account the difference in the direction of relative motion) to write the corresponding equation for x in terms of x' and t'.

$$x = k(x' + vt') \tag{2}$$

The factor k must be the same in both frames of reference since there is no difference between S and S' other than in the sign of v. Consequently, the equation (2) assumes that k depends on the even degrees of v.

As in the case of the Galilean transformation, there is nothing to indicate that there might be differences between the corresponding co-ordinates y, y' and z, z' which are normal to the direction of v. Hence, we have

$$y' = y \tag{3}$$

$$z' = z \tag{4}$$

The time co-ordinates t and t', however, are not equal. We can see this by substituting the value of x' given by equation (1) into equation (2) we obtain:

$$x = k^2(x - vt) + kvt'$$

from which we find that

$$t' = kt + [(1 - k^2)x]/kv \tag{5}$$

Equations (1) and (3) to (5) constitute a co-ordinate transformation that satisfies the first postulate of special relativity that the laws of physics may be expressed in equations having the same form in all frames of references moving at constant velocity with respect to one another.

At this juncture we introduce the phase coincidence of De Broglie quantum waves. The linear momentum of the point particle along X'-axis is exactly defined in S'. As a consequence, the mathematical description of De Broglie quantum y-wave equivalent of the point particle, in question, of total energy E' and linear momentum p' moving with $(-v)$ velocity along X'-axis in S' is given by

$$y = A\exp\left\{-(i2p/h)(E't' + p'x')\right\}$$

where h is Planck's constant, $i = \sqrt{-1}$ and A is the amplitude of the y-wave.

Here, it is postulated that this y-wave must travel with phase velocity $(E'/p' = c^2/v)$. Consequently, this y-wave takes the following form.

$$y = A\exp\left\{-(i2p/h)E'(t' + vx'/c^2)\right\} \tag{6}$$

This plane y-wave accounts for the infinite uncertainty in the position of point particle along X'-axis in S' in accordance with the Heisenberg's uncertainty principle, as linear momentum is exactly defined in S'. This implies that the emission of the light photon by point particle in S' at time instant $t' = 0$ must not necessarily take place at $x' = 0$ but can take place at any point on X'-axis in S'. This can be mathematically shown when we substitute $t' = 0$ in the above y-wave equation (6) and we get

$$y = A\exp\left\{(i2p/h)E'(-vx'/c^2)\right\} \tag{7}$$

Substituting $t' = 0$ in equation (2) we get

$$x = kx' \text{ or } x' = x/k \tag{8}$$

Substituting the above equation (8) in earlier stated y -equation (7), we get:

$$y = A\exp\{(i2p/h)(E'/k)(-vx/c^2)\} \tag{9}$$

This y -wave equation (9) associated with point particle at rest in S at $t=0$ is a single pulse because De Broglie wavelength of point particle (at rest in S) is infinite in frame S. At time instant t, this y -wave equation (9) takes the following form:

$$y = A\exp\{(i2p/h)(E'/k)(t-vx/c^2)\} \tag{10}$$

If the distance involved between the point of actual emission of light photon at $t'=0$ on X'-axis in S' and the origin of S' is $(x_1 = kx_1')$ in S, the above y -wave pulse equation (10) in S exhibit same phase at point x_1 as is displayed by it at the origin of S. This is possible if and only if this distance x_1 is covered by y -wave pulse in equation (10) with same phase velocity c^2/v. Mathematically, this condition is satisfied when we substitute $x = x_1$ and $t'=0$ in equation (5) and get:

$$0 = kt + [(1-k^2)/kv]x_1$$

and solving for x_1,

$$x_1 = c^2t/v\{1/[(k^2c^2 - c^2)/k^2v^2]\} \tag{11}$$

In equation (11), the distance x_1 is covered by De Broglie y -wave pulse with phase velocity (c^2/v) in time t in S provided that the quantity in the bracket { } equals unity. Therefore,

$$\left\{1/[(k^2 / c^2 - c^2)/k^2 v^2]\right\} = 1$$

$$k = 1/(1 - v^2 / c^2)^{0.5}$$

Here the choice of positive sign in solutions for k follows from the consideration that it leads to the possibility of reducing equation (1) to the equation $(x' = x - vt)$, which we know to be correct in ordinary mechanics.

Thus, when $k = 1/(1 - v^2 / c^2)^{0.5}$, we have from equation (11) above

$$t - vx_1 / c^2 = 0 \tag{12}$$

When $x = x_1$ is substituted in equation (10) and the above equation (12) is substituted in resulting equation, it is mathematically proved that the phase of y -wave pulse in S at point x_1 at time t is same as is displayed by it at the origin of S at $t = 0$ if and only if the distance x_1 is covered by y -wave pulse in S with phase velocity c^2 / v.

Inserting the above calculated value of $k = 1/(1 - v^2 / c^2)^{0.5}$ in equations (1) and (5), we have complete information about the distance x' covered by the light photon along X'-axis after its emission at $t' = 0$ in S', *i.e.*,

$$x' = (x - vt)/(1 - v^2 / c^2)^{0.5} \tag{13}$$

$$t' = (t - vx / c^2)/(1 - v^2 / c^2)^{0.5} \tag{14}$$

Therefore, the equation (13), (3), (4) and (14) comprise the Lorentz transformation. In order to obtain inverse Lorentz transformation, primed and unprimed quantities in equation (13), (3), (4) and (14) are exchanged and v is replaced by $(-v)$.

Conclusions

(1) We observe that in the above derivation of Lorentz transformation there is a De Broglie wave pulse in S associated with point particle's linear momentum measured exactly along X-axis in every possible frame S'. Since, this linear momentum can have infinite number of positive as well as negative exact values in different S'. This implies that there are infinite number of De Broglie wave pulses associated with point particle in S. These infinite De Broglie wave pulses constitute the wave group or wave packet of the Quantum Mechanics. This De Broglie wave packet obviously has zero expectation value for linear momentum along X-axis in S in quantum mechanics corresponding to the rest state of a point particle in S.

Moreover, in Feynman's proposal to formulate quantum mechanics in terms of a sum over histories, each De Broglie wave pulse represent unique history of the particle in the sense that every possible frame S' represents every possible path in space-time that the point particle is supposed to follow to arrive at the origin of S at $t = 0$. In the derivation of Lorentz transformation the De Broglie wave pulse associated with point particle travels backward in time when the distance x_1 is on the positive X-axis and travels forward in time when x_1 is on the negative X-axis. This has an interesting effect on space-time: the distinction between space and time disappears completely and there is no difference between the time direction and directions in space. Such a space-time may have imaginary values of the time coordinate assigned to events and is said to be Euclidean space-time, the metric of which is well known Euclidean metric in the quantum field theory $x_0 \to x_4 = ict$.

(2) Taking the ratio of equations (7) and (8), we get

$$x'/t' = (x - vt)/(t - vx/c^2)$$

Substituting $x = ct$ in above equation, we get

$$x'/t' = c = \text{light speed in free space}$$

This implies that the derivation of Lorentz information does not need the postulate of constancy of light speed in free space when the postulate of phase coincidence of De Broglie quantum waves is introduced.

References

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[2] Stephen Hawking, *A Brief History of Time*, Bantam Press, Great Britain, (1988), p. 13.

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