Inactive Portion of the Radiative Part of the Liénard-Wiechert Field

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A point charge in motion generates the Liénard-Wiechert energy-momentum tensor. Teitelboim [1] showed that this tensor splits into its bounded part $\underset{B}{T}{}_{ac}$ and its radiative part $\underset{R}{T}{}_{ac}$. All the terms in $\underset{B}{T}{}_{ac}$ are know to contribute to the matter-field energy-momentum balance. In this paper the inactive part of $\underset{R}{T}{}_{ac}$ is found, i.e., the terms do not contribute to the energy-momentum fluxes are shown.

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We shall employ the notation and quantities explained in detail in refs.[2-9].

A classical point charge q in arbitrary motion in Minkowski space generates the Liénard-Wiechert field [10-12]; Teitelboim[1] showed that the corresponding Maxwell tensor admits the splitting:

$$T_{ij} = \underset{B}{T}_{ij} + \underset{R}{T}_{ij} \tag{1}$$

where $\underset{B}{T}_{ij}$ and $\underset{R}{T}_{ij}$ are the bounded and the radiative parts, respectively. The bounded part has been studied in [4-7,13-15]. On the other hand, we consider here the radiative part in relation with its contribution to the electromagnetic energy-momentum fluxes quantified through a Bhabha tube [16,17] around the charge. The approach is important [17] to determine the equation of motion [2,10,18-21] for the charge q.

The radiative portion $\underset{R}{T}_{ac}$ is given by [1-3,8,9,22]:

$$\underset{R}{T}_{bc} = q^2 w^{-4}(a^2 - w^{-2}W^2)K_b K_c \tag{2}$$

we now show that it can be written as:

$$\underset{R}{T}_{ij} = \underset{A}{T}_{ij} + \underset{I}{T}_{ij} \tag{3}$$

where

$$\underset{A}{T}_{ij} = 2q^2 w^{-6} W^2 K_i K_j \tag{4}$$

$$\underset{I}{T}_{ij} = q^2 w^{-4}(a^2 - 3w^{-2}W^2)K_i K_j \tag{5}$$

which are dynamically independent because they separately satisfy the Villarroel conditions [23] for tensors of radiation:

$$\underset{A}{T}_b{}^c{}_{,c} = \underset{I}{T}_b{}^c{}_{,c} = 0 \tag{6}$$

$$\underset{A}{T}_{bc}K^c = \underset{I}{T}_{bc}K^c = 0$$

The physical meaning of the splitting (3) is the following: As we enclose the world-line of the point charge by a Bhabha cylinder and calculate the energy-momentum fluxes of $\underset{I}{T}_{ij}$ across this cylinder it is found that the fluxes vanish. This means that if the Bhabha surface is used to determine the equation of motion for q the tensor (5) will not contribute at all to such equation. Hence, the fluxes of (3) through the Bhabha tube are due only to (4); then we say that $\underset{I}{T}_{bc}$ is the inactive portion of $\underset{R}{T}_{bc}$. It is easy to show this result using the Synge expressions [2,4,11,17,24,25] for the fluxes of linear and angular momentum:

$$\int_{w=\mathit{cons}\tan t} \underset{I}{T}_{bc}\, ds^{c} = w^{2} \int_{t_1}^{t_2} dt \int \underset{I}{T}_{b}{}^{c}\, w_{,c}\, d\Omega = 0$$

$$\int_{t=\mathit{cons}\tan t} \underset{I}{T}_{bc}\, ds^{c} = -\int_{w_1}^{w_2} w\,dw \int \underset{I}{T}_{bc} K^{c}\, d\Omega = 0 \qquad (7)$$

$$\int_{w=\mathit{cons}\tan t} \underset{I}{M}_{abc}\, ds^{c} = \int_{t=\mathit{cons}\tan t} \underset{I}{M}_{abc}\, ds^{c} = 0 \quad ,$$

where $\underset{I}{M}_{abc} = X_a \underset{I}{T}_{bc} - X_b \underset{I}{T}_{ac}$.

Hence $\underset{A}{T}_{ij}$ – active part of $\underset{R}{T}_{ij}$ – is equivalent to (2) in connection with the Bhabha tube. If (5) do not contribute to electromagnetic fluxes, which is then the reason for its presence in equation (3) ? Perhaps it is due to the non-unicity [2,17,26-29] of any energy-momentum tensor.

The differential properties (6) imply the existence of electromagnetic superpotentials as generators for (4) and (5), in fact:

$$\underset{A}{T}_{ij} = \underset{A}{K}_{i}{}^{c}{}_{j,c}\ ,\ \underset{I}{T}_{ij} = \underset{I}{K}_{i}{}^{c}{}_{j,c} \tag{8}$$

such that:

$$\underset{I}{K}_{bjc} = \frac{q^2}{4} w^{-2}\,{}^{\cdot\cdot}[w^{-2}W^2\left(g_{cj}K_b - g_{cb}K_j\right) + w^{-1}\left(v_b\ xK_j\right)^{\circ} \circ\ \left(3w^{-2}WK_c - a_c\right) + \left(a_b\ xK_j\right)\left(a_c - 4w^{-2}WK_c\right)]; \tag{9}$$

$$\underset{A}{K}_{bjc} = -2qF_{bj}p_{(s)}\,p_{(g)}\left[\int_0^t a_{(s)}a_{(g)}v_c\,dt + p_{(b)}\int_0^t a_{(s)}a_{(g)}e_{(b)c}\,dt\right], \tag{10}$$

With sums over $s\ , b, g\ = 1,2,3$ are implied. Thus we see that (10) depends of integrals along the world-line of the charge, which means that the process of measuring the radiation rate is intrinsically non-local [30, 31]. However, (9) is local because the inactive part $\underset{I}{T}_{ij}$ does not participate in the equation of motion of q. Therefore (3) is an exact divergence:

$$\underset{R}{T}_{ij} = \left(\underset{A}{K}_{i}{}^{c}{}_{j} + \underset{I}{K}_{i}{}^{c}{}_{j}\right)_{,c} = \underset{R}{K}_{i}{}^{c}{}_{j,c}, \tag{11}$$

Then (9) and (10) give us alternative Cartesian expressions for $\underset{R}{K}_{i}{}^{c}{}_{j}$ to those obtained in [32] using Newman-Unti coordinates [33].

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