Lanczos Superpotential for Kinnersley Spacetimes

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We obtain the Lanczos spintensor for the eleven type D vacuum Kinnersley spacetimes.

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The Lanczos potential K_{abc} is a generator [1-4] for the Weyl tensor in four dimensions. Here, using the Newman-Penrose formalism [5-7] we will obtain K_{ijr} for any type D vacuum space by studying each one of the eleven Kinnersley's metrics [8-10]. For each of them it is possible to select the null tetrad such that:

$$\begin{gathered} k = s = n = l = 0,\ t = p\ ,\ a = b\ ,\ g = qe\ , \\ r - \bar{r} = 2(e - \bar{e})\ ,\ p + \bar{p} = 2\left(b + \bar{b}\right)\ ,\ m = q\,r\ ,\ q = \pm 1 \\ y_2 = 4(gr - pb)\ ,\ db + \overline{db} + Dg + \Delta e = 0\ , \end{gathered} \tag{1}$$

this special tetrad appears when we perform the scale-rotation changes [7,11] defined by $m_c \to e^{-iB} m_c$, $l_c \to e^{-A} l_c$ and $n_c \to e^{A} n_c$, for convenient scalar functions A and B onto the Kinnersley's tetrads.

The Weyl-Lanczos equations [3,12-18] under (1) are now solved to give the solution:

$$\begin{gathered} \Omega_0 = \Omega_7 = q\frac{p}{4}\ ,\ \Omega_4 = q\Omega_3 = \frac{r}{4}\ , \\ \Omega_1 = q\Omega_6 = \frac{e}{3} + \frac{r}{12}\ ,\ \Omega_2 = \Omega_5 = \frac{b}{3} + \frac{p}{12}\ , \end{gathered} \tag{2}$$

which contains as a particular case the Lanczos spintensor published in [19] for the Kerr metric [7,20,21]; from (2) the corresponding K_{ijr} is given by

$$\begin{aligned} K_{abc} = {} & \Omega_0\left(U_{ab} l_c + V_{ab} n_c\right) + \Omega_1\left[M_{ab} l_c - U_{ab} m_c + q\left(M_{ab} n_c - V_{ab}\bar{m}_c\right)\right] + \\ & + \Omega_2\left(V_{ab} l_c - M_{ab} m_c + U_{ab} n_c - M_{ab}\bar{m}_c\right) - \\ & - \Omega_3\left(V_{ab} m_c + q U_{ab}\bar{m}_c\right) + c.c. \end{aligned} \tag{3}$$

where *c.c.* denotes the complex conjugate of all previous terms. The spintensor obtained fulfill the Lanczos gauges:

$$K_{a\ \ b}^{\ b} = 0\,, \qquad K_{ab\ \ ;c}^{\ \ c} = 0\,, \tag{4}$$

and therefore it is valid the Lanczos-Illge wave equation [4,22-24] $\Box K_{abc} = 0$.

Also, the superpotential (3) has the remarkable structure:

$$K_{abc} = A_{ca;b} - A_{cb;a} + g_{ca} A_b - g_{cb} A_a \tag{5}$$

where

$$A_{ij} = \frac{1}{4}\left[q\left(l_i l_j + n_i n_j\right) - m_i m_j - \overline{m}_i \overline{m}_j\right], \qquad A_c = \frac{1}{3} A_{c\ \ ;r}^{\ r}$$

$$A_c = \frac{1}{6}\left[\left(p - 2b\right)\left(m_c - \overline{m}_c\right) + \left(r - 2e\right)\left(l_c - q n_c\right)\right], \quad A^c_{\ ;c} = 0\,, \tag{6}$$

thus, the Lanczos potential showed at [25-27] in Kerr geometry is a special case of (5) and (6) for $q=1$ and Boyer-Lindquist coordinates [7,9,28]. It should be interesting to study whether A_c, as given in (6), is the gradient of a scalar function as it was the case in [25]. Besides, we point out that the structure (5) also appears [29] in the Gödel cosmological model [30,31] with $A_i = 0$ and $A_{ij} = -\frac{1}{9} R_{ij}$.

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