More About the Claimed Identity Between Inertial Mass and Gravitational Mass

Raúl A. Rapacioli
Universidad Nacional del Comahue
and Fundación Julio Palacios
Buenos Aires 1400, 8300-Neuquén, Argentina.
E. Mail: rrapacio@uncoma.edu.ar

Following the work recently developed by Guala-Valverde in this journal, we search for the conditions which constrain the choice of arbitrary sets of standards in order to make inertial mass numerically equal to gravitational mass.

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Metric Principle, Coherence And Dimensional Formulae

The *metric principle*, put forward by Fourier and clearly stated by Palacios [1], [2], [3] enables us to apply mathematical machinery to physics:

$$(quantity) = measure * (unit)$$

Symbolically,

$$(A) = A \;\; U_A = A' \;\; U'_A = \cdots = A^{(n)} \;\; U_A^{(n)} \tag{1}$$

Example 1

$$(1 \;\; mile) = 1.609 \;\; (km) = 1{,}609 \;\; (m) = 16{,}090 \;\; (cm)$$

Example 2

$$(G) = 6.67 \;\; 10^{-11} \;\; m^3 \;\; kg^{-1} \;\; s^{-2} = 6.67 \;\; 10^{-8} \;\; cm^3 \;\; g^{-1} \;\; s^{-2}$$

Now we remember that a given set of *units* is *coherent* with the set of independent laws which define a theory, only if the above laws remain unchanged when its symbols are replaced by the corresponding measurements. Both the well-known MKS and cgs systems are coherent with the laws of mechanics.

In the MKS system we adopt the meter, the kilogram and the second as the *independent* primary standards (basic units) which suffice to develop the entire theory without ambiguity. The MKS system is a subset of the 7-dimensional system named *SI* (Système International d'Unités).

In the second law of Newton, $f = md^2/dt^2$, the newton (N) is the *only* coherent unit of force in the MKS system. If we wish to work with meters, kilograms, seconds and poundals, the second law becomes $f \;\; (poundal) = (1/0.138255) \;\; md^2/dt^2 \;\; (kg \;\; m \;\; s^{-2})$ The price to pay for this *arbitrary* choice is the introduction of a *parasite coefficient*, $(0{,}138255)^{-1}$, in the second law.

The actual meaning of dimensional formulas such as $[X] = L^l \;\; M^m \;\; T^t$ is [2]

$$U_X/U'_X = (U_L/U'_L)^l \;\; (U_M/U'_M)^m \;\; (U_T/U'_T)^t \tag{2}$$

where U_L, U_M, U_T are the units (*i.e.*, physically defined standards) employed to measure length, inertial mass and time with the aid of a coherent system of units, say the MKS. The set $\{U'_r\}$, $r = L, M, T$ means the corresponding units pertaining to another coherent system such as the cgs.

Note that each parenthesis is a *real positive number* [4], which means that the quotient U_X/U'_X is also a positive real number. Let $\{U_r\}$ be the set of basic units valid in the MKS system (*i.e.*, meter, kilogram, second) and $\{U'_r\}$ the set which defines the cgs system (*i.e.*, centimeter, gram, second). Let X be energy. Using eq. (2) we get:

$$U_E/U'_E = (1 \;\; m/1 \;\; cm)^2 (1 \;\; kg/1 \;\; g)^1 (1 \;\; s/1 \;\; s)^{-2} = (100)^2 (1000) = 10^7$$

which means that the MKS unit of energy (Joule) is 10^7 times greater than the cgs unit of energy (erg).

If X labels *viscosity*, m, then:

$$U_m/U'_m = L^{-1} \;\; M^{1} \;\; T^{-1} = (100)^{-1}(1{,}000) = 10$$

That is, the MKS unit of viscosity is ten times greater than the cgs unit.

Ad Hoc Systems Of Units

From time to time some authors attempt to make *numerically equivalent* the measures of the quantities of two different magnitudes: mass and energy are one example.

Let us consider Einstein's mass-energy relation,

$$E = m \;\; c^2 \tag{3}$$

In the above equation c^2 plays the same role as G in gravitation and $(k/2)$ in statistical mechanics, being k the Boltzmann's constant

[5]: all are *numerical and dimensional links* between two physically different magnitudes. Here, $c = 3\ 10^8\ \ m\ \ s^{-1}$ in the MKS system and $3\ 10^{10}\ \ cm\ \ s^{-1}$ in the cgs system. If, for some purposes, we wish to write eq. (3) in the form $E = m$ then we are compelled to build a *special system* of standards. On account of eqs. (1) and (2) we get

$$c'/c = L^1\ \ T^{-1} ... = (U_L/U'_L)\ \ (U_T/U'_T)^{-1} \tag{4}$$

and we may modify the MKS system, also preserving the meter and the kilogram. In such a case, *necessarily* we must adopt a new standard for time, U'_T, defined by eq. (4). Now it will be $c' = 1$, $c = 3\ 10^8\ \ m\ \ s^{-1}$, $U_L = U'_L = 1\ \ m$, $U_T = 1\ \ s$. With the above, eq. (4) gives us $(1/c) = (U'_T/U_T)$. Thus, $U'_T = U_T/c = (1/3\ 10^8)\ \ s$.

Keeping the kilogram and the second, if we modify the unit of length, we get $U'_L = 3\ \ 10^8\ \ m$.

Both the new set of units, (m, kg, U'_T) and (U'_L, kg, s) ensure $c' = 1$ and, thus, $E = m$. It must be emphasized that, despite $c' = 1$ in the above *ad hoc* systems, c' also retains its dimension. In the first case the quantity *velocity of light in vacuum amounts* $c' = 1\ \ m/1\ \ U'_T$. If we coin the name *tau* for U'_T, then we would express the velocity of light as being equal to 1 *meter/tau*. Analogously, If we coin the name *lambda* for U'_L in the second case, then we would read the velocity of light as being equal to 1 *lambda/s*.

The Gravitational Constant G

As Schrödinger, Palacios and Guala-Valverde have shown [4] [6], the gravitational constant G is a dimensional quantity, with measurement different from 1 in both the customary MKS and cgs systems. Nevertheless, we are free to modify the above systems in order to get $G' = 1$.

Let us start with the MKS system and search for a new unit of mass which ensures $G' = 1$, also preserving the meter and the second as primary standards of length and time, respectively. Now our startpoint is $G = 6.67 \; 10^{-11}$ *MKS units*, $G' = 1$ *modified units*, $U_L = U'_L = 1 \; m$, $U_T = U'_T = 1 \; s$, $U_M = 1 \; kg$. From the dimensional formula for G, $[G] = L^3 \; M^{-1} \; T^{-2}$, and eqs. (1) and (2) we get:

$$G'/G = L^3 \; M^{-1} \; T^{-2} = 1 \; (U_M/U'_M)^{-1} \; 1$$

from which it follows that $U'_M = U_M \; (1/G) = (10^{11}/6.67) \; kg$, *i.e.*, some ten million tons.

Coining the name Einstein, *Ein*, for this new standard of mass, then we can express the gravitational constant G' as $1 \; m^3 Ein^{-1} s^{-2}$.

Other authors prefer to change the *second* in the cgs system, also preserving the *centimeter* and the *gram*. In this case it will be $G = 6.67 \; 10^{-8}$ cgs units and $U'_T = U_T/\sqrt{G} = (10^4/\sqrt{6.67}) \; s$.

Unwanted Problems with *Ad Hoc* Systems

As is well known, the usual MKS and cgs systems are *coherent* with the entire set of laws and definitions usually employed in the realm of mechanics. Thus, when we write $f = m \quad a$, the force is expressed in newtons (N), while length, mass and time are respectively measured in meters, kilograms and seconds (MKS). How does the second law of motion read when we choose another set of fundamental units such as the (*meter, Ein, s*)? The alteration of the mass standard is, of course, reflected in the related unit of force. The dimensional formula of force governs the above alteration. In fact, from $\left[F\right] = L \quad M \quad T^{-2}$ we get $U'_F / U_F = 1 \quad (Ein/kg) \quad 1$, from which it follows that $U'_F = \left(10^{11}/6.67\right) \quad U_F \approx 1.5 \quad 10^{10}\, N$. In other words, the alteration of the mass standard in the MKS system, in order to get $G' = 1$, obliges us to reject the *newton* as the derived coherent unit of force and adopt U'_F in order to achieve coherence with the second law of motion. Coining the name Galilei (*Ga*) for the new coherent standard, we get $1 \quad Ga \approx 1.5 \quad 10^{10} \quad N$.

The same above argument can be applied, *mutatis mutandis*, to the remaining magnitudes as energy, linear momentum, torque, viscosity, surface tension, *etc*.

Briefly speaking, the arbitrary alteration of at least one of the basic standards in the systems in use, in order to get numerical equivalence between two different magnitudes, *destroys* the coherence for the remaining magnitudes.

Dimensional Analysis In Gravitational Theory

Despite its usefulness, Dimensional Analysis is a little acknowledged branch of *mathematical analysis* [3] [4], often forgotten by physicists. As a matter of fact, this tool has been successfully applied by Guala-Valverde in the realm of Relational Mechanics, a recently developed far-reaching theory consistent with Mach's principle [7], [8], [9]. In its original version the cosmological function Φ provides the link between gravitational and inertial mass becomes $\Phi_{ASSIS} = \left(2 \ p \ \ x \ \ r_{go} \ \ G/3 \ \ H_o^2\right)$ and, in order to recover classical mechanics, we are obliged to make $\Phi = 1$, dimensionless. With the above constraint, inertial mass becomes the same as gravitational mass. A later version of Relational Mechanics [5] offers us $\Phi_{GV} = \left(2 \ p \ \ x \ \ r_{go}/3 \ \ H_o^2\right)$, without including G. The choice between the two above formulations is not merely a question of taste since it carries physical consequences.

a) Taking $\Phi = 1$ as dimensionless, hides the true dependence of inertial mass density on the square of gravitational mass density, $r_i \propto r_g^2$, as recently stated in this journal [4], [5]. Moreover, the choice $\Phi = 1$ leads to $G \propto H_0^2 / r_{go}$ [7], [8], [9], when indeed we have $G \propto H_0^2 / r_{io}$, according to Dirac [10].

b) Taking $\Phi = 1$ as dimensionless, the Newtonian gravitational force reads as $G \ \ m_{g1} \ \ m_{g2}/r_{12}^2$ and, therefore, scales as $1/r_{go}$. The above fact is undesirable since in order for Relational Mechanics to make sense, local forces (*i.e.*, *near zone* gravitational, elastic, electromagnetic, nuclear, *etc.*) cannot depend upon the whole distant material Universe. Only the inertial reaction to local forces must be non local, *i.e.*, governed by distant matter. This ambiguity disappears at once when we take Φ_{GV}, since in this case the influence of non local parameters is embodied in the $m_i = \Phi_{GV} \ \ m_g$ term.

Concluding Remarks

It is strange that today, without any valid reason, and despite its manifest validity, Dimensional Analysis remains for many people a rather obscure and metaphysical branch of mathematical physics. It is worthwhile to recall classical statements on this issue [11] [12]. Fortunately, modern authors begin to pay attention to Dimensional Analysis in elementary textbooks [13] and we sincerely hope this tendency will gain momentum.

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