A New Theorem in Relational Mechanics

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We probe a theorem recently advanced in this journal.

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Introduction

In a couple of recent papers we pointed out that gravitational mass, a *primary magnitude,* has intrinsic, essential, differences from inertial mass, a derived, *secondary* magnitude [1], [2], [3]. A primary magnitude cannot be derived, up to now, from other previously known properties. Moreover, we get [1] the right connection between inertial mass density, r_i, and gravitational mass density, r_g.

$$r_i = \Phi_{GV}\ r_g \propto r_g^2 \tag{1}$$

Wherein, in obvious notation, $\Phi_{GV} \equiv \left(2\, p\, x\ r_g / 3\, H_0^2\right)$. Equation (1) comes from the following relation, valid for every material particle k in the universe [1], [2].

$$m_{ik} = \Phi_{GV}\, m_{gk} \propto r_g \qquad (2)$$

Equation (1) has a clear physical meaning: an increase in the density of inertial mass arises from two different causes:

Theorem

1. An increase in the number of galaxies also increases r_g and, consequently, r_i (“cumulative effect”, taken into account in both classical and Einsteinian mechanics).
2. Given equation (2), the increase in the density of gravitational mass also increases the *individual inertial mass* of each particle (a “Machian” effect which only derives from relational mechanics [3]).

Proof

Adding equ. (2) for the N particles contained in an arbitrary volume V, we get $\sum_{k=1}^{N} m_{ik} = \Phi_{GV} \sum_{1}^{N} m_{gk} \propto r_g \sum_{k} m_{gk}$. If, keeping the volume and Hubble´s constant unchanged, N changes to $N + dN$, then $d\left(\sum_{k} m_{ik}\right) \propto d\left(r_g\right) \sum_{k} m_{gk} + r_g\, d\left(\sum_{k} m_{gk}\right)$. Bearing in mind that $r_g \equiv (1/V) \sum_{k} m_{gk}$, the above relation becomes

$d\left(\sum_k m_{ik}\right) \propto V\left(2\,\mathbf{r}_g\,d\mathbf{r}_g\right) = V\,d\left(\mathbf{r}_g^2\right)$ which means that $(1/V)\sum_k m_{ik} \equiv \mathbf{r}_i \propto \mathbf{r}_g^2$. QED.

Equation (1) must be considered a straightforward consequence of the *Theory of Relational Mechanics*, an *entirely relativistic* mechanics, recently developed by Assis [4], [5].

Our above considerations enhance the role of Dimensional Analysis [6] in the formulation of straightforward algorithms able to describe physical facts without ambiguities. These algorithms must preserve the relevant distinction which really does exist between two related different magnitudes. The differences can be qualitative, epistemological, and dimensional.

Thermodynamics provides us another related interesting example: Following Carnot we know that $Q_1/T_1 = Q_2/T_2$, wherein Q_1 and Q_2 mean the input and output heat in an ideal cyclic machine working between the absolute temperatures T_1, T_2. The above ratios can be expressed in *cal/abs.degree, J/ºK*, etc. As far as we know, no author has never adopted an *ad hoc* system of standards in order to get the meaningless equation $Q = T$. As everybody knows, the core of thermodynamics is anchored to the once largely ignored distinction between heat and temperature. The above distinction has been clarified by the lasting works of Black, Davy, Rumford, Mayer, Joule, Thomsom, Helmholtz, and others.

In statistical mechanics, *one half* of the *Boltzmann constant* provides us the link between average mechanical energy per degree of freedom and absolute temperature, $\langle E\rangle = (k/2)T$, where $k = 1.38\,10^{-16}$ *erg/absolute degree.* [1], [2]. Just as it would be a mistake to identify Q or $\langle E \rangle$ with T, it is a mistake to identify inertial with gravitational mass; a mistake that hides the possibility of a consistent Machian (relational) mechanics.

Appendix

From eqs. (1) and (2) we derive [1] $\left(3\,H_0^2/2\,p\ x\ r_i\right) \equiv G = 6{,}67\,10^{-8}\ cm^3 g^{-1} s^{-2}$, in agreement with the calculation advanced by Dirac in 1938 [7]. Now we rewrite equ. (2) as $m_g = \sqrt{G}\ m_i$. The last equation allows us to grasp the "size" of the standard of gravitational mass in terms of the most familiar standards of inertial mass [2]. Thus, in the cgs system, a body having 1 *Unit* of gravitational mass has an inertial mass amounting to $1/\sqrt{G} \approx 4\,10^3\,g$, i.e. some 4 *kg*. From the first law of Newtonian gravitational theory [1], $F = m_{g1}\,m_{g2}/r^2$ and $[F] = L^1 M^1 T^{-2}$ we deduce the *dimensional formula* for gravitational mass [6],

$$[m_g] = L^{3/2} M^{1/2} T^{-1} \tag{3}$$

Wherein the Maxwell's bracket means the ratio of the standards employed to measure gravitational mass in two *coherent* systems of units (such as the cgs and the MKS). Thus, $[m_g] \equiv U'_{gm}/U_{gm} = a\ positive\ \ real\ \ number.$ The symbols $L \equiv U'_L/U_L$ for length, and M and T, for inertial mass and time, respectively, have the same meaning [6].

On account of equ. (3) we get $U'_{gm}/U_{gm} = (100/1)^{3/2}(1000/1)^{1/2}(1/1)^{-1} = 3.1623\,10^4$, for U'_{gm} and U_{gm} the MKS and cgs standards of gravitational mass, respectively.

Due to historical arguments [6], [8] we propose the name *schrödinger* (*Sch*) for the cgs standard, U_{gm}, and the name *palacios* (*Pal*) for the MKS standard of gravitational mass, U'_{gm} Thus, 1 *Pal* = 31,623 *Sch*, despite the fact that 1 *kg* = 1,000 *g*.

Einstein [9] based his GTR upon the relation $m_g = m_i$, which, according to our above analysis, is physically and dimensionally untenable. He said: *"We achieve the equality of the two masses by choosing a suitable set of standards."* But, in practice, he forgot to describe his suitable set of standards.

The very strong constraint $m_g = m_i$ precludes the positive implementation of Mach's Principle, which requires that $m_i / m_g = f\left(\mathbf{r}_g, H_0\right)$, $\mathbf{r}_g$ being the average matter density of the distant universe (galaxies) and H_0 the Hubble's constant [4], [2], [10], [11].

We cannot refrain from quoting Maxwell in reference to the Priestley-Coulomb law of electrostatics [13]:

> *We may now write the general law of electrical action in the simple form* $F = e e' r^{-2}$*... If [Q] is the concrete electrostatic unit of quantity itself, and e, e′ the numerical values of particular quantities, if [L] is the unit of length,..., then the equation becomes*
>
> $$[Q] = \left[L^{3/2} M^{1/2} T^{-1}\right] \tag{4}$$
>
> *Other units may be employed for practical purposes, and in other departments of electrical science, but in the equations of electrostatics, quantities of electricity are understood to be estimated in electrostatic units, just as in*

> *physical astronomy we employ a unit of mass which is founded on the phenomena of gravitation, and which differs from the units of mass in common use.*

The view advocated by Maxwell was embodied by Schrödinger in his relevant (but little known) work in which he acomplished the first mathematical implementation of Mach´s Principle [8]. In fact, he wrote the modified gravitational potential energy in terms of gravitational masses, without including G.

Palacios was able to develop a sound and rigorous vectorial theory of dimensional analysis based upon the ideas of Fourier [6], [13]. In his theory, the squared brackets mean the *ratio* of two *coherent* units (i.e. they are *positive real numbers*), instead of the units themselves, as claimed by Maxwell.

The ideas of Maxwell concerning dimensional analysis, when properly updated, are entirely consistent with our actual views. Translating equ. (4) to modern symbolism [14], [15], [16], [17] we get, according to Maxwell (Ref. 13, chapter 1): $U_Q = (U_L)^{3/2}(U_T)^{-1}(U_M)^{1/2}$, a *symbolic operationally undefined* relation between coherent units, say the cgs ones [18].

Taking another coherent system of units, such as the MKS, it will be $U'_Q = (U'_L)^{3/2}(U'_T)^{-1}(U'_M)^{1/2}$. On account of the above relations we get: $U'_Q/U_Q = (U'_L/U_L)^{3/2}(U'_T/U_T)^{-1}(U'_M/U_M)^{1/2}$, an *algebraic, operationally defined* equation, nowadays written in the form $[Q] = L^{3/2}M^{1/2}T^{-1}$. It is worthwhile to compare the last equation with equ. (3).

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