A Simple "Classical" Interpretation of Fizeau's Experiment

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It is well known that *Stokes's aether dragged theory* is one of the best classical aether theories, since it is in agreement with almost all experimental results (see for instance R. Resnick's popular textbook: *Introduction to Special Relativity*). This theory is usually dismissed on the grounds of *two* natural phenomena, which are said to be "unexplainable" in Stokes's conceptual context: *Bradley's astronomical annual aberration*, and the speed of light in moving water (*Fizeau's experiment*). In this paper, a simple "time-delay" model for the behaviour of light in a transparent medium is given, which at least gets rid of the second of the two previous objections.

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It is well established that (in an "aether-frame"—which could be a terrestrial laboratory, at least in a first approximation, according to Stokes's hypothesis) the speed of light *in vacuo* c becomes c/n (where $n > 1$), when light travels through a transparent medium, for instance water (the *refraction index n* depends on wave length—henceforth we shall speak of a *monochromatic* beam). So, if light travels through a given length L in water, then, instead of the time L/c, it would take a time:

$$\frac{L}{\left(c/n\right)}=\frac{nL}{c}.$$

Thus, we can introduce the delay:

$$\frac{nL}{c}-\frac{L}{c}=\frac{(n-1)L}{c}, \tag{1}$$

and make the natural assumption that this delay is the contribution of many *single delays*, due to the total number of "obstacles" that light meets during its travel.

If we call $\mathbf{t}$ this single delay, and N the number of obstacles of the given medium for unit length, we can write:

$$\textit{total delay}=\frac{(n-1)L}{c}=NL\mathbf{t}, \tag{2}$$

from which we get:

$$\mathbf{t}=\text{single delay}=\frac{(n-1)}{cN}. \tag{3}$$

Let us suppose now that, in the given length L, the water *moves* with some (uniform) speed v (for instance in the same direction as the light), with respect to the fixed aether-frame, and ask: *what will be the delay of the light be then*?

In 1817 Fresnel theorized that, because the aether present in the water would have been "dragged," the light would have been dragged too, and for this reason the delay would have been *less* than in the previous case.

In 1851 Fizeau confirmed Fresnel's prediction, showing that the speed of light $c(v)$ is, in this case, experimentally compatible with the expression:

$$c(v) = \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right). \tag{4}$$

This would imply only a *partial dragging* of the aether, since one gets (4) instead of the (*a priori* more naturally expected?!) expression:

$$c(v) = \frac{c}{n} + v\,. \tag{5}$$

Fizeau's result is still used for two important, though different, purposes:

(A) to show that any aether theory must acknowledge the fact that the aether cannot be "completely dragged" by heavy bodies, and so to disprove Stokes's theory with one more argument (the other is Bradley's aberration);

(B) to give further evidence in favour of the relativistic composition of velocities, since the "sum" of the two speeds c/n and v is indeed, from the relativistic point of view, not equal to (5), but to:

$$c_{rel}(v) = \frac{\left(v + \frac{c}{n}\right)}{\left(1 + \frac{v}{nc}\right)} = \frac{c(1 + n\boldsymbol{b})}{n\left(1 + \frac{\boldsymbol{b}}{n}\right)} \tag{6}$$

(where, as usual, $\boldsymbol{b} = v/c$, and the subscript "*rel*" stands, of course, for "relativistic").

As far as (A) is concerned, we prefer to conjecture that the possibility that the aether is being dragged by the Earth, during its motion around the Sun, is quite *unlikely*, and that on the contrary one should perhaps better suppose that *it is the aether that drags the Earth*

(*Descartes-Leibniz vortex theory*), and so Fizeau's experiment cannot say anything about this case.

With regard to (B), one must indeed acknowledge that (6) is a fairly good result in favour of Special Relativity (SR), since one can approximate this expression in order to get:

$$\frac{c(1+nb)}{n\left(1+\frac{b}{n}\right)} \cong \frac{c}{n}(1+nb)\left(1-\frac{b}{n}\right) \cong \frac{c}{n}\left(1+nb-\frac{b}{n}\right)$$

(up to higher order terms in b), which yields indeed:

$$\frac{c}{n}+v-\frac{v}{n^2},$$

a formula which is, surprisingly, identical with the experimental *datum* (4)!

But the very important question to ask is: *what would be a very good aether-theory prediction for the value of c(v)*, different from (5)?

As a matter of fact, one should perhaps conjecture that *the aether is not dragged at all by the moving water*, and that the only physical phenomenon we are dealing with in this case is that the light, during its travel through the moving water (say for a time Δt), simply meets *fewer* obstacles, and that the single delay for each obstacle is (for instance in the case of water moving in the same direction as the light) *less* than (3).

Let us now try to compute the delay of light with the two aforesaid assumptions, obviously assuming $N(L - v\Delta t)$ as the total number of obstacles, and the following expression as a possibly correct value for each single delay:

$$t(v) = \frac{(n-1)(c-v)}{c^2 N} = t(1-b). \qquad (7)$$

This is in truth the simplest *linear* (in the parameter v) function of $t(v)$, such that it does coincide with (3) when $v = 0$, and which is zero when $v = c$.

Thus, with this value for $t(v)$, we have:

$$\Delta t = \frac{L}{c} + \text{delay} = \frac{L}{c} + N\left(L - v\Delta t\right) t\left(v\right)$$
$$= \frac{L}{c} + \frac{(n-1)\left(L - v\Delta t\right)\left(c - v\right)}{c^2},$$

which implies:

$$\left[c^2 + v\left(n-1\right)\left(c - v\right)\right]\Delta t = L\left[v + n\left(c - v\right)\right]$$
$$\Delta t = \frac{L\left[b + n\left(1-b\right)\right]}{c\left[1 + \left(n-1\right) b \left(1-b\right)\right]}$$

and then, at last:

$$c_{cls}\left(v\right) = \frac{L}{\Delta t} = \frac{c\left[1 + \left(n-1\right) b\left(1-b\right)\right]}{\left[b + n\left(1-b\right)\right]}$$
$$= \frac{c\left[1 + \left(n-1\right) b\left(1-b\right)\right]}{n\left[1 - \frac{\left(n-1\right) b}{n}\right]} \tag{8}$$

(in the previous formula, the subscript "*cls*" obviously stands for "classical").

From this last identity, we can deduce the following approximation, again up to higher order terms in b:

$$c_{cls}(v) \cong \frac{c}{n}\left[1+(n-1)b\right]\left[1+\frac{(n-1)b}{n}\right]$$

$$\cong \left[1+\frac{(n^2-1)b}{n}\right] = \frac{c}{n}\left[1+nb-\frac{b}{n}\right] = \frac{c}{n}+v-\frac{v}{n^2},$$

which is once again equal to the experimentally supported value (4)!

At this point, one could even note that:

(*i*) (7) is indeed an approximation, up to higher order terms in b, of the expression:

$$t_0(v) = \frac{(n-1)}{(c+v)N}, \tag{9}$$

and that, if one makes use of this value, instead of (7), in the previous computations, then one would get as a final result *exactly* (6), that is to say the relativistic expectation, in place of (8);

(*ii*) both (6) and (8) give, of course, the "correct" *limit value*, namely, $c_{rel}(c)=c$ and $c_{cls}(c)=c$ (the first identity is a consequence of the invariance of the speed of light in all *inertial* frames, according with SR; the second, due to the fact that in the proposed model, when the water travels at speed c, there would be no more collisions with the obstacles).

Conclusion

Summing up, the previous very simple "logical" argumentation shows, in one more case, how completely different theories (actually, SR and a very "natural" aether theory) can sometimes give the *same* experimental previsions.

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Remark 1

This paper is inspired to the ideas contained in G. Antoni's: "Una nuova interpretazione dell'esperienza di Fizeau, relativa al trascinamento della luce da parte del mezzo rifrangente in moto," *Atti della Fondazione G. Ronchi*, Anno VIII, N. 1, Pubblicazioni dell'Istituto Nazionale di Ottica, Serie IV, N. 143, Arcetri, Firenze, 1953.

Remark 2

Information about Fizeau's experiment can be found, for instance, in E.T. Whittaker's: *History of the Theories of Aether and Electricity*, Dublin University Press Series, 1910, Chap. IV. Fizeau's experiment was shortly followed (1868) by an attempt by Hoek to detect, using the same "idea," a possible "absolute" speed of Earth, which we may call w. As Whittaker points out, if formula (4) holds, then Hoek's experiment should indeed give a "null result," as it did(!), even if w was different from zero; but, of course, a null result for this experiment should be foreseen in the case $w = 0$ too, which is Stokes's hypothesis. Once again, we should conclude that some experimental evidence might not be sufficient to discriminate between very different theoretical interpretations.