Is the Theory of Relativity Self-consistent?

Reply by the author to the comment by Vladimir Onoochin on the paper

"Remarks about Correspondence of Relativity and Causality Principles"

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It is shown that the comment by Vladimir Onoochin to the mentioned paper contains some irrelevant assertions. At the same time, the comment recalls a recognized statement that the theory of relativity is internally consistent theory, and the revealed in [1] paradox comes into a certain contradiction with this statement. A goal of the present paper is to answer on the entitled question.

In a recent issue of Apeiron my paper "Remarks about correspondence of relativity and causality principles" has been published [1], as well as a comment by Vladimir Onoochin [2], where he claims that a new relativistic paradox revealed in the paper seems to be questionable. His particular remarks can be formulated in short form as follows:

1. An expression for the metrics of space-time in the case of a constant homogeneous gravitational field seems to be questionable.
2. What do different observers actually see in the case of a uniformly accelerated frame? Onoochin considers a case where the chain of re-emitters is in an inertial reference frame, while an external observer moves at a constant acceleration.
3. It is impossible to create an experimental setup on the current technological level to test the revealed paradox.

Finally, he makes a general remark already mentioned in the abstract:

This paper is in certain contrast with the majority view that the relativistic theory is an internally consistent theory, which can only be falsified by experiment.

Before consideration of the latter statement, I would like to give a reply to the remarks 1-3.

1. The expression for metrics of space-time (formula (13) in [1]) has been derived not for a case of constant gravitation field, but for a uniformly accelerated rigid frame, proceeding from its definition (10) [1]. The same definition of uniformly accelerated rigid frame and the same expression for metrics are used in [3]. In the case of the constant homogeneous gravitation field I even do not introduce any expressions for the metrics of space-time, I directly use a well-known equation (6), describing a change of rate of clock in a weak gravitation field.
2. Vladimir Onoochin writes: “It is difficult to understand from the content of the paper [1] which frames the observer and the chain of the emitters are being in.” However, it seems obvious that the section 2.3 deals with a case where the chain of the emitters and

corresponding non-inertial observer are in the same rigid accelerated frame. In any other case it would be impossible to ascribe any physical meaning to Eqs. (18)-(23). It is also clear that the inertial observer introduced is external with respect to this chain of the emitters. Moreover, there is an exact definition of his inertial frame: namely, we choose an external inertial reference frame in which an observer sees the simultaneous arrival of two light pulses (left and right) at RL_0 and RL_1, correspondingly. In contrast, Vladimir Onoochin considers a case, where a chain of re-emitters is in inertial frame, while an observer moves at some constant acceleration with respect to this chain. Thus, Onoochin considers a different problem from the one in the paper, and his calculations are irrelevant.

3. Paradox in [1] was not invented for purposes of practical realization; it is actually impossible on the current technological level. A crucial test of relativity can be made in other experiments, *i.e.*, those proposed in [4, 5]. Nevertheless, the paradox has its own significance, indicating a contradiction between relativistic postulates and causality principle.

It is obvious that the paradox requires a resolution. *Indeed, if the theory of relativity is self-consistent, then an error in my arguments does exist. If there are no errors in the paradox, then the theory of relativity should be not self-consistent.* In this case it is necessary to find logical errors in the foundations of special theory of relativity (STR). In the author's opinion, such a logical error actually exists.

Let us write Einstein's two well-known postulates:

1. All inertial frames have equal rights, they are equivalent.
2. The velocity of light in vacuum *c* does not depend on the velocity of the emitter.

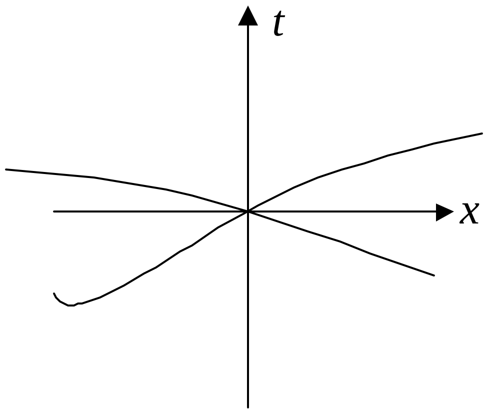

Fig. 1. World lines for two particles intersecting in the origin of coordinates of some inertial reference frame

It has been recognized that a direct inference of these postulates is an invariance of space-time interval

$$s^2 = c^2 t^2 - r^2, \quad (1)$$

where t, $\vec{r}$ are the time and space intervals, correspondingly. In turn, invariance of s leads to the Lorentz transformations, which describe all effects of relativistic kinematics.

Now let us reconstruct a derivation of STR from Einstein's postulates. Following Einstein, let us choose two points A and B separated by some distance $\vec{r}$ in an inertial reference frame K. Then the propagation time of light between the point A and B is equal to r/c, and the space-time interval between the events of emission of the light pulse from the point A and arrival of this pulse at point B is equal to zero. Due to invariance of c, an observer in any other inertial reference frame K' measures the time of light propagation between the points A and B as r'/c, where r' is the difference of space coordinates between the events of emission of light from the point A and arrival of light at the point B in K'. Hence, $s'=0$, and the space-time interval goes to zero simultaneously in all inertial reference frames. From there one can prove by well-known arguments, that the space-time interval is invariant.

We state that this evidence is not complete. Indeed, from a formal point of view, vanishing of s is possible in two different ways: 1) $t=r/c$, 2) t, $r=0$, and the second way has been lost in the evidence. At the first sight, the case t, $r=0$ seems to be trivial. Nevertheless, its careful analysis leads to some additional questions. Indeed, the equality t, $r=0$ corresponds to intersection of two world lines from two particles or short light pulses. This situation is depicted for the one-dimensional case in Fig. 1.

There arises a question: is the same intersection detected in all other inertial frames? It is clear that the answer on this question should be positive due to the requirement of the causality principle*. Thus, the space-time interval actually simultaneously vanishes for all inertial observers, which allows us to prove a theorem about invariance of s. However, now we have to recognize, that this theorem follows not only from Einstein's postulates exclusively, but under additional involvement of the causality principle. Now let us ask the next question: do we have a right to apply simultaneously Einstein's postulates and the causality principle, in developing the mathematics of STR? It is obvious that we would get this right only in a single case: namely, if we prove that the postulates and the causality principle are in accordance with each other. However, no one has presented such a proof. A finiteness of light velocity provides

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* The causality principle can be defined by two requirements: 1. A cause-consequence order of events is absolute. 2. The events, which could cause essential inferences (for example, collision of particles), are absolute.

conformity only with requirement 1 of the causality principle, but there is no reason to assert a correspondence of the relativity theory with requirement 2, as the paper [1] demonstrates. Thus, a logical chain of STR is not perfect; this theory involves the causality principle without sufficient justification for construction of mathematical apparatus of this theory. Therefore, a search for a possible contradiction between the causality principle and the theory of relativity seems to be quite a correct problem from the standpoint of formal logic. Moreover, the effectiveness of this search can be greatly increased in the following way.

Thus, we revealed two ways for vanishing of s:

$t=r/c$ – here a simultaneous implementation of the equality $s=0$ in all inertial reference frames follows from the Einstein's postulates;

t, $r=0$ – here a simultaneous implementation of the equality $s=0$ in all inertial reference frames follows from the causality principle (its requirement 2).

Now let us introduce the definitions:

1. we will call two events "related", if they belong either to the same world line, or to different world lines, but which had points of intersection at an absolute past;
2. we will call two events "unrelated", if their world lines did not have points of intersection at an absolute past.

According to the causality principle these definitions are invariant in relation to any transformations of reference frames. An example of "related" events is the emission of a light pulse from one point A and its arrival at another point B. An example of "unrelated" events is shown in Fig. 2.

It can be shown that inside of class of "related" events a theorem about invariance of space-time interval can be proved without

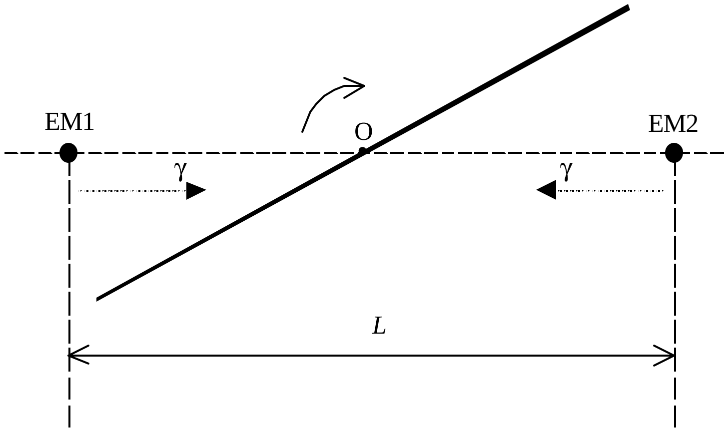

Fig. 2. A rod with the proper length L, in rotation in some inertial reference frame, simultaneously touches at its opposite ends the emitters of light EM1 and EM2. At that instant both emitters emit short light pulses toward one another. The instants of contact represent "unrelated" events, a meeting of both light pulses means a vanishing of s by means of the equality t,r=0. It can also be seen that for "unrelated" events the space-time interval is always space-like.

involving the causality principle. Inside the class of "unrelated" events, a vanishing of s is possible only through the equality t, r=0, and a simultaneous vanishing of s for all inertial observers follows from the causality principle. *Thus, a possible contradiction between the relativistic postulates and the causality principle may exist only in phenomena dealing with "unrelated" events.*

A search for such allowed phenomena led me to formulate a problem considered in ref. [1]. It can be seen that before the hypothetical meeting of both light pulses the events of absorption and emission of these pulses by different re-emitters are "unrelated". This gives an additional reason to conclude that a contradiction between the relativity theory and causality principle revealed in [1] does not follow from erroneous calculations; it only means that a joint application of Einstein's postulates and the causality principle in developing the mathematical apparatus of the theory of relativity was not justified.

Finally, I would like to make a short self-comment on the paper [1]. Its sections 2.1 and 2.2. can be considered as an introduction, and their results contain no contradiction with a recognized viewpoint on the theory of relativity. A crucial section is 2.3, where a contradiction with the causality principle has been found. In my opinion, all questions and doubts should be focused on the formula (11)-(23), which indicate a meeting of two short light pulses in a uniformly accelerated frame. I continue to believe that the expressions (11)-(23) are quite correct.

Conclusion

Thus, an implicit application of the causality principle in deriving STR has been revealed. Hence, in order to close a logical chain of the theory of relativity, it is necessary to prove the compatibility of the Einstein's postulates and the causality principle. Moreover, such a proof should be made without any calculations, because the mathematical apparatus of the theory of relativity is already an inference from the joint application of the relativity and causality principles. A particular example—the problem considered in section 2.3 of ref. [1]—indicates that such a general proof is impossible. Indeed, we can say nothing about possible intersection (or non-intersection) of two light pulses before any calculations. Under these conditions there is only a single natural way to develop STR: *to propose* that the Einstein's postulates and the causality principle are compatible, *to develop* a mathematical apparatus of STR under joint application of these principles, and *to check* a conformity of relativity and causality principle *using* this apparatus. The paper [1] shows that the result of such a test is *negative.*

At the same time, regardless of the result of such a test, in a restricted meaning the STR continues to be a self-consistent theory. Indeed, a reverse statement would be true, if we find an internal contradiction with the Einstein's postulates—for example, if we derive an existence of an "absolute" frame using relativistic calculations. However, it seems impossible. In contrast, a problem about correspondence of relativity and causality principle belongs to the problems about physical validity of STR, and its present resolution in [1] indicates that STR seems to be physically incorrect. Hence, we have to propose another system of postulates, and again to check their compatibility with the causality principle, using a mathematical apparatus derived from these postulates. In principle, there exists another way to construct a space-time theory, where the original statements are already in accordance with the causality principle. The author applied this method [4, 6] in developing "covariant ether theories", which, like STR, satisfy all known experimental facts gathered up to now.

Acknowledgment

I thank Vladimir Onoochin to the attention to my paper [1] and for his comment in spite of disagreement with his conclusions.

References

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