Remarks on the Correspondence of the Relativity and Causality Principles

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A particular problem about special kind of two light pulses propagation has been considered in cases of inertial motion, constant homogeneous gravitation field and progressive non-inertial motion with constant acceleration. A contradiction between the causality principle and relativity theory has been revealed.

1. Introduction

It is well-known that two Einstein's postulates form a basis of special theory of relativity (STR):

1. All inertial reference frames are equivalent to each other.
2. A light velocity in vacuum does not depend on a velocity of emitter.

At present time one can see some restrictions on application of these postulates. For example, the second postulate formally is not

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valid in arbitrary co-ordinates. In order to overcome the restrictions, ref. [1] proposes a relativistic postulate in the following short form: a geometry of empty physical space-time is pseudo-Euclidean. Indeed, such a postulate allows to operate with any arbitrary co-ordinates of inertial reference frames and additionally includes into STR a case of non-inertial motion. The latter statement follows from the obvious fact that any arbitrary motion does not influence on a properties of geometry of physical space-time, it continues to be pseudo-Euclidean. (From a formal point of view it means that the curvature tensor is equal to zero in both inertial and non-inertial frames). According to the author's opinion, such a formulation of relativistic postulate allows to deeper understand a physical essence of relativity theory. For example, it allows to advance a problem about correspondence of STR to the causality principle. At the first sight, this problem seems to be trivial due to a finity of light velocity in STR. Nevertheless, the present paper finds some additional questions in this topic in the following particular physical problem.

2. A special case of two light pulses propagation: in inertial reference frame, in constant homogeneous gravitation field, and in rigid non-inertial frame.

2.1. Case of inertial motion

Let us consider the following problem in Cartesian inertial reference frame.

Let a short light pulse be emitted from the point $x=0$ along the axis x. Let a number of re-emitters of light RL_m be located along the x-axis in some points x_m (RL_0 is located in the point $x=0$ and, for simplicity, all $\Delta x_m = x_{m+1} - x_m$ are equal to each other). When a light pulse arrives at each re-emitter, it is absorbed by it, and after a fixed interval of its own time Δt_0 is emitted by RL along the x-axis again.

Further, let the second light pulse be emitted from the point $x=0$ at such moment of time (taken as t=0), when the first light pulse has a coordinate

$$0\langle\Delta x \le x_1. \qquad (1)$$

One requires to find the times t_1 and t_2, where t_1 is the moment of time when the first (right) light pulse is emitted by RL_n, while t_2 is the moment of time when the second (left) pulse is reaching RL_n, and n is some number.

Due to the condition (1), the most general expression for t_1 can be written as

$$t_1 = t_{x_1-\Delta x} + \sum_{m=1}^{n-1} t_m + \sum_{m=1}^{n} \Delta t_m, \qquad (2)$$

where $t_{x_1-\Delta x}$ is the propagation time of the first (right) light pulse from the point Δx to point x_1, t_m is the propagation time of the right pulse from RL_m to RL_{m+1}, and Δt_m is the time interval Δt_0 for RL_m, remitting the right pulse. The general expression for t_2 is:

$$t_2 = t'_{x_1-0} + \sum_{m=1}^{n-1} t'_m + \sum_{m=0}^{n-1} \Delta t'_m, \qquad (3)$$

where t'_{x_1-0} is the propagation time of the second (left) light pulse from the point $x=0$ to point x_1, t'_m is the propagation time of the left pulse from the RL_m to RL_{m+1}, and $\Delta t'_m$ is the time interval Δt_0 for RL_m, emitting the left pulse. From (3) and (2)

$$t_2 - t_1 = \Delta t + \sum_{m=1}^{n-1} (t'_m - t_m) + (\sum_{m=0}^{n-1} \Delta t'_m - \sum_{m=1}^{n} \Delta t_m), \tag{4}$$

where $\Delta t = t'_{x_1 - 0} - t_{x_1 - \Delta x}$.

Now let us ask the question: is it possible to implement the equality $t_2 - t_1 = 0$? It is obvious, such an equality would mean an absolute event: a meeting of both light pulses considered in the spatial point x_n.

This problem has a trivial solution in inertial reference frame. Here $t_m = t'_m$, $\Delta t_m = \Delta t'_m$, and all Δt_m are equal to each other for any m. Hence, $t_2 - t_1 = \Delta t$, and the equality of t_1 and t_2 is impossible. This result means that at the moment of time when the second (left) pulse is reaching RL_n in x_n point, the first (right) pulse already has a space coordinate x_n+*Dx*.

2.2. Case of constant homogeneous gravitation field

In this case

$$t_m = t'_m \tag{5}$$

due to independence of metric tensor on time co-ordinate, and

$$\Delta t_0 \approx \Delta t_m (1 + \frac{j_m}{c^2}) \tag{6}$$

in the approximation of weak gravitation field. Here c is the light velocity in vacuum, and j is gravitation potential. (Further we take j_0=0).

It follows from (6) that all values Δt_m and $\Delta t'_m$ are equal to each other, and

$$\sum_{m=0}^{n-1}\Delta t'_m - \sum_{m=1}^{n}\Delta t_m = \Delta t'_0 + \sum_{m=1}^{n-1}\Delta t'_m - \sum_{m=1}^{n-1}\Delta t_m - \Delta t_n = \Delta t_0 - \Delta t_n = \\ = \frac{\Delta t_0\left(j_n/c^2\right)}{1+\left(j_n/c^2\right)} \approx \Delta t_0 \frac{j_n}{c^2} \quad . \tag{7}$$

in this approximation. Substituting (7) and (5) into (4), one gets:

$$t_2 - t_1 = \Delta t + \Delta t_0 \frac{j_n}{c^2} . \tag{8}$$

Hence, the equality of t_1 and t_2 is implemented under the condition

$$\Delta t = -\Delta t_0 \frac{j_n}{c^2} . \tag{9}$$

Thus, we conclude that for appropriate choice of the parameters in (9), we are able to observe the absolute event: a meeting of two short light pulses in the spatial point x_n.

Case of rigid non-inertial frame

In this Section we will consider the problem in rigid non-inertial frame moving along the axis x at constant (in relativistic meaning) acceleration a, and we will perform the exact calculations due to importance of the results obtained.

By definition, in a rigid frame the proper distance between two spatial points (measured by means of a scale being at rest in this frame) does not depend on time. Let us define such a rigid frame by the relationships

$$x^a = x'^a ; x^0 = t , \tag{10}$$

where x'^a are the space coordinates in successive instantaneously co-moving inertial reference frames, while t stands for the proper time at the origin of coordinates. In such definition, for the case of constant (in instantaneously co-moving inertial frames) acceleration a along the axis x, a relationship between space-time coordinates in a fixed reference frame (T, X, Y, Z) and (t, x, y, z) takes the form [3]:

$$dT = dt(1+\frac{ax}{c^2})ch\frac{at}{c}+\frac{dx}{c}sh\frac{at}{c}, \tag{11}$$

$$\begin{aligned} &dX = cdt(1+\frac{ax}{c^2})sh\frac{at}{c}+dxch\frac{at}{c}; \\ &dY = dy; dZ = dz. \end{aligned} \tag{12}$$

The metrics of space-time determined by (11), (12), is the following:

$$ds^2 = c^2dt^2\left(1+\frac{ax}{c^2}\right)^2-(dx)^2-(dy)^2-(dz)^2. \tag{13}$$

The corresponding components of the metric tensor are:

$$g_{00} = \left(1+\frac{ax}{c^2}\right)^2; g_{0a} = 0; g_{11} = g_{22} = g_{33} = -1, \text{ all others } g_{ab} = 0. \tag{14}$$

The physical values are defined as

$$\begin{aligned} &dx_{\text{ph0}} = \sqrt{g_{00}}dx^0+\frac{g_{0a}dx^a}{\sqrt{g_{00}}}, \\ &\sum x^2_{\text{ph}\acute{a}} = \left(-g_{ab}+\frac{g_{0a}g_{0b}}{g_{00}}\right)dx^a dx^b. \end{aligned} \tag{15}$$

Substituting the components of metric tensor from (14) to (15), we obtain

$$dx_{\text{ph}} = dx, dy_{\text{ph}} = dy, dz_{\text{ph}} = dz; \tag{16}$$

$$dt_{\mathrm{ph}} = dt\left(1+\frac{ax}{c^2}\right). \tag{17}$$

Due to independence of the metric tensor on time, we again get the equality (5), from there

$$\sum_{m=1}^{n-1}(t'_m - t_m) = 0. \tag{18}$$

The intervals of physical time at different points are determined by (17). Hence, the physical values

$$\Delta t_0 = \int_0^{\Delta t_m} dt_{ph}(x_m) = \Delta t_m\left(1+\frac{ax_m}{c^2}\right), \tag{19}$$

and

$$\Delta t_m = \frac{\Delta t_0}{1+\dfrac{ax_m}{c^2}}. \tag{20}$$

Therefore, $\Delta t'_m$ and Δt_m are equal to each other, and the third term in (4) is equal to

$$\begin{aligned}\sum_{m=0}^{n-1}\Delta t_m - \sum_{m=1}^{n}\Delta t_m &= \left(\Delta t_0 + \sum_{m=1}^{n-1}\Delta t_m\right) - \left(\sum_{m=1}^{n-1}\Delta t_m + \Delta t_n\right) = \\ &= \Delta t_0 - \Delta t_n = \Delta t_0 \frac{ax_n}{c^2}\left(\frac{1}{1+\dfrac{ax_n}{c^2}}\right).\end{aligned} \tag{21}$$

Substituting the obtained values (19), (21) into (4), one gets:

$$t_2 - t_1 = \Delta t + \Delta t_0 \frac{a x_n}{c^2}\left(\frac{1}{1+\frac{a x_n}{c^2}}\right). \tag{22}$$

Hence, the left and right light pulses will meet in the point

$$x_n = -\frac{\Delta t c^2}{a\left(\Delta t_0 + \Delta t\right)}. \tag{23}$$

Thus, an observer in an accelerated frame will detect the absolute event: the left and right light pulses will meet in the point defined by (23) (under negative sign of the acceleration a). This conclusion is in agreement with the result of Section 2.2 and the equivalence principle. However, here we meet a quite difficult problem: for observer in inertial frame both light pulses will never intersect.

Indeed, let the process of light pulses propagation in the accelerated frame be observed from some inertial reference frame. Furthermore, let us choose for observing the light pulses propagation process an inertial frame K, such that at the time moment when an observer sees the appearance of the left light pulse in the point $x = 0$, he simultaneously sees an arriving right pulse to RL_1 (such a choice is always possible due to (1)). For this time moment, let us introduce into consideration the second inertial frame K_s shifted along the axis x at such a distance (with respect to K) which is equal to the distance between RL_0 and RL_1. (The relative velocity of K and K_s is equal to zero). Due to the space homogeneity in inertial frames, such a shift is equivalent to re-numeration of the re-emitters in K_s: the RL_m (in K) be RL_{m-1} (in K_s). Hence, the propagation time from RL_0 to RL_{n-1} for the left pulse is exactly equal to the propagation time from RL_1 to RL_n for the right pulse in both K and K_s frames (since the RL_1, RL_n in K are the RL_0, RL_{n-1} in K_s). Hence, at the moment of time (in K) when the right pulse is emitted by RL_n, the left one is emitted by RL_{n-1} for any

n. Therefore, the light pulses considered will never meet in the inertial frame K, that means a contradiction with the causality principle.

Conclusions

Thus, a consistent relativistic consideration of the problem about special kind of two light pulses propagation in a non-inertial reference frame contradicts with the causality principle. One can show that a resolution of this contradiction is possible only under supposition that a geometry of empty space-time is not pseudo-Euclidean [4]. However, it is in a deep disagreement with the relativity theory. It seems that a full resolution of this contradiction within the scope of relativity theory is impossible.

Bibliography

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