Remarks on the Correspondence of the Relativity and Causality Principles

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A particular problem about special kind of two light pulses propagation has been considered in cases of inertial motion, constant homogeneous gravitation field and progressive non-inertial motion with constant acceleration. A contradiction between the causality principle and relativity theory has been revealed.

1. Introduction

It is well-known that two Einstein’s postulates form a basis of special theory of relativity (STR):

1. All inertial reference frames are equivalent to each other.
2. A light velocity in vacuum does not depend on a velocity of emitter.

At present time one can see some restrictions on application of these postulates. For example, the second postulate formally is not

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valid in arbitrary co-ordinates. In order to overcome the restrictions, ref. [1] proposes a relativistic postulate in the following short form: a geometry of empty physical space-time is pseudo-Euclidean. Indeed, such a postulate allows to operate with any arbitrary co-ordinates of inertial reference frames and additionally includes into STR a case of non-inertial motion. The latter statement follows from the obvious fact that any arbitrary motion does not influence on a properties of geometry of physical space-time, it continues to be pseudo-Euclidean. (From a formal point of view it means that the curvature tensor is equal to zero in both inertial and non-inertial frames). According to the author’s opinion, such a formulation of relativistic postulate allows to deeper understand a physical essence of relativity theory. For example, it allows to advance a problem about correspondence of STR to the causality principle. At the first sight, this problem seems to be trivial due to a finity of light velocity in STR. Nevertheless, the present paper finds some additional questions in this topic in the following particular physical problem.

2. A special case of two light pulses propagation: in inertial reference frame, in constant homogeneous gravitation field, and in rigid non-inertial frame.

2.1. Case of inertial motion

Let us consider the following problem in Cartesian inertial reference frame.

Let a short light pulse be emitted from the point $x = 0$ along the axis $x$. Let a number of re-emitters of light $RL_m$ be located along the $x$-axis in some points $x_m$ ($RL_0$ is located in the point $x = 0$ and, for simplicity, all $\Delta x_m = x_{m+1} - x_m$ are equal to each other). When a light
pulse arrives at each re-emitter, it is absorbed by it, and after a fixed interval of its own time \( \Delta \tau_0 \) is emitted by RL along the \( x \)-axis again.

Further, let the second light pulse be emitted from the point \( x = 0 \) at such moment of time (taken as \( t=0 \)), when the first light pulse has a coordinate

\[
0 < \Delta x \leq x_1. \quad (1)
\]

One requires to find the times \( t_1 \) and \( t_2 \), where \( t_1 \) is the moment of time when the first (right) light pulse is emitted by RL\(_n\), while \( t_2 \) is the moment of time when the second (left) pulse is reaching RL\(_n\), and \( n \) is some number.

Due to the condition (1), the most general expression for \( t_1 \) can be written as

\[
t_1 = t_{x_1-\Delta x} + \sum_{m=1}^{n-1} t_m + \sum_{m=1}^{n} \Delta t_m, \quad (2)
\]

where \( t_{x_1-\Delta x} \) is the propagation time of the first (right) light pulse from the point \( \Delta x \) to point \( x_1 \), \( t_m \) is the propagation time of the right pulse from RL\(_m\) to RL\(_{m+1}\), and \( \Delta t_m \) is the time interval \( \Delta \tau_0 \) for RL\(_m\), remitting the right pulse. The general expression for \( t_2 \) is:

\[
t_2 = t'_{x_1-0} + \sum_{m=1}^{n-1} t'_m + \sum_{m=0}^{n-1} \Delta t'_m, \quad (3)
\]

where \( t'_{x_1-0} \) is the propagation time of the second (left) light pulse from the point \( x = 0 \) to point \( x_1 \), \( t'_m \) is the propagation time of the left pulse from the RL\(_m\) to RL\(_{m+1}\), and \( \Delta t'_m \) is the time interval \( \Delta \tau_0 \) for RL\(_m\), emitting the left pulse. From (3) and (2)
where $\Delta t = t'_{x_1-0} - t_{x_1-\Delta x}$.

Now let us ask the question: is it possible to implement the equality $t_2 - t_1 = 0$? It is obvious, such an equality would mean an absolute event: a meeting of both light pulses considered in the spatial point $x_n$.

This problem has a trivial solution in inertial reference frame. Here $t_m = t'_m$, $\Delta t_m = \Delta t'_m$, and all $\Delta t_m$ are equal to each other for any $m$. Hence, $t_2 - t_1 = \Delta t$, and the equality of $t_1$ and $t_2$ is impossible. This result means that at the moment of time when the second (left) pulse is reaching $RL_n$ in $x_n$ point, the first (right) pulse already has a space coordinate $x_n + \Delta x$.

2.2. Case of constant homogeneous gravitation field

In this case

$$t_m = t'_m$$

(5)

due to independence of metric tensor on time co-ordinate, and

$$\Delta \tau_0 \approx \Delta t_m \left(1 + \frac{\varphi_m}{c^2}\right)$$

(6)

in the approximation of weak gravitation field. Here $c$ is the light velocity in vacuum, and $\varphi$ is gravitation potential. (Further we take $\varphi_0=0$).

It follows from (6) that all values $\Delta t_m$ and $\Delta t'_m$ are equal to each other, and
\[ \sum_{m=0}^{n-1} \Delta t'_m - \sum_{m=1}^{n} \Delta t_m = \Delta t'_0 + \sum_{m=1}^{n-1} \Delta t'_m - \sum_{m=1}^{n-1} \Delta t_m - \Delta t_n = \Delta t_0 - \Delta t_n = \frac{\Delta \tau_0 \left( \phi_n / c^2 \right)}{1 + \left( \phi_n / c^2 \right)} \approx \Delta \tau_0 \frac{\phi_n}{c^2}. \] (7)

in this approximation. Substituting (7) and (5) into (4), one gets:

\[ t_2 - t_1 = \Delta t + \Delta \tau_0 \frac{\phi_n}{c^2}. \] (8)

Hence, the equality of \( t_1 \) and \( t_2 \) is implemented under the condition

\[ \Delta t = -\Delta \tau_0 \frac{\phi_n}{c^2}. \] (9)

Thus, we conclude that for appropriate choice of the parameters in (9), we are able to observe the absolute event: a meeting of two short light pulses in the spatial point \( x_n \).

**Case of rigid non-inertial frame**

In this Section we will consider the problem in rigid non-inertial frame moving along the axis \( x \) at constant (in relativistic meaning) acceleration \( a \), and we will perform the exact calculations due to importance of the results obtained.

By definition, in a rigid frame the proper distance between two spatial points (measured by means of a scale being at rest in this frame) does not depend on time. Let us define such a rigid frame by the relationships

\[ x^\alpha = x'^\alpha; x^0 = \tau, \] (10)

where \( x'^\alpha \) are the space coordinates in successive instantaneously co-moving inertial reference frames, while \( \tau \) stands for the proper time at the origin of coordinates. In such definition, for the case of constant (in
instantaneously co-moving inertial frames) acceleration \( a \) along the axis \( x \), a relationship between space-time coordinates in a fixed reference frame \((T, X, Y, Z)\) and \((t, x, y, z)\) takes the form [3]:

\[
dT = dt(1 + \frac{ax}{c^2})ch \frac{at}{c} + dx sh \frac{at}{c},
\]

\[
dX = cdt(1 + \frac{ax}{c^2})sh \frac{at}{c} + dxch \frac{at}{c};
\]

\[
dY = dy; dZ = dz.
\]

The metrics of space-time determined by (11), (12), is the following:

\[
ds^2 = c^2 dt^2 \left(1 + \frac{ax}{c^2}\right)^2 - (dx)^2 - (dy)^2 - (dz)^2.
\]

The corresponding components of the metric tensor are:

\[
g_{00} = \left(1 + \frac{ax}{c^2}\right)^2; g_{0\alpha} = 0; g_{11} = g_{22} = g_{33} = -1, \text{ all others } g_{\alpha\beta} = 0.(14)
\]

The physical values are defined as

\[
dx_{ph0} = \sqrt{g_{00}} dx^0 + \frac{g_{0\alpha} dx^\alpha}{\sqrt{g_{00}}},
\]

\[
\sum x_{ph\alpha}^2 = \left(-g_{\alpha\beta} + \frac{g_{0\alpha} g_{0\beta}}{g_{00}}\right) dx^\alpha dx^\beta.
\]

Substituting the components of metric tensor from (14) to (15), we obtain

\[
dx_{ph} = dx, dy_{ph} = dy, dz_{ph} = dz;
\]
Due to independence of the metric tensor on time, we again get the equality (5), from there

\[ \sum_{m=1}^{n-1} (t'_m - t_m) = 0. \]

The intervals of physical time at different points are determined by (17). Hence, the physical values

\[ \Delta \tau_0 = \int_0^{\Delta t_m} dt_{ph}(x_m) = \Delta t_m \left( 1 + \frac{ax_m}{c^2} \right), \]

and

\[ \Delta t_m = \frac{\Delta \tau_0}{1 + \frac{ax_m}{c^2}}. \]

Therefore, \( \Delta t'_m \) and \( \Delta t_m \) are equal to each other, and the third term in (4) is equal to

\[
\sum_{m=0}^{n-1} \Delta t_m - \sum_{m=1}^{n} \Delta t_m = \left( \Delta \tau_0 + \sum_{m=1}^{n-1} \Delta t_m \right) - \left( \sum_{m=1}^{n-1} \Delta t_m + \Delta t_n \right) = \\
= \Delta \tau_0 - \Delta t_n = \Delta \tau_0 \frac{ax_n}{c^2} \left( \frac{1}{1 + \frac{ax_n}{c^2}} \right).
\]

Substituting the obtained values (19), (21) into (4), one gets:
\[ t_2 - t_1 = \Delta t + \Delta \tau_0 \frac{ax_n}{c^2} \left( \frac{1}{1 + \frac{ax_n}{c^2}} \right). \]  

(22)

Hence, the left and right light pulses will meet in the point

\[ x_n = -\frac{\Delta tc^2}{a(\Delta \tau_0 + \Delta t)}. \]  

(23)

Thus, an observer in an accelerated frame will detect the absolute event: the left and right light pulses will meet in the point defined by (23) (under negative sign of the acceleration \( a \)). This conclusion is in agreement with the result of Section 2.2 and the equivalence principle. However, here we meet a quite difficult problem: for observer in inertial frame both light pulses will never intersect.

Indeed, let the process of light pulses propagation in the accelerated frame be observed from some inertial reference frame. Furthermore, let us choose for observing the light pulses propagation process an inertial frame \( K \), such that at the time moment when an observer sees the appearance of the left light pulse in the point \( x = 0 \), he simultaneously sees an arriving right pulse to \( RL_1 \) (such a choice is always possible due to (1)). For this time moment, let us introduce into consideration the second inertial frame \( K_s \) shifted along the axis \( x \) at such a distance (with respect to \( K \)) which is equal to the distance between \( RL_0 \) and \( RL_1 \). (The relative velocity of \( K \) and \( K_s \) is equal to zero). Due to the space homogeneity in inertial frames, such a shift is equivalent to re-numeration of the re-emitters in \( K_s \): the \( RL_m \) (in \( K \)) be \( RL_{m-1} \) (in \( K_s \)). Hence, the propagation time from \( RL_0 \) to \( RL_{n-1} \) for the left pulse is exactly equal to the propagation time from \( RL_1 \) to \( RL_n \) for the right pulse in both \( K \) and \( K_s \) frames (since the \( RL_1, RL_n \) in \( K \) are the \( RL_0, RL_{n-1} \) in \( K_s \)). Hence, at the moment of time (in \( K \)) when the right pulse is emitted by \( RL_n \), the left one is emitted by \( RL_{n-1} \) for any
Therefore, the light pulses considered will never meet in the inertial frame $K$, that means a contradiction with the causality principle.

**Conclusions**

Thus, a consistent relativistic consideration of the problem about special kind of two light pulses propagation in a non-inertial reference frame contradicts with the causality principle. One can show that a resolution of this contradiction is possible only under supposition that a geometry of empty space-time is not pseudo-Euclidean [4]. However, it is in a deep disagreement with the relativity theory. It seems that a full resolution of this contradiction within the scope of relativity theory is impossible.

**Bibliography**


