Mass, Energy, Space

Henrik Broberg
Djürsholm, Sweden

This paper deals with the conceptual origin of particle mass and its relation to energy and space. It has been impossible, along the way, to avoid the question of the unification of the forces, and some results have been achieved in this area as well.

The Cosmological Redshift and Vacuum Space

While employed at the European Space Agency, I was given the opportunity to undertake my own scientific research with a grant from the ESA to spend a year (1982-83) at the *Laboratoire d'Astrophysique Theorique* (LAT) in Paris, where I worked on cosmology with professor J-C Pecker, former Head of the *Paris Observatory*. During that period, in the unorthodox surrounding of Pecker, Vigier and other followers of de Broglie, who did not take the "standard model" for granted—least of all "Big Bang" cosmology—I was encouraged to continue my own thinking in new directions.

I would like to argue that from time to time we need to look at problems from a new angle in order to advance our scientific knowledge and our technology. The key ingredient in my work at the LAT was the assumption that the cosmological redshift must involve some interaction between photons and the vacuum space. As I saw it, whatever the mechanism causing the redshift, even if it were due only to the (hypothetical) expansion of the Universe, information about the mechanism must be transferred to the photon via the vacuum in one way or another.

I investigated this process, without further knowledge about its inner nature, by defining a cross-section for the photon-vacuum interaction. This cross-section has the property of a surface proportional to the probability of the interaction between the vacuum and the photon during any photon cycle.

According to the rough distance relation from Hubble's observation of the redshift in the 1920's, later confirmed and improved by numerous other observers, the photon loses energy approximately in proportion to its own energy level and the length of an interval of time pertinent to the observed redshift. This indicates an exponential law behind the redshift, which would lead to an average loss of energy for any photon during any cycle-time of a constant value equal to hH, where h is Planck´s constant and H is Hubble's constant.

In cosmology, H is often assumed to be a variable, proportional to the inverse of the "age" or "scale factor" of the expanding Universe. It could just as well be a constant related to the density or curvature applicable to a "steady state" Universe. On our more limited human time scale it does not matter much what the case is, and both H and h can be treated as constants. The distinction, however, will become important on the cosmological scale.

If such a small quantity of energy as hH is exchanged between the photon and the vacuum space, it would be the smallest amount of energy observed to participate in any physical process, corresponding to the energy of an electromagnetic wave with a wavelength of the same order of magnitude as the "scale factor" (or radius) of the Universe according to the current theories in cosmology. It is hardly possible to imagine the existence of any smaller quantum of energy in our Universe, and it is, therefore, tempting to think of this as a minimal energy quantum.

The loss of such a minimal quantum of energy as *hH* would therefore be the average result of the interaction between a photon and the vacuum space during any photon cycle-time. Even if this energy loss were an unavoidable physical attribute of each cycle of the photon vacuum oscillator, the result could also be seen as a stochastic process due to the requirements of Heisenberg's Uncertainty Principle. The probabilistic Schrödinger wave function, normalised over each wavelength, would then generate the surface corresponding to the cross-section for the photon-vacuum interaction. The photon cross-section σ_γ so defined becomes a surface proportional to the energy of the photon, for which we introduce the parameter *A* expressed in m^2/kg:

$$s_g = A\frac{E}{c^2} \tag{1}$$

From this it follows that the cross-section of each photon covers a certain constant volume during each cycle; this quantum volume in the vacuum V_q becomes:

$$V_q = A\frac{h}{c} \tag{2}$$

Since the Planck Radius is assumed to be the scale at which our known physical laws break down, it sets a lower limit for the concepts we can deal with. It is therefore interesting to note that the above defined cross-section applied to the minimal quantum *hH* becomes approximately equal to the square of the Planck radius, or more precisely:

$$s_q = 2p\, R_p^2 \tag{3}$$

This relation is true for H equal to the inverse of 15 billion years. The above expression can also be translated into a relation between the parameters A, h, H, c and Newton's constant G:

$$G = A\frac{Hc}{4p} \tag{4}$$

We observe that the latter expression gives G as a function of H, if these two properties are variables and the others true constants. This can be interpreted in different ways. One possible interpretation is to regard H as the inverse of a scale-factor, in which case it serves as something which can be thought of as a ground state resonance frequency of the Universal space. In the latter case we might be able to identify different Universes of different sizes, and accordingly also with different values of the gravitational parameters.

The results from my earlier work are reflected in two ESA reports. The first report (Broberg 1981), written just before my time there, now appears as a naive effort to construct a cosmologically based particle theory on a non-relativistic foundation, while the second report (Broberg 1984), written afterwards, includes relativistic concepts, using the Schwarzschild and Kerr singularities as the fundamental vehicles to a conceptualization of particle rest mass. Though lacking some of the concepts presented here, these are necessary for a deeper understanding. In that report, the interpretation of the relation between H and G developed here is used. This relation leads to a model of fundamental particles as miniature "universes", obeying the same laws as the large-scale Universe.

The Singularities

Theories about particles and forces are often linked to "singularities", which could perhaps be defined as extreme mathematical solutions leading to abrupt changes in parameters, manifesting themselves as physical discontinuities when a variable approaches a certain value or limit. A simple example of a singularity is the change in friction coefficient when a car wheel becomes blocked during braking, leading to a sudden loss in braking power—an effect we try to avoid by using anti-blocking systems (ABS).

The Einstein Singularity

Within the framework of General Theory of Relativity, Einstein tried to show that the elementary particles could be understood as resulting from singularities in space-time, or in other words that the particle masses (energies) were so dense that they curved the space in their surrounding to the extent that their energies were trapped or locally confined in the warp of space-time. They would then behave like "black holes". However, the radius of this border, or event horizon, calculated as the Schwarzschild radius turns out to be something like 10^{40} times smaller than the measured radii of particles, and consequently Einstein's idea was never accepted.

In one early study, Einstein indicated (1919) the "possibility of a theoretical construction of matter out of gravitational field and electromagnetic field alone", suggesting that particle energies could be accounted for by means of a modification to the field equations of General Relativity. More specifically, in a discussion of the electron, he proposed that the gravitational constant ("the scalar of curvature") could have another value in the system of a particle than in the space outside the particle. It now appears that Einstein was on the right track when he made this proposal, and the analysis presented in this paper clearly leads us in the same direction.

The "Big Bang" Singularity

The presently accepted "standard model" in physics (Weinberg 1972), invokes the idea that time, space, matter and energy all have developed from one moment of origin about 15 billion years ago—

the original singularity—to the Universe of our time. The changing scale from almost zero dimensions to the present large scale is used to explain the creation of the particles and elements; the lighter ones (up to the size of Helium) emanated from the hot and small early Universe, while the heavier elements are supposed to be created in the cores of the stars.

The main evidence for the "Big Bang" model is normally stated to be (1) the observation of the redshift of the light from the distant stars, (2) the 3°K background radiation measured by the radio telescopes in all directions in space and (3) the mathematical requirement that the Universe is dynamic and not static.

Some years before Hubble discovered the redshift, Alexander Friedmann had built a model of the Universe based on General Relativity and the assumption that the Universe looks the same from anywhere and in whatever direction, as is the case with the background radiation. His model also predicted the expansion in a way that fits with Hubble's observation. The Friedmann model is still the main foundation for the "standard model" in cosmology, the three variants of which all lead back to an original singularity, beyond which we cannot know anything.

However, more recently within the framework of the theory of quantum gravity, Hawking (1987a,b) has proposed that the surface of space-time would be closed and without a boundary, like the surface of the Earth, but with more dimensions. He suggests that it would be possible to move from one singularity to another, by starting at a point at the north pole and expanding into a circle that would grow towards the equator and slim down towards the south pole, where the circle would then a become a point again. It is questionable whether the points at the poles really are singularities (any more so than other points on the surface) or whether the entire surface should be seen as a self-supporting singular system, without the need for any particular beginning or end.

The Electron Singularity Problem

The electron is, as far as we know, the smallest fundamental rest-mass quantum in the Universe. It does not decay, and it carries a minimal quantum charge and a spin-quantum. About its inner structure, nothing is known, and it is normally treated as point-like. Measurements indicate that if the electron has a size, its radius is not much larger than 10^{-16} m.

However, treating the electron as a point contradicts the fact that it is charged, since the potential energy of a charged sphere would become infinite when the radius becomes zero. This difficulty is circumvented in the textbooks of physics by stating that the known laws of physics don't apply in the very small. That means that Coulomb's law would be truncated at some point. Often the Planck radius 10^{-35} m is referred to as the length beyond which the known laws break down. But at that scale the confined potential electrical energy would be about 10^{20} times the energy of the electron mass, which is hardly possible, as that energy would presumably have to be part of the mass.

One solution to the electron dilemma would be to assume the geometry of the electron to be like a fibre rather than a sphere. As an indication, before going into a more detailed analysis, we will suppose that the charge is distributed in small dots with units of Q/N along a fibre of length L, the distance between two nearby dots being L/N. The force between any of two nearby dots will, independent of N, become:

$$F = k \cdot \frac{\left(Q/N \right)^2}{\left(L/N \right)^2} \equiv k \frac{Q^2}{L^2}$$

Even if the forces of all dots *vis-a-vis* one dot are added, the sum converges as the series:

$$F = k \frac{Q^2}{L^2} \cdot \sum_n \frac{1}{n^2}$$

and therefore a limited value of the tension in the fibre is achieved. This tension can be treated as the energy stored in an elastic string and be recalculated in mass units. It turns out that a string with a length of about 10^{-15} m will have energy on the order of the rest-mass of the electron. If nothing holds the string back, it will simply extend itself lengthwise. If it is tied into a circle instead, the tension will produce an outward expanding force on the periphery. If the electric field lines are curved along the same circle as the fibre, such that each dot-charge is influenced only by its neighbours, the potential energy stored in the tension of the ring becomes:

$$E = \frac{Q^2}{8pe L}$$

This little exercise is hardly an exhaustive explanation of the electron mass—that is not the point. It does, however, show the possibility offered by a string-like geometry. It might therefore be interesting to look into string theories as a means to investigate the properties of the electron. It will later be shown that this approach is very fruitful for an understanding of particles and forces generally.

String Theories

Since the mid-80s, super-string theories, first introduced by Scherk and Schwarz (1974), have been much used in efforts to describe and unify the concepts of physics. Strings are supposed to represent particles, and they can be open or curved lines. Over time, the strings generate surfaces, or world-sheets:

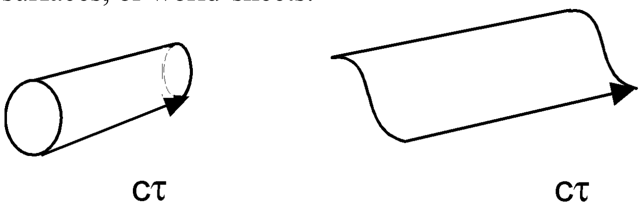

Figure 1—A String and a Sheet

The world-sheet of a closed string is a cylinder, while its cross-section is a flat disk, for example a circle, representing the position of the string (or particle) as a function of time. When particles absorb one another, strings and world-sheets simply become joined. A particle can be modelled as a wave travelling along the string world-sheet as the string vibrates.

A problem with the current super-string theories is that they have become increasingly complicated and now require many more than the normal four dimensions of space-time to work consistently. Yet they still have not achieved the anticipated success in explaining the particle properties.

In the following, the concept of strings will be used, though not in the framework of the super-string theories. Instead we will start from an analysis of singularities generated by a rotating closed string. The importance of these new singularities is that they are set up without the use of the standard gravitational theory and therefore become independent of the current gravitational parameters; for example, we will see that Newton's constant may not be a constant at all.

The Rotating String

We now introduce the concept of a constant surface-to-mass ratio for rotating strings, a relationship that will be fundamental to quantized systems.

From experience we know that the surface-to-mass relation of a nucleon is of the order of about one square meter per kilogram of nucleonic matter, and that a similar relation can be found for the mass-content of the large scale Universe (say, the mass of 10^{11} galaxies within a radius of 10^{15} light-years). We find the same result again if we consider a certain surface to characterize the cosmological Hubble redshift. In an earlier study (Broberg 1984), I found that a value of 0.7 m^2/kg can be calculated from the redshift for a scale-factor of 15×10^9 light-years, a figure that also fits well with the dimensions of the elementary particles.

As an introduction, consider a string possessing mass that rotates around its centre, shown in Figure 2.

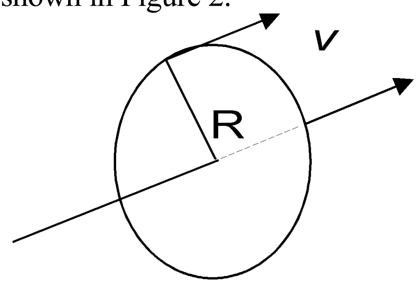

Figure 2—A Rotating String

The interaction between the magnetic and the electric fields in a photon can be treated as two rotating strings, like circles sliding with a certain displacement on the surface of a sphere, from points at the poles to large circles at the equator, and back (as in the Hawking model), while their total surface sum is constant and equal to the surface of the sphere. The surface corresponding to each field component in the photon would then have to be proportional to the cross-section of its string, while its average surface over a cycle would be half that of the entire photon.

In the case of a particle with a rest-mass, there will also be a characteristic time constant for the system, and the cyclic time sheet will be expressed as a closed three-dimensional surface, like a sphere or a toroid (or surface of a four-dimensional sphere).

The kinetic energy of the rotating string is:

$$E_k = m_0 c^2 (g - 1)$$

The surface across the string is a large circle given by:

$$f = p\left(\frac{vt}{2p}\right)^2$$

where τ is the cycle-time (one turn). By analogy with the cross-section for the photon-vacuum interaction from equation (1) above, the relation between the surface across the string loop and the kinetic mass of the rotating string is:

$$A = \frac{p\left(\frac{vt}{2p}\right)^2}{m_0 (g - 1)} \tag{5}$$

Hence, if A is a constant parameter, we have the following relation between surface and mass:

$$A m_0 g = \frac{g + 1}{g} \cdot p\left(\frac{ct}{2p}\right)^2 \tag{6}$$

The limiting value when the rotational velocity $v \to 0$ becomes:

$$Am_0 = 2p\left(\frac{ct}{2p}\right)^2 \tag{7}$$

The limiting value when the rotational velocity $v \to c$ becomes:

$$Am_\infty = p\left(\frac{ct}{2p}\right)^2 \tag{8}$$

Hence, vis-a-vis its radius, and for the same energy and rotational time constant, the fibre ring with zero velocity has twice the surface compared to the ring rotating with velocity *c*.

The two cases represent two singularities. When the rotational velocity drops close to zero the radius must also approach zero for the string to be able to complete its revolution during its cycle time. When the rotational velocity becomes *c* the periphery shrinks to zero due to Lorentz contraction. The result is apparently highly relativistic both for the rest mass and the “relativistic” string.

A relativistic (massless) particle can generate two rest-mass particles, as follows:

$$\begin{aligned} &\text{I)} \quad Am_\infty = p\left(\frac{ct}{2p}\right)^2 \Rightarrow 2p\left(\frac{ct}{2p}\right)^2 \\ &\text{II)} \quad 2p\left(\frac{ct}{2p}\right)^2 \Rightarrow 4p\left(\frac{ct}{4p}\right)^2 + 4p\left(\frac{ct}{4p}\right)^2 \end{aligned} \tag{9}$$

In simple terms, the above mechanism would be a flat electromagnetic wave with unitary spin, which, once stopped and in a state of rest, first becomes an integer spin system with a double surface based on the same radius as before. This system in turn collapses into two spherical half-integer spin particles. The interim system could correspond to a mesonic state and the final system, baryons. Another possibility is a collapse of the original wave directly into any pair of ½ spin particles, such as a positron and an electron.

We will now give two examples, very briefly at first, to show how the above relations work with particles.

Example 1. A Spherical Stationary System—the Neutron

To set up a simple self supporting stationary system in three-dimensional space, we need a minimum of 6 string components grouped in pairs, one pair for each one of the three dimensions. The entire system has a surface equal to the sum of the surfaces of the field components:

$$Am = 4p\, R^2 \tag{10}$$

The space-time volume of the system is equal to the sum of the volumes of the field components:

$$V_0 \equiv \frac{Ah}{c} = \frac{4}{3} p\, R^2 \tag{11}$$

The mass of the entire system becomes:

$$m = \sqrt[3]{\frac{p\, h^2}{Ac^2}} = 1.68 \cdot 10^{-27}\, kg\ \left(940\ \mathrm{MeV}\right) \tag{12}$$

It should be noted that the same mass for the system is achieved if all the surfaces of the string components, as well as the volumes, are assumed to be folded over one another, and the total is calculated as the mass of one string quantum, for which A is divided by 6.

With a value of $A = 0.7$ m^2/kg, the mass of the system becomes approximately equal to that of the neutron. As it turns out, this value of the constant (because it seems really to be a constant) will be consistent with all the particle and other concepts analysed in the following. However, the above calculated mass formula is an oversimplification of the neutron mass, and therefore it cannot be justified for a more precise calculation of A.

Example 2. A Charged String—the Electron

The concept of a charged rotating string will here be used to describe another stationary system, which will turn out to have the mass of the electron when the charge is equal to the electron charge.

Let the string be a thin toroid, on the surface of which a current flows, induced by a charge moving in a spiral pattern, in analogy with a toroidal electrical coil (Figure 3):

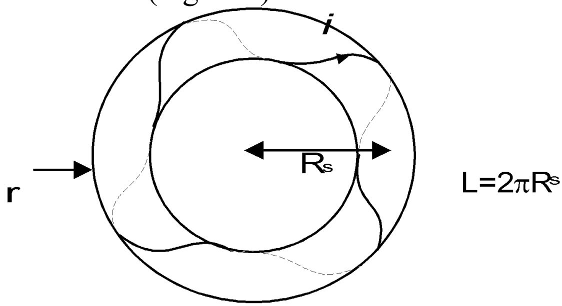

Figure 3—The toroidal electron string

The length of the coil is L, its large radius being

$$R = \frac{L}{2p} \tag{13}$$

The current is i, the number of turns of the current around the coil before closing the loop is N, and the small radius is r.

With the charge Q and the magnetic permeability of vacuum space μ_0, the energy of the magnetic field in such a coil becomes:

$$E = \frac{1}{2} m_0 \left(\frac{iN}{L}\right)^2 \cdot V \tag{14}$$

where V is the volume of the coil:

$$V = p\, r^2 \cdot L \tag{15}$$

The distance the charge goes before closing one loop (N turns) around the coil is:

$$\ell = 2p\, rN\sqrt{1+\left(\frac{L}{2p\, rN}\right)^2} \tag{16}$$

The current becomes:

$$i = \frac{Qc}{\ell} \tag{17}$$

Therefore:

$$N \cdot i = \frac{Qc}{2p\, r\sqrt{1+\left(\frac{L}{2p\, rN}\right)^2}} \tag{18}$$

The magnetic energy stored in the coil now becomes:

$$E_B = \frac{1}{2} m_0 \frac{Q^2 c^2 V}{(2p\, r)^2 \left[1+\left(\frac{L}{2p\, rN}\right)^2\right] \cdot L^2} \tag{19}$$

or, the mass equivalent:

$$m_B = \frac{m_0 Q^2}{8p\, L\left[1+\left(\frac{L}{2p\, rN}\right)^2\right]} \tag{20}$$

When $2\pi Nr >> L$ (an assumption which is justified in Appendix A), we get an expression for the mass corresponding to the magnetic field, which in combination with Equation 7 gives:

$$\begin{cases} m_B = \dfrac{mQ^2}{4p} \cdot \dfrac{1}{4p R_S} \\ Am_B = 4p R_S^2 \end{cases} \tag{21}$$

The potential electric energy stored in the loops of the string disappears when N becomes large, because of the division of the charge by N and the relation between N and the thickness of the fibre. Therefore, ignoring the gravitational field, the only contribution to the mass is from the magnetic field.

Eliminating R gives a value for the rest-mass of the string:

$$m = \frac{1}{4p} \sqrt[3]{\frac{\left(m_0 Q^2\right)^2}{A}} \approx 9.10 \cdot 10^{-31} kg \text{ (0.511MeV)} \tag{22}$$

This gives the radius

$$R_S = \frac{1}{4p} \sqrt[3]{Am_0 Q^2} \approx 2.25 \cdot 10^{-16} \text{ m} \tag{23}$$

and the length of the string:

$$L = \frac{1}{2} \sqrt[3]{Am_0 Q^2} \tag{24}$$

A is expected to be a Universal constant. Using the expression given in equation (4), A is estimated from the parameters applicable in the large scale (G and H), as well as Planck's constant:

$$A = \frac{4p G}{Hc} \approx 0.7 \text{ m}^2/\text{kg} \tag{25}$$

Here, Hubble's constant is given as $15 \times 10^9 \text{ yrs}^{-1}$.

This gives the mass from equation (23) to be 9.10×10^{-31} kg, which is indeed equal to the electron rest mass.

The Fundamental Particle System

In the above, we have derived a few concepts concerning singularities and strings. We will now introduce them into a general theory for particle rest mass.

First some postulates will be made:

1. There exists a fundamental elementary ground stage particle with a rest mass.
2. The existence of particle rest mass is based on the existence of a singularity in space-time
3. The masses of the other particles will follow logically as consequences of the properties of the fundamental elementary particle.

As will be shown in the following, the electron is found to be the fundamental elementary particle.

Above, we have identified a string singularity which seems capable of explaining the property of particle mass.

In the framework of general relativity, the Schwarzschild and the Kerr metrics contain singularities, which in popular terms lead to the concepts of "black holes". The radius of the so called event horizon of the Schwarzschild singularity is the distance from the centre that defines a boundary, from within of which no matter or energy can escape, hence the name "black hole". However, it has been shown by Hawking and others (Hawking 1987a,b) that black body radiation will be emitted from the black holes and that they are therefore not "totally black".

Following the same idea as I presented in an earlier study (Broberg 1984), we will now see how the Schwarzschild singularity can be related to the rotating string described in the preceding section.

The Schwarzschild radius is:

$$R_g = \frac{2gm}{c^2} \tag{26}$$

In large-scale Universal space the gravitational parameter g is equal to Newton's constant G.

For the moment we will here leave the value of g as an open parameter—we will come back to the evaluation of G later.

If the particle is a "black hole" singularity, the mass should be seen as a "ghost image" of the absorbed energy, hovering over the event horizon, due to the time dilation towards infinity at the event horizon *vis-a-vis* a distant observer. For a stable particle, the rest mass should be in an equilibrium state, where the inflow of energy from the surrounding vacuum space is equal to the black body radiation from the surface of the particle.

As an indication of how to understand the latter process, if protons radiate energy to the surrounding vacuum space at the same rate as the photons are redshifted according to Hubble's law, and if the surface characterising the black body spectrum of the radiation is a sphere with the electron Compton wavelength as its radius, the result would be a 3 Kelvin spectrum, in accordance with the observed background radiation:

$$\begin{cases} T = \sqrt[4]{\dfrac{Hm_p c^2}{\left(5.67\cdot 10^{-8}\right)4\mathbf{p}\, r^2}} = 2.9^\circ\,\mathrm{K} \\ r = {h}/{cm_e} \end{cases} \tag{27}$$

Therefore it is possible that the observed background radiation emanates mainly from the hydrogen clouds in space.

Returning to the particle model, as seen from the outside, the particles would have their masses concentrated in a shell at, or just above, the event horizon. For the electron, for example, this would mean that the rest mass, seen from the outside, would be concentrated to a shell built by stored potential electric energy:

$$m = \frac{mQ^2}{4p\, R_g} \tag{28}$$

Comparing this result with the mass from Equation (20), we see now that the relation between the string radius and the radius of the Schwarzschild singularity from equation (26) above becomes:

$$R_g = 4p\, R_S \tag{29}$$

This radius corresponds to the dilated electron time constant seen by a distant observer.

Although the outside observer sees the rest energy as being stored at the event surface, an observer travelling with the inflow of energy into the electron singularity would cross the surface and enter an inner structure, where the potential electric energy has collapsed into a charged string, as the one described above.

The more detailed calculation in Appendix A yields the mass of such a charged string:

$$m = \frac{1}{4p} \sqrt[3]{\frac{\left(mQ^2\right)^2}{A} \cdot \left(\frac{2N}{2N+1}\right)^2} \tag{30}$$

The parameter N is equal to the number of twists of the charged electron string. It is obvious that the mass converges very quickly towards the proper rest mass of the electron when the number of twists increases. As the number of twists increases, the potential electrical energy on the string disappears, being converted into magnetic energy stored in the core of the string. Hence the string soon becomes like an almost infinitely thin fibre of magnetic field energy.

The radius of the string becomes:

$$R_S = \frac{1}{4p}\sqrt[3]{Am_0Q^2\left(\frac{2N}{2N+1}\right)} \tag{31}$$

This radius quickly converges towards its limit value 2.25×10^{-16} m.

The Schwarzschild radius or R_g derived above may be called the radius of the electron system. Its value is:

$$\underset{N\to\infty}{R_g} = \sqrt[3]{Am_0Q^2} = 2.826\cdot10^{-15}\ \text{m} \tag{32}$$

We can also set up g as a function of the mass contained in the singular electron system:

$$\begin{cases} g = \sqrt{\dfrac{p\,Ac^4}{m}} \\ gR_g = 2p\,Ac^2 \end{cases} \tag{33}$$

If our constants A and c are true constants, it is obvious that g depends on the mass of the system within which it is applicable as a force constant. This is a reasonable solution because on a large scale, using the value of Newton's constant for g, it corresponds to a Universe containing mass corresponding to 10^{11} galaxies, which is in line with our observations. At the other end of the scale, the above relation gives a force for the nuclei, which is about 10^{40} times the gravitational force in the large-scale Universe—also in agreement with observations.

With the above established function for *g,* the true nature of the force becomes that of a surface-energy relation, and the force in kilonewtons needed to change the surface radius becomes:

$$\begin{cases} E = 4p\,R^2 \cdot \dfrac{c^2}{A} \\ F = \dfrac{\mathrm{d}E}{\mathrm{d}R} \end{cases} \Rightarrow F = 8p\,R \cdot \frac{c^2}{A} \approx 3\mathrm{KN} \qquad (34)$$

for $R = 10^{-15}$ m. In its form and amplitude, this force agrees with what is assumed for the quarks by present day theories.

The above relations also result in identical amplitudes of the electrical and gravitational interactions inside the system of the electron:

$$\begin{cases} \dfrac{2GM^2}{r^2} = \dfrac{1}{4pe} \cdot \dfrac{Q^2}{r^2} \\ G = \sqrt{\dfrac{p\,Ac^4}{M}} \end{cases} \Rightarrow M = m_e \qquad (35)$$

Solving out the mass from the above relation simply gives back the electron rest mass.

In the electron system, the strong force, the gravitational force and the electric force therefore appear to be unified. The fourth force, the weak force responsible for the radioactive particle decay, is here not seen as a force, but the result of decaying unstable particle systems.

Hence, it is possible that the "forces" of Nature become unified in the system of the fundamental elementary particle—the electron.

The Electron and the Hydrogen Spectrum

The hydrogen spectrum is used as a basic source for measurement of fundamental constants of Nature, such as, for example, Planck's and Rydberg's constants. The dimensionless fine structure constant is also an important parameter of this spectrum.

As it turns out, the hydrogen spectrum shows an interesting structure of at least four strings superimposed upon each other, from the event horizon of the electron outwards, separated in size only by multiples of the fine structure constant.

The fine structure constant can be given as:

$$a = \frac{m_0 e^2 c}{2h} \tag{36}$$

This constant is also, in relation to the Bohr model of the hydrogen atom, identical with the relation v/c, where v is the velocity of the electron in its inner orbit around the nucleus. The following radii of string-like concepts are identified:

1. The electron event horizon from equation (29)
2. The Compton wavelength of the electron
3. The inner de Broglie orbit in the Bohr model of the hydrogen atom
4. The Rydberg wavelength (the inverse of Rydberg's constant)

The size of each of the above objects is given by multiplying the earlier in the series by the inverse of the fine structure constant, except for the last one, which needs an additional factor of two. This series may be considered as a number of string resonances from the rest-state of the electron mass, the relation between object 3 and object 4 involving a transition from a zero velocity string to a string with velocity c, thus splitting into two objects, *i.e.* the two electromagnetic quanta emitted by the two inner electrons when falling in from a large distance to their places in the H atom.

The existence of the fine structure constant obviously depends on the coupling between the photons and the quantum volumes of vacuum space, identified earlier in equation (2), which in turn is superimposed on the force interactions in the electromagnetic field.

From the earlier developed expression for the electron mass, equation (23), a relation between the quantum volume and the Rydberg wavelength can be established:

$$V_0 l_R = \left(\frac{\hbar}{cm_e} \right) \tag{37}$$

This relation can be understood such that the quantum volume V is the three-dimensional surface to a four-dimensional volume in space-time, which describes the electron as a quantum oscillator with Rydberg's constant as its unique parameter.

How the electron gets its specific mass can now also be seen from the point of view of geometry. The unfolded time constant of the electron oscillator is given by equation (38) above as:

$$ct_0 = \frac{\hbar}{cm_e} \tag{38}$$

A circle in space-time set up by this time-constant would have too large a surface compared to the energy of the oscillator. Therefore it has to curl itself together in a number of loops to reduce its surface. This process introduces the fine structure constant α. From equations (26) and (32), we see the relation between the surface corresponding to the electron mass and the radius of its event horizon. The curling will reduce the radius of the system to the latter, and the relation between mass and surface gives the following:

$$\begin{cases} R_g = a \cdot \dfrac{\hbar}{cm_e} \\ R_g^2 = 4p\, AM \end{cases} \tag{39}$$

From this equation system we can again calculate the mass, and as a result we can now give the following three identical expressions for the electron mass:

$$\begin{aligned} m_e &= \frac{1}{4p}\sqrt[3]{\frac{m^2Q^4}{A}} \equiv \frac{1}{4p}\sqrt[3]{\frac{4a^2h^2}{Ac^2}} \\ &\equiv \frac{mQ^2}{a^3} \cdot R_{Ryd} \end{aligned} \tag{40}$$

This concludes, for the time being, our linking of constants measured from the hydrogen spectrum with the electron rest-mass.

The Masses of the Basic Particles

As it turns out, the basic charged elementary particle masses follow in a rather uncomplicated way from the geometry of the electron. The guiding principle seems to be that the electron string in some different configurations serves as the geometrical norm for the particle time constant. The interference between the electromagnetic coupling and the vacuum quantum volume generates the charged particle systems.

Of the uncharged particles, the neutron mass was already introduced by equation (12) in the preceding on the simple basis of a three-dimensional spherical cross-section to a system of vacuum waves in the plus and minus directions of the three axes. The uncharged pion mass follows from an equally simple relation. A more detailed exposition of the hadron mass spectrum is given in Appendix C.

The Charged Pion

Using the periphery of the electron system from the radius given in Equation (34) as a wavelength generates a wave with half the energy of the pion restmass. Two such waves interacting with each other therefore yields the pion mass. (Due to its spin-½ particle nature, the periphery from the electron system is probably folded twice over itself.) Hence the mass becomes:

$$m_{p^{\pm}} = 2 \cdot \frac{h}{2p\, R_g c} \quad , \quad \text{or}$$

$$= \frac{h}{p\, c \sqrt[3]{AmQ^2}} \tag{41}$$

$$= 2.489 \cdot 10^{-28}\ \text{kg}\ (139.5\text{MeV})$$

This calculated mass agrees with the observed mass to three to four significant digits.

The Neutral Pion

Following the example of the electron and the form for the solution for the charged pion, we get the following:

$$\begin{cases} 2R^3 = V_0 \\ m = \dfrac{h}{cR} \end{cases} \tag{42}$$

Using the radius of the particle system as wavelength gives half the mass of the neutral pion. Hence, the mass of the system becomes:

$$m_{p_0} = \sqrt[3]{\frac{2h^2}{Ac^2}} = 2.407 \cdot 10^{-28} kg\ (135.0\ \text{MeV}) \tag{43}$$

This calculated mass exhibits four figure agreement with the observed mass.

The Generalised Pion Group

From the above it is found that the pion-system occupies one (three-dimensional) quantum volume in space-time. In the generalized solution given here, the two subsystems (quarks) also occupy together two spherical quantum volumes in three-dimensional space.

The volume in three-dimensional space becomes:

$$2V_0 = \frac{4}{3} p\, R^2$$

Hence,

$$R = \sqrt[3]{\frac{3Ah}{2p\,c}}$$

The mass of the system is given by:

$$m = \frac{h}{cR}$$

The mass becomes:

$$m = \sqrt[3]{\frac{2p\,h^2}{3Ac^2}} \cong 2.44 \cdot 10^{-28}\,\mathrm{kg}\,\left(137\ \mathrm{MeV}\right) \qquad (44)$$

This mass falls between the observed masses of the charged and uncharged pions.

The Proton

In accordance with the solutions for the neutron and the charged pion we can now also find the proton mass.

The system is based on an assembly of three charged strings, each charge equal to one third of the electron charge. Each string is twisted twice. The length of each string is then, from equation (31):

$$L = \frac{1}{2}\sqrt[3]{Am_0\left(\frac{Q}{3}\right)^2\left(\frac{2N}{2N+1}\right)} \tag{45}$$

Therefore, the mass of each, following the same pattern as the charged pion, becomes:

$$m_q = \frac{h}{4p\,cL\left(Q=\frac{1}{3}, N=2\right)} \tag{46}$$

The mass of the assembly of three such components is:

$$m_p = \frac{3h}{p\,c}\cdot\frac{\sqrt[3]{\frac{9\cdot 5}{4}}}{\sqrt[3]{Am_0Q^2}} = 1.673\cdot 10^{-27}\,\text{kg}\,\left(938\text{ MeV}\right) \tag{47}$$

This mass is equal to the observed mass of the proton with three to four significant figure accuracy.

If the same relation applies directly as a modification to the observed mass of the charged pion, we get:

$$m_p = 139.566\cdot 3\cdot\sqrt[3]{\frac{9\cdot 5}{4}} = 938.2\text{ MeV}$$

which gives an accuracy of at least four significant figures. Therefore the geometrical framework seems to be in good agreement with the charged baryons.

The Neutron and the Generalised Nucleon group

The neutron mass was given already in equation (12). The accuracy of the calculation in respect of the observed mass is about three significant figures. It is therefore a less accurate figure than those calculated for the other particles. However, the geometrical framework is similar to that of the proton above, the three wave-pairs of the neutron system corresponding to the three double twisted strings of the proton-system.

The mass of equation (12) can therefore be considered as a generalised solution for the nucleon group, based on the principle that each nucleon occupies one spherical quantum volume in three-dimensional space, while the three subsystems (quarks) together occupy three quantum volumes in space-time.

The Muon

The muon is the particle within the lepton group that is most closely related to the electron by its physical properties. Its mass follows directly from the geometry of the electron. It is a particle without a quark substructure. Its normative time constant is equal to twice the length of an electron spiral which has two loops. It can be seen as an aborted effort to create an electron. Its mass is that of a ground state field quantum.

The length of the spiral (*cf.* Appendix A) is:

$$\ell = L \cdot \sqrt{1+2N}$$

The mass of the muon is:

$$m_m = \frac{1}{2} \cdot \frac{h}{c^2 t}$$

The complete expression for the mass becomes:

$$m_m = \frac{h}{2c\sqrt[3]{Am_0 Q^2 \cdot \frac{4}{5}} \cdot \sqrt{5}} \qquad (48)$$

$$= 1.883 \cdot 10^{-28}\,\text{kg}\,(105.55\ \text{MeV})$$

This agrees with the observed muon mass to four figure accuracy.

Results and Prospects

In this paper I have shown how micro-particles might be approximated as singularities in spacetime. The most important results are the evidence of the unification of the forces in the electron system and a solution to the problem of particle rest-masses in general.

Another interesting possibility is that the description of the electron geometry presented above may offer new insights into the problem of superconductivity, which should benefit from a more in-depth knowledge of electron properties.

Many things remain to be said, perhaps in another paper. In Appendix B, I have given an overview of the particle spectrum, showing how the resonances are generated from the ground states.

This is neither the beginning, nor the end. My driving force has always been curiosity. And I am still very curious. There will always be new frontiers to cross. I hope we will help each other to challenge them. May I therefore be excused if I take the liberty of using the remaining lines for some speculations.

For example, the introduction of the cross-section for the interaction between the particles and the vacuum makes it possible to describe inertial mass as a local phenomenon, without any reference to Mach's principle, simply by the energy needed to change the pressure inside the closed particle surface as a function of the Lorentz compression.

The introduction of a process by which particles exchange energy with the vacuum space paves the way for a new description of gravitation, not necessarily in contradiction with the geometrical picture given by General Relativity, but possibly with some additional qualities of comprehension and simplification.

An understanding of the nature of the graviton may come within our grasp if the mass-particles, as well as the photons, are indeed subject to an energy loss with the same rate as the cosmological redshift, the result of which is seen as the background microwave radiation. The stable particles should then stay in their equilibrium state by absorbing and radiating energy with the same rate.

If the photons lose a minimal energy quantum each cycle, the rest-mass particles may pick up such quanta from the vacuum in an ecological renewal process. Every forced absorption of such a quantum of energy can be seen as the ejection of a hole of negative energy into the vacuum space. These holes, small enough to have an uncertainty in their position of cosmic size, can act as carriers of the gravitational long range action.

The energy absorption and radiation by the particles will make them serve as entropy regenerators, absorbing extremely long-waved quanta from the background and emitting a microwave spectrum. The super-particles existing as neutron stars could then be effective regenerators of waves powerful enough to produce new mass particles in large quantities from the vacuum.

The energy of the Sun might be due to the energy absorption by its particles from the vacuum, feeding the core of the Sun with the returned radiation from the particles in quantities sufficient to account for the entire radiation of the Sun and the fusion of the lighter elements into heavier ones all the way up to the complete neutron stars. This might also explain why the underground facilities for neutrino detection have failed, after more than 20 years, to detect any neutrinos released from the assumed fusion process on the Sun—perhaps the radiated energy does not come from fusion at all, but has a cosmological origin instead.

References

Bondi, Herman, 1961. *Cosmology*, Cambridge University Press.

Broberg, Henrik, 1981. *ESA STM*-**223**.

Broberg, Henrik, 1984. *ESA STM*-**233**.

Einstein, Albert, 1919. "Do Gravitational Fields Play an Essential Part in the Structure of the Elementary Particles of Matter", in *The Principle of Relativity*, Dover.

Hawking, Stephen W., 1987a. *Physica Scripta*, **T15**, 151.

Hawking, Stephen W., 1987b."Quantum Cosmology", in Hawking and Israel eds., *300 Years of Gravitation*, Cambridge.

Scherk, J. and Schwarz, J.H., 1974. *Nucl. Phys.*, **B81**, 118.

Weinberg, Steven, 1972. *Gravitation and Cosmology*, J. Wiley and Sons.

Appendix A1: Further Analysis of the Electron String Geometry

The mass of the magnetic field stored on the electron string/coil was given in the text by equation (20):

$$m_b = \frac{mQ^2}{8p\,L\left(1+\left[\frac{L}{2p\,rN}\right]^2\right)} \qquad \text{(A1.1)}$$

To solve this expression, we must establish a relation between the large radius of the particle string, *R,* and the small radius, *r*, of the loops. Consequently, we need to have some understanding of the geometry of the system composed of the string and its loops.

In the following, we refer to the parameters of the strings with the subscript "*S*" and the loops with "*L*". The lenth of the spiral is defined as $l = c\tau$.

The relationship between the kinetic energy of the string and the surface set up in the loop-system becomes

$$A \cdot m_S \left(g_S - 1\right) = k\left({}^{ct}\!/_{g_r} \right)^2 \tag{A1.2}$$

Similarly, the relation between the kinetic energy of the loops and the surface set up by the cycle-time in the string system becomes:

$$N \cdot Am_L \left(g_L - 1\right) = k\left({}^{ct}\!/_{g_r} \right)^2 \tag{A1.3}$$

In the above relations, N is the number of loops and k is introduced as an arbitrary parameter, which in the text of the paper was defined as l/2*p*.

From the spiral structure, we have the following relation:

$$1 = \frac{1}{g_S} + \frac{1}{g_L} \tag{A1.4}$$

Using these relations, the above formulae can be transformed into:

$$Am_S \cdot g_S = \frac{\left(g_S + 1\right)}{g_S} k\left(ct\right)^2 \tag{A1.5}$$

and

$$Am_L \cdot g_L = \frac{\left(g_L + 1\right)}{g_L} k\left(\frac{ct}{N}\right)^2 \tag{A1.6}$$

Hence, we have restored the original form of the string from formula (6) of the paper, which applies to the loops as well as the charged string itself.

The relation between the surface of the string and the sum of the surfaces of the loops therefore becomes:

$$\frac{f_S}{f_L} = \frac{g_S}{g_L} \cdot \frac{\left(g_S + 1\right)}{\left(g_L + 1\right)} \cdot N \tag{A1.7}$$

If only singularities are allowed as basic particle components, there can only be *c*-strings and 0-strings. Instead of assuming different peripheral velocities, we suppose that the time is dilated differently in each system, *i.e.* the string system and the loop system. In the terminology of General Relativity, we have introduced a metric system.

If the particle rest-mass is identified with the 0-string, its relativistic content of loops forms a system of *c*-strings. We interpret this as meaning that we have found a unique solution to the relationship between the surface of the rest-mass particle string and the surfaces of the loops, yielding a factor $2N$. This relation turns out to be the key to the different particle geometries.

For example, it is possible that in the system of the baryons, the geometry requires one 0-string to interface with two *c*-strings. The same logic applied to the mesons would dictate that two elements from one group interface with each other.

In the case of the electron system, the particle surface was assumed to be a sphere of radius R. This surface contains the energy of the N relativistic loops. If the energy of a *c*-string had a surface πr^2, the sum of N such surfaces would be $N\pi r^2$. The energy of the N loops makes up the energy of the particle. Therefore the sum of the surfaces of the loops is half the surface of the particle string:

$$N \cdot p\, r^2 = \frac{1}{2} \cdot 4p\, R^2 \tag{A1.8}$$

The relation between R and r is therefore:

$$r = \sqrt{\frac{2}{N}} \cdot R \tag{A1.9}$$

By inserting this relation in (A1.1), we find the expression for the mass and the radius of equations (30) and (31) in the main paper.

Appendix A.2: The Disappearance of the Electrical Energy in the Electron String

The remaining problem is to show that the electrical energy disappears in the little loops. If we consider one of these loops as having a fraction of the charge on its surface, we get the potential energy:

$$E_n = k\frac{(Q/N)^2}{r} \equiv k\frac{Q^2}{\sqrt{2}R_S}\cdot\frac{1}{N\sqrt{N}} \tag{A2.1}$$

Summing over all the N loops gives:

$$\sum_{n=1}^{N} E_n = k\frac{Q^2}{\sqrt{2}R_S}\cdot\frac{1}{\sqrt{N}} \tag{A2.2}$$

Hence, when N is large the potential electric energy of the loops disappears. The remaining energy is the tension along the string, which is equivalent to the magnetic energy.

Appendix B: The Mass Spectrum of the Hadrons

The generalized mass formula for a particle singularity composed of electromagnetic energy rotating about a surface on a Schwarzschild radius, adapted from equation (12) in the text, is:

$$E(N) = 86\cdot(N)^{2/3}\ \text{MeV} \tag{B1}$$

The integer N indicates the number of waves with a wavelength equal to the circumference of the particle that are needed to set up the particle energy that fulfills the Schwarzschild singularity condition in the system of waves. Choosing specific numbers for N gives the masses of ground-state particles, such as N=2 for the pion and N=36 for the *n*/*p* group.

Here we explain the use of the N values and indicate how particles may be formed using excitations to a quantum oscillator as a model.

The mesons

In the flat case where all N waves are in one plane, the particle ground-state will correspond to a squared integer number of waves, because the square of the number of wave-nodes (P) on the surface of the particle (ignoring half-waves) will be equal to the number of single waves (N) assumed to envelop the singularity in setting up the mass formula. A particle with four nodes will correspond to sixteen basic one-node waves:

This particle can also be split up into two two-node flat wave packets, which will have half the radius of the free particle.

The two sub-wave packets cannot exist as free ground-states themselves, since they do not satisfy the mass formula above. However, they may serve as building blocks for larger particles. In the same way, the two double-node packets can be broken down into two single-node waves.

The lowest level wave packets have a mass that is 1/4 of the 16-wave particle. Such a mass is possible in free form, according to the mass formula, since this gives a particle with quantum number N=2. A particle with this mass does indeed exist: the pion. The four-node (N=16) particle also exists in the form of the η-particle.

Taking the π and η particles as ground-states of quantum oscillators generates practically the whole meson spectrum, with the η particle as the ground-state for most of the spin mesons (though it has no spin itself).

The energy of the quantum oscillator can be written as:

$$E = \hbar w_0 \left(\frac{1}{2} L + n \right) \tag{B2}$$

where L is the number of degrees of freedom and n is the excitation quantum number. If the ground-state corresponds to $n = 0$, the ground oscillator is given by:

$$E_0 = \hbar w_0 \cdot \frac{1}{2} \tag{B3}$$

and the spectrum is given by:

$$E = E(N)\left(1 + \frac{2}{L} \cdot n\right) \tag{B4}$$

A spectrum that scans through most of the meson family is generated by $E(2)$ as the ground-state and L=4 degrees of freedom, corresponding to a system of two-dimensional oscillators. This spectrum is given in Table 1.

Among the particles listed above, there is a mixture of different spin states. By choosing a large number for the degrees of freedom, the particles with spin different from zero will be separated out. In Table 2, the η-particle has been used as a ground-state. Here, $E(16)=4E(2)$ and L=10, corresponding to a system of two oscillators, each with 5 degrees of freedom.

Only the ground-state and the D-particle have zero spin (the latter being an arithmetic coincidence). However, this model of mesonic isospin states apparently gives a fairly accurate description of reality, and additionally conforms to the quark-gluon concept of meson structure.

Table 1

Spectrum for *E*(2), *L*=4

n	Particle energy (GeV)	Nearest particle or resonance	Spin
0	0.137	π (0.135, 0.139)	0
-			
5	0.48	K (0.498, 0.494)	0
6	0.55	η (0.549)	0
-			
9	0.75	ρ (0.77)	1
10	0.82	ω (0.78)	1
11	0.89	K^* (0.892)	1
12	0.96	η' (0.958)	0
13	1.02	φ (1.02)	1
14	1.10	A_1 (1.10)	1
-			
16	1.23	B (1.231)	1
17	1.30	ε (1.30), A_2 (1.31)	0,2
18	1.37	κ (1.40)	0
19	1.44	K^* (1.43)	2
20	1.51	f' (1.52)	2
21	1.57	ρ' (1.60)	1
22	1.64	A_3 (1.64)	2
-			
24	1.78	K^* (1.78)	3
25	1.86	D (1.86)	0

Table 2

Spectrum for *E*(16), *L*=10

(2 oscillators, 5 degrees of freedom each)

n	Particle energy (GeV)	Nearest particle or resonance	Spin
0	0.55	η (0.55)	0
-			
2	0.77	ρ (0.77)	1
3	0.88	K* (0.89)	1
4	0.99	φ (1.02)	
5	1.10	A_1 (1.10)	1
6	1.23	B (1.231)	1
7	1.32	A_2 (1.31)	2
8	1.44	K^* (1.43)	2
9	1.51	f' (1.52)	2
10	1.64	A_3 (1.64)	2
11	1.78	K^* (1.78)	3
(12	1.86	D (1.86)	0)

The Baryons

A model for the baryon spectrum can be set up by analogy with the above model for the mesons, though the baryons are treated as three-oscillator systems (three quark-gluons) instead of the two oscillator-systems applicable to mesons. This gives a 6-node particle with 36 waves that has a mass identical to the neutron-proton group. The lower sub and sub-sub wave packets do not satisfy the mass formula, and therefore cannot function as free ground-states.

A quantum oscillator with energy $E(36)$ as its ground-state will be used to generate a spectrum. The formula for the energy levels is:

$$E = E(36)\left(1 + \frac{2}{L} \cdot n\right) \tag{B5}$$

The number of degrees of freedom for which resonances are found are L=12, 15 and 18, corresponding to three sub-particles with 4, 5 and 6 degrees of freedom respectively. The spectrum generated for each is given in Table 3.

However, it turns out that a generator starting with a 24-wave system (N=24) as a ground-state generates a large part of the baryon spectrum in a straightforward way, although the ground-state has not been observed to date. The formula for the excited states is then:

$$E = E(24)\left(1 + \frac{2}{L} \cdot n\right) \tag{B6}$$

Resonances are found for 12 and 18 degrees of freedom, corresponding to three four- and six-dimensional oscillators respectively. The particles and resonances generated are given in Table 4.

The top level of a hypothetical ground-state particle (N=24 waves) could be composed of three 3-dimensional wave systems with double nodes.

Table 3

Spectrum for
***E*(36), *L*=3X**
(2 oscillators, 5 degrees of freedom each)

X	Excitation number	Calculated energy (GeV)	Nearest particle or resonance
4	0	0.94	n/p (0.94)
	1	1.10	~ Λ (1.116)
	2	1.25	~ Δ 1.232
	3	1.41	Λ 1.405
	4	1.56	~ N 1.535 + other res.
	5	1.72	~ Ω (1.67)
	6	1.88	Δ 1.890, K 1.860
5	0	0.94	
	1	1.07	~ Λ (1.116)
	2	1.19	Σ (1.19)
	3	1.32	Ξ (1.32)
	4	1.44	N (1.39 - 1.47)
	5	1.57	~ multiple resonances
	6	1.69	Λ 1.690
	7	1.82	Λ 1.815
	8	1.94	Δ 1.950, R 1.940
	9	2.07	Λ 2.110 (2.050-2.150)
	10	2.19	N 2.190 (2.140-2.250)
6	0	0.94	n/p
	-		
	5	1.46	~ N 1.470 (1.390-1.470)
	6	1.56	-
	7	1.671	Ω (1.672)
	8	1.775	Σ (1.774) + multiple resonances
	9	1.88	Δ 1.890, K 1.860

Table 4

Spectrum for
***E*(24), *L*=3X**

X	Excitation number	Calculated energy (GeV)	Nearest particle or resonance
4	0	0.717	-
	-	-	
	2	0.96	~ n/p (0.94)
	3	1.08	~ Λ (1.115)
	4	1.196	Σ (1.195)
	5	1.32	Ξ (1.32)
	6	1.44	N 1.470
	7	1.55	~ Λ 1.520 + other resonances
	8	1.674	Ω (1.672)
	9	1.79	Λ 1.870
	10	1.91	Δ 1.910, Σ 1.915
	11	2.03	Σ 2.030
	12	2.15	N 2.190 + other res.
	13	2.272	Λ_c (2.273)
	14	2.39	~ Δ 2.420
	15	2.51	-
	16	2.63	~ N 2.650
6	0	0.717	-
	5	1.116	Λ (1.116)
	10	1.57	N 1.520 + other res.
	15	1.91	Δ 1.910 + other res.
	20	2.31	~ Λ_c (2.273)