Laws of Light Propagation in Galilean Space-Time*

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None of the theories proposed to date for the propagation of light in Galilean space-time has possessed the requisite experimental and theoretical coherence to merit consideration as an alternative to special relativity. The reason for this failure is that the notion of a propagating light wave, as accepted prior to and since Einstein's special relativity theory, has no validity in Galilean space-time. We propose a new concept of light waves, the distributed wave concept, and define laws of propagation.

1. Introduction

In Galilean relativity, the velocity composition law is strictly vectorial, *i.e.*

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 \quad (1)$$

regardless of whether one of the velocities is that of light.

* Editor's note: The manuscript of this text was being prepared for publication at the time of the author's death. The version published here has been edited to the best of my ability in accordance with the author's wishes.

If we denote the velocity of light measured between a source and receiver fixed relative to each other, as c_0, and call the velocity of a receiver's frame relative to the source u, and apply the same law as above, the expression for the velocity of light relative to the receiver is

$$\mathbf{c}' = \mathbf{c}_0 - \mathbf{v} \tag{2}$$

All modern relativists and anti-relativists have adhered to this procedure when discussing the velocity of light in Galilean relativity. However, this procedure for the composition of velocities leads to experimental incoherences when one of the velocities is that of light. For this reason Galilean relativity has been rejected *ipso facto.*

What we will discover in this paper is that in a composition where one of the velocities is that of light, the value c_0 (source fixed relative to receiver) is no longer the value to be used in the composition, for it stems directly from the concept of the "unique wave", as opposed to the *distributed wave* concept. After making the required corrections we will have no reason to reject Galilean space-time from this standpoint.

2. Propagation of light in Galilean space-time

We begin by setting forth the principles and hypotheses upon which the new theory will rest:

1. The principle of relativity, in its restricted sense.
2. The homogeneity of space and time.
3. The isotropy of space.
4. The principle of causality.
5. The requirement of group properties for the transformations of coordinates, time and space.

Both the Einsteinian and the Galilean relativity theories satisfy fully the above five principles and hypotheses (Lévy-Leblond 1976).

Any further hypothesis or principle is likely to set the two relativity theories apart. In fact, if we introduce the hypothesis of *constant velocity* of light, as Einstein did, we arrive at the special theory of relativity. If, on the other hand, we introduce the principle of *invariant space and time,* we obtain a neo-Galilean relativity theory. No coherent theory can admit both of these hypotheses.

For our purposes, we will postulate invariance of time (and space) as a sixth hypothesis:

6. The invariance of time (and space), T' = T.

Obviously, the Galilean transformation will now be fundamental to the new theory. But in that case, what happens to the Lorentz transformation? In its usual form, given below, it cannot hold in the new propagation theory if u is the relative velocity between reference frames.

$$\begin{aligned} x' &= (x - ut)\left(1 - \frac{u^2}{c^2}\right)^{-1/2} \\ y' &= y \\ z' &= z \\ t' &= \left(t - \frac{ux}{c^2}\right)\left(1 - \frac{u^2}{c^2}\right)^{-1/2} \end{aligned} \tag{3}$$

A mathematical definition of the Lorentz transformation is as follows:

> *A transformation of the variables X_1, X_2, X_3 and X_4 that leaves the expression*

$$X_1^2 + X_2^2 + X_3^2 - X_4^2 \tag{4}$$

invariant is a Lorentz transformation.

(Einstein 1905)

In the usual form of the Lorentz transformation, the coordinate X_4 is equal to $c_0 t$, where c_0 is the constant speed of light and t is the time. Here we introduce another transformation, one which will meet all of the six conditions given above. It is identical to the Lorentz transformation when expressed in hyperbolic functions, except that it has a different parameter. It also satisfies the mathematical definition of the Lorentz transformation for all null space-time intervals. The transformation and its inverse are given here.

$$\begin{aligned} X' &= X\cosh B - CT\sinh B & X &= X'\cosh B + C'T'\sinh B \\ Y' &= Y & Y &= Y' \\ Z' &= Z & Z &= Z' \\ C'T' &= CT\cosh b - X\sinh B & CT &= C'T'\cosh B + \sinh B \end{aligned} \tag{5}$$

where B, the parameter of the transformation, is defined as:

$$B = \frac{V}{c_0} \tag{6}$$

with c_0 a constant equal to the measured speed of light between a source and a fixed receiver. V is the relative velocity of a frame moving parallel to the XX' axis for a Galilean transformation.

In this new form, the coordinate X_4 and its transformation X'_4 are equal to CT and CT', where C and C' (usually not equal) are the new velocities of light, and where T, or T', is the time.

The transformation must now yield

$$X^2 + Y^2 + Z^2 - C^2T^2 = X'^2 + Y'^2 + Z'^2 - C'^2T'^2 \tag{7}$$

where C is the velocity of light relative to the source and in direction A. C' is the velocity of the same ray of light in the primed frame, in the direction A' (Figure 1).

As is customary, we have given only the transformation that applies to a velocity V parallel to the XX' axis. The transformations referred to the other axes follow readily.

Also, let a Galilean transformation, again for a velocity parallel to the X-axis, be expressed as:

$$\begin{aligned} X' &= X - VY \\ Y' &= Y \\ Z' &= Z \\ T' &= T \end{aligned} \tag{8}$$

Since the transformation in equation (5) is strictly Galilean, the values of C and C' are easily found by setting:

$$\begin{aligned} X' &= X \cosh B - CT \sinh B = X - VT \\ C'T' &= CT \cosh B - X \sinh B \end{aligned} \tag{9}$$

The term CT is defined as the trajectory of a signal starting out from the source S at time $T = 0$, and ending at the point P(X,Y,Z) at time T, where C is the speed of propagation of the signal (in the direction A) relative to its source S.

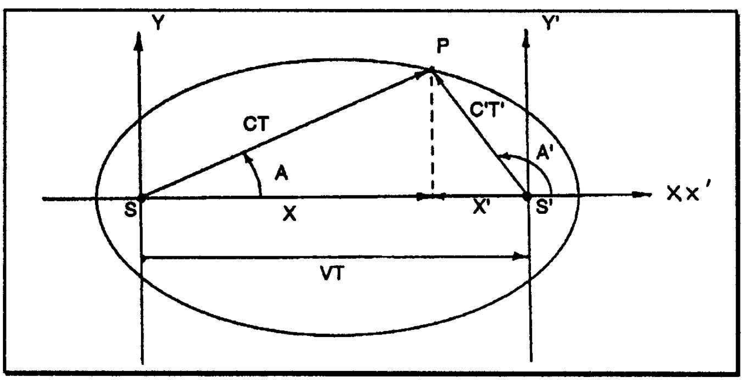

Figure 1

Equations (9) set the required conditions for satisfying both the new transformation and the Galilean transformation simultaneously.

From Figure 1 it is obvious that:

$$X = CT\cos A \quad \text{and} \quad X' = CT'\cos A' \tag{10}$$

Introducing the value of *X* in both right members of equation (9) we get:

$$CT\cosh A\cosh B - CT\sinh B = \cos A - VT \tag{11}$$

Dividing by –*T*, factoring and rearranging terms, we obtain for the value of *C*:

$$C = \frac{V}{\sinh B - (\cosh B - 1)\cos A} \tag{12}$$

A similar process on the variable *C'* gives

$$C' = \frac{V}{\sinh B + (\cos B - 1)\cos A'} \tag{13}$$

C and *C'* are the expressions, as functions of *A* and *A'*, respectively, and of *V*, for the velocity of all points on a uniformly and homethetically expanding ellipsoidal light wave relative to its S and S foci, or if we prefer, *C* and *C'* express the velocities of the signals emanating from the source S, situated at the origin of the rest frame, and from S', the virtual source located at the origin of the primed (moving) frame.

It is important to note that the signal velocity *C* is the vector sum of *C'* and V_1 where *C'* is itself a function of *V*. and not a constant.

$$\mathbf{C} = \mathbf{C} + \mathbf{V} \tag{14}$$

This velocity composition is quite different from the method used by Einsteinian relativists (and antirelativists) for the Galilean composition of the velocity of light with that of a moving frame. To an Einsteinian relativist light can have but one speed relative to the source.

It is interesting to note that the ratio C/C' is identical to the ratio

emitted frequency/received frequency

i.e., the Doppler factor in both theories. When expressed with hyperbolic functions, it is given by:

$$\frac{u}{u'} = \frac{1}{\cosh B - \sinh B \cos A} = \cosh B + \sinh B \cos A' \tag{15}$$

The values of C and C' are easily obtained for moving receivers whose trajectories pass through the source S, *i.e.,* where $A = 0°$ or $A = 180°$. They are given by:

$$C_{0°} = \frac{V \exp B}{\exp B - 1}, \quad C'_{0°} = \frac{V}{\exp B - 1} \tag{16}$$

The Doppler factor is just $C/C' = \exp B$.

$$C_{180°} = \frac{V}{\exp B - 1}, \quad C'_{180°} = \frac{V \exp B}{\exp B - 1} \tag{17}$$

Here the Doppler factor is $C/C' = \exp(-B)$. The curves for C and C' are given for this case in Figure 2.

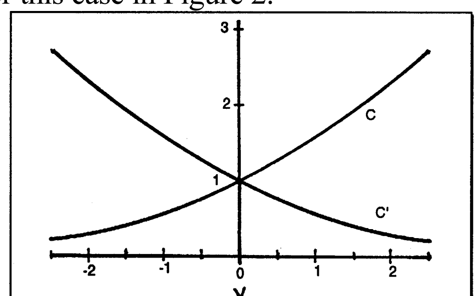

Figure 2

Note that setting $V = 0$ in equations (12) and (13) gives $C = c_0$ and $C' = c_0$.

It is remarkable that this new law of propagation of light in Galilean space-time reduces to the constant c_0 independent of A and A' when $V = 0$. Obviously the Doppler factor is equal to 1 in this case. When V tends to infinity, C also tends to infinity and C' tends to zero. When V tends to minus infinity, C' tends to infinity and C tends to zero.

When light is reflected off a body and is returned to its original emission point, the law of propagation for the return trip is the same as for the original trip. This does not mean that the value is the same in both directions, for on the return trip the value of the receiver's velocity (*i.e.*, the velocity of the original source) has now changed sign, and the angle A takes on the value of its supplement. For the return velocity of light C_R in the original source frame, these changes in values yield:

$$C_R = \frac{V}{\sinh B + (\cosh B - 1)\cos A} \tag{18}$$

Note the change of sign in the denominator.

3. Correspondence Between the Two Theories

When both theories are expressed in variables that refer to frames having their X and X' axes parallel to the velocity of the primed frame relative to the unprimed frame, and where two frames only are considered, we find a correspondence between their coordinates.

The space-time coordinates of events in the Einsteinian theory will be given in lower case, and their Galilean homologues will be in upper case. The frame velocity will therefore be v in Einstein special relativity and V in Galilean relativity. We find the following correspondences between variables:

$$T = t - \frac{x}{u}\left[1 - \left(1 - \frac{u^2}{c_0^2}\right)^{1/2}\right] \tag{19a}$$

$$T' = t + \frac{x'}{u}\left[1 - \left(1 - \frac{u^2}{c_0^2}\right)^{1/2}\right] \tag{19b}$$

$$\begin{gathered}
\frac{u}{c_0} = \tanh\frac{V}{c_0} = \tanh B \\
\frac{1}{\left(1 - u^2/c_0^2\right)^{1/2}} = \cosh B \\
\frac{u/c_0}{\left(1 - u^2/c_0^2\right)^{1/2}} = \sinh B \\
\frac{X}{x} = \frac{Y}{y} = \frac{Z}{z} = \frac{X'}{x'} = \frac{Y'}{y'} = \frac{Z'}{z'} = \frac{B}{\sinh B} \\
t = T + \frac{X}{V}\left(\cosh B - 1\right) \\
t' = T' - \frac{X'}{V}\left(\cosh B - 1\right)
\end{gathered} \tag{20}$$

4. Experimental Coherence of the New Theory

If we accept the fact that Einsteinian special relativity yields experimentally coherent results, we must also admit that the new theory produces equally coherent results, for both theories will predict the same *measurable* quantities in all experiments where only two reference frames are involved. No experiment has yet been performed for a three reference frame (or three observer) situation where only the velocity of light is brought into play.

Here we would like to make an important remark on measurable quantities. Einstein, in his presentation of the special theory of relativity, refers to clocks and rods as measuring instruments. We show here that rods cannot be involved as permissible measurable instruments in such a theory.

From his observing station an observer can only measure temporal and angular intervals directly at any one time. A rod cannot be used to measure distances, because an observer can only be at one end of the length to be measured at one time. All length measurements involve a theory: a theory of light propagation. The only way to determine a length is with the aid of a clock, a light source and a theory of light propagation. This means that all conclusions based on length measurements in the Einsteinian presentation of special relativity are not admissible, since they beg the question.

The only way to fully prove the Einstein theory is in a situation where at least three reference frames are involved. To date no such experiment has been performed. The new theory and Einstein's theory are the only two possible theories that can accommodate experimental results obtained so far. Both cannot be true to facts in all experiments, since this would imply that one theory is an isomorphism of the other, which is obviously impossible.

A three-observer experiment relies only on direct measurements for the determination of the velocity of light between moving objects. To accept interpretations where energies, deduced from the principles of special relativity, are involved is likewise begging the question. To avoid faulty procedures, we suggest an experiment where direct measurements only are involved. The only instruments required will be two clocks, a fast-moving (the faster the better) reflector and a light source.

As shown in Figure 3, a light source S is placed on the trajectory of a fast-moving reflector M. An observer is located at the source S and another one is at a point P on the trajectory of M. Sometime after M passes 5, the source emits a flash of light in the direction of M. If

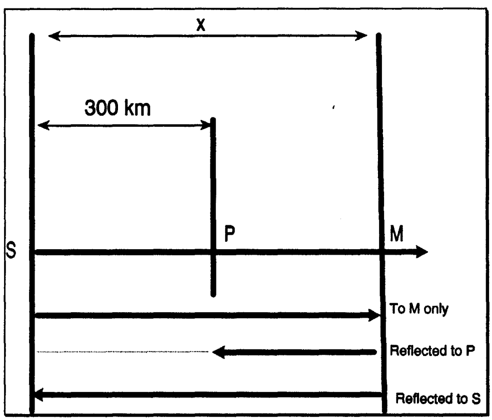

Figure 3

Diagram showing the two to-and-fro paths of light from S to P and M. The following numerical values illustrate the expected effect:

L: distance SP	300 km
T^E: emission time	0s
T_P: reception at P	$0.001s = L/c_0 = 300/300000$
T_{rPS}: echo from P at S	$0.002s = T_P + 0.001s$
T_{rMP}: echo from M at P	a function of V and X
T_{rMS}: echo from M at S	a function of V and X

the flash can be timed to be reflected from M after it has passed the observer at point P, the light returning to P and S will either have a velocity c_0 (if special relativity is valid) or a smaller velocity, if the new theory is true.

Figure 3 illustrates the experimental conditions. The values indicated are not necessarily the best for a feasible experiment. They are only given to illustrate the principles involved.

If the Einstein theory is valid, the difference between T_{rMS} and T_{rMP} would be equal to the difference between T_P and T_{rPS} If, on the other hand, we find that $(T_{rMS} - T_{rMP})$ is greater than $(T_{rPS} - T_{rP})$, then we can say without doubt that the theory presented here is experimentally coherent and that the velocity of light is not constant between all reference frames.

For a further check, we can repeat the experiment with an approaching reflector, instead of the receding one, and should find that the sign of the inequality is now reversed. The second experiment has the advantage of disposing of experimental errors when the difference is small.

5. Light Waves in Galilean Space-Time

All light waves in Galilean space-time are ellipsoidal expanding surfaces of revolution (a sphere is also an ellipsoid) about the *XX'* axis of the source and receptor frames, since the velocity *C*, in equation (12) describes an ellipsoidal surface in three-dimensional space when the angle A is varied from 0° to 360° (and the XY plane is rotated about the *X* axis) with *T* kept constant. The eccentricity *E* of such an ellipsoidal surface is given by:

$$E = \frac{\cosh B - 1}{\sinh B} = \frac{\exp B - 1}{\exp B + 1}$$

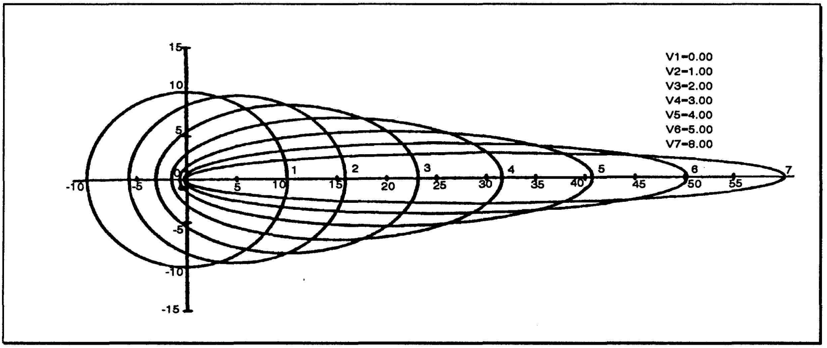

Figure 4 - Shapes of Galilean light waves for various relative velocities. The theoretical maximum velocity is infinite.

Figure 4 shows the shapes of light waves in reference frames at various relative velocities.

The angles *A* and *A'*, which are always measured from the direction of *V* to that of a light ray, are related as follows:

$$\cosh A' = \frac{\cosh B \cos A - \sinh B}{\cosh B - \sinh B \cos A}$$

$$\cos A = \frac{\cosh B \cos A' + \sinh B}{\cosh B - \sinh B \cos A'}$$

The above relations are of course identical to those given in Einstein special relativity for the same angles when they are expressed with hyperbolic functions.

In Einstein special relativity the light cone is a right circular cone with time measured on the axis perpendicular to the *XY* plane. The Galilean counterpart of this cone is an oblique elliptical cone having as its time axis the world-line of the source S. Figure 5 depicts such a cone.

6. The Distributed Wave Concept

If the abstract signals we have been analyzing in Galilean space-time are to be identified with light signals, then we must inevitably conclude that light waves have variable velocities, not only relative to the receptors, but also relative to the emitting source. But how can signals vary their velocity relative to their receptors? At first glance, it would appear that this requirement violates the principle of *causality*. Let us look at a simple case where a similar condition is met.

Assume that we have a special machine gun with a thousand barrels, and assume that in each burst it shoots bullets at different velocities from each barrel. The velocities can be distributed from near zero to some maximum value, according to a distribution function D(V). Then one shot from the gun is actually one thousand bullets. For the target that receives the bullets, the distribution function may appear quite different from the function at the machine gun. If the target is fixed relative to the gun, what was formerly a velocity distribution of the bullets will turn out to be a "reception time" distribution, with the slowest bullets arriving last. If the target moves away from the machine gun at a given velocity, it is obvious that all the bullets whose velocities are less than or equal to the velocity of the target will never reach their goal, thereby changing the distribution function of the receptions at the target.

If we regard the *average* velocity, or the root mean square, as the muzzle velocity of all the received bullets, then it will no longer appear as a violation of the principle of causality to say that the muzzle velocity of the machine gun varies to suit the relative velocity of the target.

We can apply the same principle to light emitted by a source: we assume that the light is emitted at all velocities according to a certain distribution law. The question that now arises is: Which distribution law?

A close look at equation (17) for the velocity of light relative to the receiver, in the case of a one-dimensional space, reveals a very close resemblance to the Planck distribution. Here is how Sir Edmund Whittaker describes Planck's theory of black body radiation (Whittaker):

> *The theorem corresponding to this [situation] in the quantum theory is that if the energy of an oscillator can take only the discrete values U_0, U_1, U_2, ...U_S, then the probability that the energy of a particular oscillator is US is*

$$\frac{\exp\left(\frac{-U_S}{kT}\right)}{\sum_{S=0}^{\infty}\exp\left(\frac{-U_S}{kT}\right)}$$

> *Thus if $U_S = shn$ for $s = 0, 1, 2, \ldots$, the probability is*

$$\left[\exp\left(\frac{-shn}{kT}\right)\right]\left[1-\exp\left(\frac{-shn}{kT}\right)\right]$$

> *The mean energy of an oscillator is therefore*

$$hn\left[1-\exp\left(\frac{-hn}{kT}\right)\right]\sum_{S=0}^{\infty}s\exp\left(\frac{-shn}{kT}\right)$$

> *which has the value*

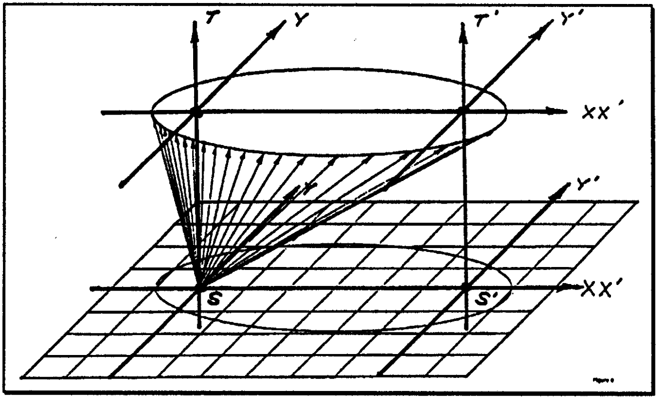

Figure 5 - The Galilean light-cone is an oblique elliptical cone with its time axis perpendicular to the X-Y plane at the source and its generator so inclined that the tangent of the angle measured from the time axis to the light ray (on the surface of the cone) is equal to C/c_0 in the given direction.

$$\frac{hn}{\exp\left(\frac{hn}{kT}\right)-1}$$

This leads at once, as before, to Planck's formula where the energy density of black body radiation in the frequency interval from n + *d*n *is*

$$E=\frac{8p\,h}{c^3}\frac{n^3\,dn}{\exp\left(\frac{hn}{kT}\right)-1}$$

If we compare the above equation for the mean energy of light quanta to equation (17) above, the resemblance is so striking that we get the feeling we are in the right ball park. To tie the two together we must establish a relation between their variables. On the one hand, we have a ratio of velocities, and on the other, a ratio of energies. We have no choice here, velocity must be related to energy if the two equations are to be connected. We may therefore posit that the energy-density (for each frequency at the source) is a linear function of the velocity "perceived" by the receptor. Then we have tied energy-density with the velocity of radiative flux.

What is most interesting about this is that the above relation was found within the confines of classical (not relativistic) quantum theory. The concept of a "distributed wave" velocity (as in special relativity) can therefore be expected to provide a key to other problems in the realm of quantum mechanics.

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