Mach's Principle and Properties of Local Structure

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An explicit formulation of Mach's principle is given, yielding a finite gravitational background. Its value appears to be the same as the local acceleration in systems of various levels of hierarchy, which indicates that Machian forces control the formation of local structure. The large-scale chain structure can be interpreted in the same way. The level and scale at which the transition from the hierarchical structure to the homogeneous and isotropic distribution occurs is explained as is the stability of the Metagalaxy. These results are based on the assumption of a coupling between the gravitational and electromagnetic interactions.

1. General remarks

In itself, inertia does not require any interpretation in terms of physics. On a general level the question can be reduced to that of causality. As a counter-example, if a mass were non-inertial, it could set itself in motion, or stop, without physical action, contrary to the principle of causality.

On the other hand, inertia calls for interpretation in the presence of gravitation, which is an attractive force presumed to cause contractions and even collapses. In other words, the question reduces to: why an equilibrium density distribution of matter? This question is significant for both local systems and cosmology.

Locally, the centrifugal force connected with rotation is known to maintain a (quasi-)equilibrium mass distribution. The problem of the origin and maintenance of the rotation is still completely open. Although we will not attempt to solve this problem in the present paper, a result closely related to it will be obtained (Sections 4 and 5). Another important effect working against gravitation involves ejections and outflows of matter from the nuclei of galaxies.

Globally, according to common belief, there is no equilibrium. Instead, a universal expansion solves the question of gravitation (and inertia against it) in an *ad hoc* manner, through the postulate of a gigantic initial explosion, the "big bang". However, the Universe is most probably stable, as indicated by properties of the redshift effect (Jaakkola 1978), by the results of the global cosmological tests (Jaakkola *et al.* 1979; Jaakkola 1983a), and by the absence of cosmological evolutionary effects (Jaakkola *et al.* 1979; Jaakkola 1983a; Jaakkola 1982a; Laurikainen and Jaakkola). How global stability is possible in the presence of gravitation is one of the central topics of the present paper, which also deals with the related problem of the homogeneity and isotropy of the global matter distribution.

2. A brief historical note

A brief historical account is in order. Analogous to Olbers' paradox for the intensity of the background radiation, there also exists a gravitational paradox: in an infinite Universe, Newton's gravitational potential is indefinite. Obviously such a system would be unstable. In order to maintain a stable Universe (as it was assumed to be at that time), at the end of the last century Seeliger (1895) and Neumann (North, 1965) added an extra factor $\exp(-ar)$ in the expression for the potential. Earlier, Laplace had posited the same law. Newton himself, and later Hall and Green, had adopted a potential of the form $f(r) = r^{p-2}$ ($0 \leq p < 1$) (North, 1965). Einstein's cosmological constant is a famous later variant of the extra factor.

The non-Newtonian formulations can be interpreted in terms of a cosmological repulsion. The notion of a repulsive force, balancing the attractive force, is quite familiar in the eastern ancient Greek and early dialectical-materialist philosophical treatments (Engels, 1972), and it was also shared by the aforementioned physicists, except Seeliger and Neumann who preferred an absorption of gravitation.

In the following we shall adopt the latter approach. Though impossible a century ago, it is now possible to identify the source of the factor $\exp(-ar)$. We observe the radiation of the distant sources diluted not only by the geometrical distance effect, *i.e.* by the factor $1/4pr^2$, but also by an additional $1/(1 + z)$ due to the redshift effect. In the case of the non-expanding Universe, redshift is an absorption-like effect which must be due to a physical interaction of some kind. This solves the paradox of the finite radiation background in a straightforward manner but does not give the ultimate answer. Evidently we must seek solutions both for the gravitational background paradox and the problem of the mechanism of the redshift effect. If we assume coupling between the gravitational and the electromagnetic interactions, both problems are solved. Redshift is caused by gravitation which becomes weakened by $\exp(-ar)$ in the process.

3. An explicit formulation of Mach's principle

From the coupling between the gravitational and the electromagnetic interactions, an explicit formulation of the gravitational interaction by the cosmic masses can be deduced.

The scalar sum of interactions by masses within a shell with radius r, thickness dr and average density $\boldsymbol{r}$ is

$$\frac{4\boldsymbol{p}\, r^2 dr G \boldsymbol{r} \cdot \exp(-\boldsymbol{a}\, r)}{r^2}$$

and therefore all the masses within a radius r affect a unit mass as

$$a_r = \int_0^r 4\boldsymbol{p}\, r^2 dr G \boldsymbol{r} \cdot \exp(-\boldsymbol{a}\, r) = -\frac{4\boldsymbol{p}\, G \boldsymbol{r} \left[\exp(-\boldsymbol{a}\, r) - 1\right]}{\boldsymbol{a}} \qquad (1)$$

It is evident in the hypothesis adopted here that the extra weakening of gravitational progresses as the redshift weakening of radiation. Then, with $\boldsymbol{a} = H/c$, we have $\exp(-\boldsymbol{a}r) = 1/(1 + z)$, and in observable parameters

$$a_z = \frac{4\boldsymbol{p}\, G \boldsymbol{r}\, c}{H} \cdot \frac{z}{1+z} \qquad (2)$$

With $z \rightarrow \infty$, we obtain the global effect of cosmic masses:

$$a_c = \frac{4\boldsymbol{p}\, G \boldsymbol{r}\, c}{H} \qquad (3)$$

Equations 1-3 can be considered an explicit formulation of Mach's Principle.

Figure 1 shows the function a_z, as well as the contributions a_i from different z-intervals. One half of the Machian interaction comes from within $z = 1$, which corresponds to $r = \ln 2\ c/H = 2450$ Mpc (if H = 60 km s^{-1} Mpc^{-1}). This scale, which also contributes one half of the cosmic background radiation, can be regarded as an "effective radius

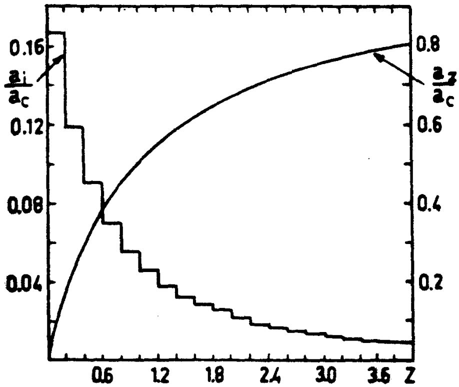

Figure 1 – Step function: contributions to the background gravitation from the different *z*-intervals. Smooth curve: contribution from within *z*.

of the Universe." The value of the Hubble radius is $c/H = 5000$ Mpc. Of course, these terms apply within the frame of an infinite Universe.

Numerically, assuming that $\rho = 5\times 10^{-30}$ g cm^{-3}, we have $a_c = 6.4\times 10^{-9}$ cm s^{-2}. In Equations 1-3, the cosmic interactions affecting a particle are integrated over the whole sky, as usual for the Machian interaction. In the vectorial presentation the forces cancel each other. For problems of local dynamics *vs*. the cosmic effect we consider the gravitational effect from a restricted solid angle, e.g. π steradians. Indeed, in view of the step function in Figure 1, the actual non-isotropic nearby distribution becomes significant. In the following, for the Machian interaction we shall use the expression

$$a_c^* = \frac{p\,G\,r\,c}{H} \simeq 1.6\times 10^{-9}\,\mathrm{cm\,s^{-2}} \tag{4}$$

Table 1

Class of objects	log M (g)	log R (cm)	a (cm s^{-2})
Supergalaxies	48.7	25.5	3.4×10^{-10}
Large clusters of ellipticals	47.9	24.5	5.4×10^{-9}
Small clusters of ellipticals	47.2	24.3	2.7×10^{-9}
Small clouds of spirals	47.0	24.3	1.7×10^{-9}
Small loose groups of spirals	46.5	24.1	1.3×10^{-9}
Small dense clusters of ellipticals	46.5	23.7	8.5×10^{-9}
Compact groups of spirals	45.5	22.6	1.3×10^{-9}
Giant ellipticals	45.5	22.35	4.2×10–7
M31 (Sb)	44.6	22.79*	$7.0\times l0^{-9}$
Milky Way (Sbc)	44.45	22.67* *	8.8×10^{-9}
The Universe (a_c)	—	—	6.4×10^{-9}

*R = 20 kpc adopted, **R = 15 kpc adopted

4. Interpretation of the structure of galaxies and systems of galaxies

Independent of the above discussion, for a stable system the local acceleration a_s subtended by the system to a unit mass lying at the edge of the system cannot smaller than a_c^* (or, what is practically the same, a_c). In a system of mass M and radius R this acceleration is

$$a_s = \frac{GM}{R^2} \tag{5}$$

Table 1 gives data for M, R, and a_s, in systems of various levels and kinds; the values of M and R are taken from de Vaucouleurs (1971; Allen 1973). It can be seen that within a range of scales extending by a factor of about 10^3, the local accelerations are similar to each other. At the same time, these are close to the cosmological value a_c^*. This implies that cosmic masses control the formation and original evolution of the large-scale structure.

In Table 1 the giant elliptical galaxies form an exception to the rule. This can be easily understood as due to their long individual dynamical evolution.

The "nests" of galaxies are interesting in this connection. These are the dense groups of interconnected galaxies first studied by Vorontsov-Velyaminov (1977). He pointed out that there is a continuous sequence from the multiple-nuclei galaxies to interconnected, and then disconnected groups of galaxies, as if the systems were formed by smooth, continuous fragmentation (and as if there were no gravitational binding of the masses). Now if the cosmic gravitational pull a_c exceeds the force by the system a_s, these interesting objects can be explained in a straightforward manner.

5. The phenomenon of rotation

The discovery that $a_s \simeq a_c^*$ systematically should be taken into account in any attempt to develop a theory of rotation. No such theory for the case of a static Universe exists as yet (except one suggested by the present author (Jaakkola 1986). In the latter hypothesis, explosions in the nuclei of galaxies act as catalysts in generating outflows of stars with proper motion in the direction of rotation; the net effect is that the random motions are transformed into rotational motion. However, this mechanism does not explain the facts in Table 1, and may also be to inefficient to explain the observed rotation. A mechanism of rotation that explains the equality of a_s and a_c^* as well cannot yet be envisioned. At the moment it suffices to say that the effect of rotation is an active effect; galaxies, stars, etc., are rotationally active systems. Certainly rotation is not a relic effect that originated in the beginning of the Universe. This is true whether the initial 'big bang' is real or not. The extremely rapid rotation of spiral galaxies, which has given rise to the problem of dark massive haloes of galaxies, speaks clearly enough.

6. The large-scale chain structure

The last few years have revealed a completely new feature of large-scale distribution: the distribution of the clusters, groups and more isolated galaxies into chain-like forms extending up to a hundred megaparsecs and having huge regions devoid of galaxies between them. The matter is not yet clear, but it seems that we cannot even speak of separate superclusters of elongated form. There appears to be a more basic structure, with the chains connected to each other in a single infinite network. Any cosmology must explain this structure.

Let us assume a supercluster with a few clusters, a total mass of 10^{49} g, radius of 15 Mpc more or less symmetrically spherical in form. Then $a_s = 3.2 \times 10^{-10}$ cm s^{-2}. Because a_s is smaller than a_c by a factor of five, the system is not stable against the cosmological pull, possibly augmented by the pull from the nearest rich system. The furthermost cluster (or a cluster in the direction of the neighbouring system) begins to escape and draws others with it. The resulting subtraction of mass causes an escape also in the opposite direction. A chain is formed. (A chain structure is nothing other than a structure being drawn from the outside.)

Obviously, any particular chain is temporary. The chains intermingle with each other, form temporarily more compact systems and fly apart again. The strength of the background gravitation keeps the chain structure as alive as we observe it. The lower limit of the time-scale of these phenomena can be obtained by integrating $a_c - a_s \approx a_c$ twice and adopting 50 Mpc as the typical scale-length of the chains. The result is 1.4×10^{10} years, identical to the Hubble time-scale and astrophysical time-scales.

7. The step from hierarchical structure to homogeneity and isotropy

The transition from local systems of various scales to a homogeneous distribution means a transition to the domain of cosmology. For the homogeneous distribution we can determine the mean values of various parameters applicable to the total infinite universe (Jaakkola 1986; Hubble 1934). Therefore, a theoretical explanation of the scale at which this happens is most important for any cosmological theory. We saw in the previous section that $a_s \leq a_s$ for the superclusters. These, and the chains into which they turn due to the Machian force, are predicted to be the largest individual structures in the Universe, and consequently should be distributed homogeneously and isotropically. This also fits the available data.

8. Global stability

Global stability can be understood in the 1ight of the preceding arguments, *i.e.*, by the finiteness of the background gravitation, the gradual weakening of the contributions from the different distances as shown in Figure 1, the local average mass density equilibrium partly caused by the Machian pull and partly by the explosive processes in the nuclei of galaxies, and the cessation of hierarchical structuring on a certain scale. Formation of a larger system, which would be implied by a global motion involved in the case of instability, is of course not allowed.

9. Discussion and summary

There are three outstanding empirical features of the observable Universe which are intimately connected with the effect of gravitation. These are: homogeneity and isotropy, the existence of local structure (galaxies and systems of galaxies), and the static state of the Metagalaxy. How do the various cosmological theories fare when confronted with these observations.

The standard big-bang cosmology cannot safely explain any of them. Homogeneity and isotropy cannot be derived from fundamental theory (Peebles 1980) and causality poses serious problems: regions now 3 degrees apart were causally disconnected at the time of recombination. The origin of galaxies in the expanding frame is extremely difficult to understand. The seeds of galaxies and their systems should have existed from the very beginning of the Universe. Still there are no signs of small-scale fluctuations of the 3 K background. Moreover, if they existed, their development into galaxies cannot be deduced from known physical principles (Peebles 1980); rather, they should have led to black holes. However, some 10^{11} galaxies are observable, and not one black hole has been reliably observed. Third, while the big-bang theory cannot be required to interpret the staticity of the Metagalaxy (as it takes expansion for granted) one might insist on an explanation of the assumed expansion directly from fundamental theory (*i.e.*, gravitation). However, fundamental theory points to a contraction of the Universe, and expansion is obtained only by an *ad hoc* assumption of an initial "big-bang."

Therefore, in terms of explanatory power, the theory formulated above appears superior to the standard cosmology. The hypothesis of coupling between the gravitational and the electromagnetic interactions solves both the gravity paradox and, through the redshift

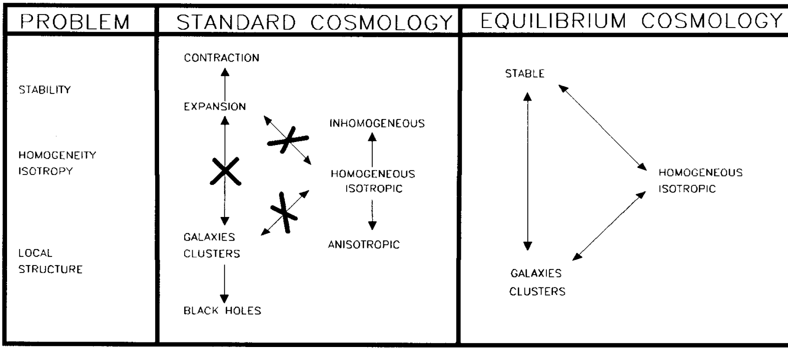

Figure 2 – Comparison of two theories. Crossed arrows in the left indicate that all three major features are mutually inconsistent; the short arrows show what follows from the fundamental theory of the standard cosmology. In the equilibrium theory all three major features are mutually consistent.

effect, Olbers' paradox. (However, the latter solution is only partial; the complete solution must also take into account the other interactions.) Furthermore, it explains the major features of local structure at the level of galaxies, groups, clusters and superclusters, as well as the lattice structure on the largest scale. It sets the scale of the transition from hierarchical to homogeneous structure in the correct place. As a corollary, it can also yield an interpretation of the very significant similarity of local and cosmological radiation energy densities, and an explanation of the dipole anisotropy of the radiation (Jaakkola 1982b). In fact, this hypothesis is essential for the cosmology of equilibrium evolutionary processes required by a non-expanding Universe. The empirical facts can be seen to converge with different parts of the theory. The situation in the "Triangle of Big Problems" is shown graphically in Figure 2.

Turning to the questions posed in the introduction, the inertia resistance of matter to collapse results both locally and globally from the gravitational interaction of cosmic masses working in an explicit manner, as described by Equations 1-4. If the inertia of matter is understood as resistance to collapse, Mach's intuition that the cause lies in the cosmic masses was right.

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