Neo-Hertzian Wave Equation for Variable Detector Velocity

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It is shown that a d'Alembertian solution previously obtained for the neo-Hertzian wave equation under the restriction that detector (absorber) velocity is constant holds likewise with minor modification in the more general case in which that velocity is an arbitrary function of time. Remarks are made also on the two-way averaging of light speed in neo-Hertzian electromagnetism.

Keywords: neo-Hertzian electrodynamics; variable detector velocity

Introduction

It has been argued previously [1-3] that a first-order invariant covering theory of Maxwell's electromagnetism (EM) originally proposed by Hertz [4] provides not only a formally more comprehensive approach to EM description but one that is more competent to describe the weak-field (single photon) limit. This claim depends on a specific physical interpretation of the Galilean invariant formalism, whereby the Hertzian fields are operationally defined in a different way from the Maxwellian ones. That is, Maxwellian fields are measured at a field point stationary with respect to the inertial observer, by a field detector or "test charge" that is by definition also at rest at that field point; whereas Hertzian fields, likewise referred to a stationary field point, are measured by a detector or test particle that can move with arbitrary velocity $\mathbf{v}_d$ with respect to that point (so that it momentarily passes through the field point at the instant of measurement). This introduces formally a new "detector velocity" parameter into EM theory, so that Hertzian theory parametrizes both source and sink motions, whereas Maxwellian theory leaves sink motions unparametrized. Mathematically, invariance in the Hertzian case and covariance in the Maxwellian case express the relativity principle with respect to inertial motions.

The claim of Hertzian superiority in the weak-field limit arises from the fact that Hertz's theory is adequately parametrized to describe from the viewpoints of *n* different inertial observers the relative motions of a single "public" instrument of field detection; whereas *n* Maxwellian observers require *n* different macro instruments, one comoving with each inertial observer (permanently fixed at his field point), in effect the "private property" of that observer. Both alternative ways of treating the relative motions of observers and detectors are equally valid if the signal to be detected is carried by a superabundance of field quanta. In this case all Maxwellian detectors, as their field points instantaneously coincide, receive enough quanta to register significant numbers—*i.e.*, those quantifying covariantly related field components. But in the single-quantum limit it is obvious that at most a single macro

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detection instrument can absorb the quantum. Any other macro instruments passing through the momentarily shared field point will register zero and thus cannot describe covariantly related field components. (Such localized detection *event uniqueness* lies at the heart of the "quantum revolution.") Thus the Hertzian invariant (single-instrument) approach remains valid in the limit of few quanta, whereas the Maxwellian covariant (many-instrument) approach fails completely.

Previous papers [1-3] have treated the special case in which the Hertzian detector velocity parameter $\mathbf{v}_d$ is a constant. Here we shall extend this to the case of arbitrary detector velocity $\mathbf{v}_d = \mathbf{v}_d(t)$ relative to the observer. We treat arbitrary motions of the detector but not of the observer (since the latter remains at rest with respect to his inertially moving frame and given field point). Our analysis will be limited, as before, to the case of vacuum EM. In order to treat general motions of the test charge or detector (including those at high speed), we need to take account of the time dilatation associated with detector motion. Such dilatation we shall consider to be an empirical fact, based on CERN evidence [5], without reference to any theory involving an assumption of spacetime symmetry. (On the contrary, our assumed higher-order spacelike invariant of kinematics will be the Euclidean length interval, in the absence of empirical evidence for dimensional noninvariance of moving extended material structures.) We limit ourselves to the (irrotational) point particle detector—that is, we do *not* consider a more general sort of "mollusk" detector described by a *velocity field* $\mathbf{v}_d = \mathbf{v}_d(x,y,z,t)$. (Since photon detection or absorption occurs always at a point, or so near it as to make little difference, there is no physical gain or justification for considering different "parts" of a photon detector to be simultaneously in different states of motion. The readings of such a detector would moreover be ambiguous and its construction an exercise in perversity.)

The detector proper time τ_d, measured by a clock comoving with the test charge, is related to the observer's frame time *t* by the defining differential relationship

$$d\tau_d{}^2 = dt^2 - \left(dx^2 + dy^2 + dz^2\right)/c^2 \ , \tag{1}$$

wherein the differentials are delimited by event points lying on the detector or test charge trajectory. From this follows

$$\frac{d}{d\tau_d} = \frac{dt}{d\tau_d}\cdot\frac{d}{dt} = \gamma_d \frac{d}{dt} \ , \tag{2a}$$

where

$$\gamma_d = \frac{1}{\sqrt{1 - v_d{}^2/c^2}} \text{ and } \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v}_d \cdot \nabla \ , \tag{2b}$$

the last being the total time derivative, whose Galilean invariance has been repeatedly proven [1-3]. Our postulated neo-Hertzian field equations for the vacuum are [2,3]

$$\nabla \times \mathbf{B} - \frac{1}{c}\frac{d\mathbf{E}}{d\tau_d} - \frac{4\pi}{c}\mathbf{j}_m = 0 \tag{3a}$$

$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{d\mathbf{B}}{d\tau_d} = 0 \tag{3b}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{3c}$$

$$\nabla \cdot \mathbf{E} - 4\pi\rho = 0 \ . \tag{3d}$$

These are supposed to be valid at higher orders in (v_d / c), whereas the Hertzian equations [1], employing *d/dt* for $d/d\tau_d$, are valid only at first order. Here $\mathbf{j}_m$ differs from the Maxwellian current density **j** at first order by a convective term, $\mathbf{j}_m = \mathbf{j} - \rho\,\mathbf{v}_d$. (The higher-order form of this velocity composition law, discussed in [2], is not needed in treating the wave equation, for which $\mathbf{j}_m = 0$.) In the limit $\mathbf{v}_d \to 0$ it is seen that $\mathbf{j}_m \to \mathbf{j}$ and $d/d\tau_d \to d/dt \to \partial/\partial t$, so that the neo-Hertzian equations (3) reduce to Maxwell's and thus constitute a covering theory. In other words Maxwell's theory is the special case of Hertz's theory in which the field detector is held stationary at the field point, $\mathbf{v}_d = 0$. Higher-order invariance of (3) follows from the fact that only kinematic invariants appear there. It is due to the appearance of total rather than partial time derivatives that Hertzian and neo-Hertzian theories are incompatible with spacetime symmetry (since a total time derivative is not mathematically symmetrical with partial space derivatives—consequently symmetry holds only in the Maxwell special case, $\mathbf{v}_d = 0$).

We now turn to solution of the neo-Hertzian wave equation for $\mathbf{v}_d = \mathbf{v}_d(t)$.

Analysis

Taking the curl of Eq. (3b) and applying (3a), under the condition that the field point is located in free space ($\mathbf{j}_m = 0$), we obtain the neo-Hertzian wave equation,

$$-\nabla^2 \mathbf{E} + \frac{1}{c^2}\frac{d^2}{d\tau_d^{\,2}}\mathbf{E} = 0 \ . \tag{4}$$

To facilitate finding a d'Alembertian solution, let us introduce a quantity *p*, defined as

$$p = \mathbf{k}\cdot\mathbf{r} - f(t) = xk_x + yk_y + zk_z - f(t) \ , \tag{5}$$

where **k** is a constant and $f(t)$ is an arbitrary smooth function that generalizes the previous [2-3] ωt. Seeking a solution of (4) of the form $\mathbf{E} = \mathbf{E}(p)$, we then find

$$\nabla^2 \mathbf{E}(p) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\mathbf{E}(p) = \left(k_x^{\,2} + k_y^{\,2} + k_z^{\,2}\right)\mathbf{E}''(p) = k^2\mathbf{E}'' \ , \tag{6}$$

where double-prime denotes two differentiations with respect to *p*. [The argument variables (*x,y,z,t*) are treated as mutually independent, as in all field theory.] From (2a)

$$\frac{d^2}{d\tau_d^{\,2}}\mathbf{E}(p) = \gamma_d \frac{d}{dt}\,\gamma_d \frac{d}{dt}\mathbf{E}(p) \ . \tag{7}$$

In order to evaluate this we need to know *dp/dt*. From (2b) and (5)

$$\begin{aligned} \frac{dp}{dt} &= \left(\frac{\partial}{\partial t} + \mathbf{v}_d\cdot\nabla\right)\left(xk_x + yk_y + zk_z - f(t)\right) \\ &= -\dot{f} + v_{dx}k_x + v_{dy}k_y + v_{dz}k_z = -\dot{f} + \mathbf{v}_d\cdot\mathbf{k} \end{aligned} \ , \tag{8}$$

where $\dot{f}$ denotes *df/dt*. Proceeding from (7), we obtain

$$\gamma_d \frac{d}{dt}\left(\gamma_d \frac{d}{dt}\mathbf{E}(p)\right) = \gamma_d \frac{d\gamma_d}{dt}\frac{dp}{dt}\mathbf{E}' + \gamma_d^2 \frac{d^2 p}{dt^2}\mathbf{E}' + \gamma_d^2 \left(\frac{dp}{dt}\right)^2 \mathbf{E}'' \ . \tag{9}$$

Using (6), the wave equation (4) then yields

$$\left[-k^2 + \frac{\gamma_d^2}{c^2}\left(\frac{dp}{dt}\right)^2\right]\mathbf{E}'' + \frac{\gamma_d^2}{c^2}\left[\frac{d^2 p}{dt^2} + \frac{1}{\gamma_d}\left(\frac{d\gamma_d}{dt}\right)\frac{dp}{dt}\right]\mathbf{E}' = 0 \ . \tag{10}$$

In order for a d'Alembert solution to exist it is necessary that the square-bracketed coefficients of both $\mathbf{E}'$ and $\mathbf{E}''$ vanish. Let us first examine the former. Its vanishing implies that

$$\frac{d^2 p}{dt^2} = -\frac{1}{\gamma_d}\left(\frac{d\gamma_d}{dt}\right)\frac{dp}{dt} \ . \tag{11}$$

Let $y \equiv dp/dt$. Then

$$\frac{dy}{dt} = -\frac{1}{\gamma_d}\left(\frac{d\gamma_d}{dt}\right)y \ \text{ or } \ \frac{dy}{y} = -\frac{d\gamma_d}{\gamma_d} \ . \tag{12}$$

With *b* an integration constant, we have $y = b/\gamma_d$ or from (8)

$$y = \frac{dp}{dt} = -\dot{f} + \mathbf{v}_d \cdot \mathbf{k} = \frac{b}{\gamma_d} \ . \tag{13}$$

Going back to (10), we see that the vanishing of the coefficient of $\mathbf{E}''$ implies that

$$\frac{ck}{\gamma_d} = \left|\frac{dp}{dt}\right| = \left|-\dot{f} + \mathbf{v}_d \cdot \mathbf{k}\right| \ . \tag{14}$$

Taking the absolute value of (13), we see from (14) that $|b| = ck$ or

$$b = \pm ck \ . \tag{15}$$

This is the condition for simultaneous vanishing of the square-bracketed coefficients of both $\mathbf{E}'$ and $\mathbf{E}''$ in (10). We assume this condition (15) to be satisfied. Eq. (13) implies

$$\dot{f} = \pm\frac{ck}{\gamma_d} + \mathbf{v}_d \cdot \mathbf{k} \ , \tag{16}$$

whence

$$f(t) = \int_0^t \dot{f}dt = \int_0^t \left(\pm\frac{ck}{\gamma_d} + \mathbf{v}_d \cdot \mathbf{k}\right)dt \ . \tag{17}$$

Here, as we recall, $\mathbf{v}_d$ is a function of time. The d'Alembertian wave function argument being of the form $\mathbf{r}\cdot\mathbf{k}/k - ut$ (in effect the definition of phase velocity *u*), a form that can alternatively be written as $p/k = \mathbf{r}\cdot\mathbf{k}/k - f(t)/k$, it follows that $ut = f(t)/k$; so

$$u = \frac{f(t)}{kt} = \pm\frac{1}{t}\int_0^t \sqrt{c^2 - v_d^2}\,dt + \frac{1}{kt}\int_0^t \mathbf{v}_d \cdot \mathbf{k}dt = \pm\left\langle\sqrt{c^2 - v_d^2}\right\rangle_{av} + \left\langle\mathbf{v}_d\right\rangle_{av} \cdot \frac{\mathbf{k}}{k} \ , \tag{18}$$

where $\langle \ \rangle_{av}$ denotes a time average over the interval zero to *t*. This is the generalization we have sought of the previous [1-3] result; namely,

$$u = \frac{\omega}{k} = \pm\sqrt{c^2 - v_d^2} + \frac{\mathbf{v}_d \cdot \mathbf{k}}{k} \ , \tag{19}$$

which was shown to be valid for the case of constant $\mathbf{v}_d$.

Although the mathematics of (18) seems straightforward, there remains a problem of interpretation. If t is the wave "detection time," what is "time zero"? One may speculate that it suffices to time-average over a large number of periods of the radiation, and that "ancient history" (*e.g.*, in the case of stellar radiation) can be ignored. If limited time-averaging makes u constant, then ω in $u \approx \omega / k$ is effectively constant, so (19) suffices, with $\mathbf{v}_d$ interpreted as detector speed at the instant of detection. This is supported by the apparent success of the neo-Hertzian theory of stellar aberration [6] with $\mathbf{v}_d$ treated as constant, so that (19) applies and the d'Alembert solution for the **E**-field is

$$\mathbf{E} = \mathbf{E}_1\left(\frac{\mathbf{k}\cdot\mathbf{r}}{k} + \left[\sqrt{c^2 - v_d{}^2} - \frac{\mathbf{v}_d\cdot\mathbf{k}}{k}\right]t\right) + \mathbf{E}_2\left(\frac{\mathbf{k}\cdot\mathbf{r}}{k} - \left[\sqrt{c^2 - v_d{}^2} + \frac{\mathbf{v}_d\cdot\mathbf{k}}{k}\right]t\right), \tag{20}$$

where $\mathbf{E}_1$ and $\mathbf{E}_2$ are arbitrary smooth functions.

Thus the result obtained previously [2,3] for the more restricted case $\mathbf{v}_d =$ constant can be considered roughly valid also for the more general detector motion case $\mathbf{v}_d = \mathbf{v}_d(t)$. The inference that constancy or non-constancy of $\mathbf{v}_d$ in most cases makes little difference implies the following regarding light propagation: In effect, all that matters is the instantaneous detector velocity at the moment it passes through the field point and makes its detection there. Detector velocities at other times than the moment of detection, or detector accelerations, do not affect apparent propagation. The only exception might be the case of very low-frequency EM radiation (wavelength of the order of light seconds), such that the detector can change its motion appreciably during the time-averaging period indicated in (18). This averaging period being uncertain, we can do little more here than insert a cautionary question mark. [As usual, we see from (18) or (19) that if $\mathbf{v}_d = 0$ the Maxwell special case is recovered.]

It will surely occur to the reader to ask: How does the photon on emission "know" how the detector is going to be moving at the "future" moment of a detection (absorption) that has "not yet" occurred? Such extreme acausality (putatively associated with "propagation") is hinted at in quantum theory but not in the Maxwell-Einstein version of field theory. It has been discussed previously [2-3]. It can be rationalized by considering advanced as well as retarded effects, or by observing that the photon propagates in zero proper time, $\Delta\tau = 0$; so that when emitted it is "already present" at its destination and can thus condition its apparent propagation (inference! inference!) in observer frame time by determining for itself ("by contact") the velocity of the absorber. This fits precisely with the old idea (Fokker, G. N. Lewis, Wheeler-Feynman) of *the absorber as the mechanism of radiation*. However, it does not matter how we rationalize, since the mathematics holds the key, and Potier's principle [2,7] assures us that nothing observably or *measurably* acausal will come of the mathematical-descriptive acausality. This completes our discussion of the neo-Hertzian wave equation.

Two-way Average Light Speed

We add here a sort of postscript to show that not only is one-way phase velocity of light affected by absorber motion in both Hertzian [1] and neo-Hertzian [2,3] theory, but two-way average speed is also affected in neo-Hertzian theory (though not in Hertzian theory).

We make use of the (one-way) phase velocity expression (18) or its simpler form (19). Suppose a light signal is emitted from the coordinate origin of the observer's inertial system (laboratory) "to the right" and is reflected from a stationary mirror, fixed at some arbitrary distance, back to the origin, which is the "field point" where a detector moving to the right with speed v_d intercepts it. The initial signal propagation to the right takes place at speed *c*, because its absorber-reflector (mirror) is stationary in the lab. (This means $v_d = 0$ for this signal, so the Maxwell special case applies.) The reflecting (mirror) source speed for the return signal makes no difference, because *in all theories* source speed has no effect on light speed. But the return signal propagation to the left is affected (in Hertzian and neo-Hertzian theory) by the speed v_d of the detector. Since the detector speed to the right is directed counter to the leftward light propagation direction, $\mathbf{v}_d \cdot \mathbf{k}$ is negative and (19) yields for the two-way speed an average of *c* for the outward speed and $\sqrt{c^2 - v_d^{\,2}} - v_d$ for the return. That is, for the two-way propagation,

$$\text{Speed average} \quad = \left(c + \sqrt{c^2 - v_d^{\,2}} - v_d \right) / 2 = c - \frac{v_d}{2} - \frac{v_d^{\,2}}{4c} + O\left(v_d^{\,4} \right) \ . \tag{21}$$

Had the detector been moving to the left at the moment of detection, $\mathbf{v}_d \cdot \mathbf{k}$ would have been positive and the sign of v_d would have been changed:

$$\text{Speed average} = \left(c + \sqrt{c^2 - v_d^{\,2}} + v_d \right) / 2 = c + \frac{v_d}{2} - \frac{v_d^{\,2}}{4c} + O\left(v_d^{\,4} \right) \ . \tag{22}$$

It is of some interest to consider the case of averaging of these averages, such as would occur if the detector were set into simple harmonic motion (SHM) parallel to the light propagation direction, symmetrically about the origin, so that its maximum speed v_d occurs there. The average of the two-way average speeds, (21) and (22), is

$$\text{SHM average speed} = c - \frac{v_d^{\,2}}{4c} + O(v_d^{\,4}) \approx c\sqrt{1 - \frac{v_{d(rms)}^{\,2}}{c^2}} = \frac{c}{\gamma_{d(rms)}} \ , \tag{23}$$

where $v_{d(rms)} = v_d / \sqrt{2}$ is the root-mean-square (*rms*) speed of the detector's harmonic motion. This apparent slowing of propagation by a factor of $1/\gamma_{d(rms)}$, attributed by the lab observer to the speed of light, *could not be measured* by the moving detector itself or by a comoving observer—because, at least to second order, it would be canceled by detector clock-time dilatation (quantified by the factor $\gamma_{d(rms)}$, dependent on *rms* detector speed), which enhances measured speed values *in the absence of Lorentz contraction* by just the amount necessary to compensate the convective slowing in (23). If, on the other hand, a detector stationary at the origin were used in an attempt to verify the "speed decrease" effect alleged by (23), that effect would vanish—for the stationary detector brings us back to the Maxwell case of no light convection.

Thus we have a sort of light-speed analogue of the Complementarity principle: The attempt to *measure* effects that entail or attribute alteration to the two-way (as well as the one-way) speed of light destroys such effects. (This does not mean necessarily that such effects do not "exist." They may exist "rationally," as *attributed* effects, for limited purposes of descriptive consistency. Such an effect, for example, is the Lorentz contraction in Maxwell-Einstein theory.) In neo-Hertzian theory it is only the lab observer or other non-participant in the detector motion who attributes (inference! inference!) variations to *c* or convections to the photon. The actual detector *cannot detect* any departure from constancy of *c*. This result, here shown valid to second order, seems likely to be valid to all orders and possibly for arbitrary detector motions.

We remark (a) that SHM of the sort considered here may be considered itself to *constitute* a clock of average running rate determined by $\gamma_{d(rms)}$, and (b) that it is not surprising that our neo-Hertzian analysis yields a result that fits with time dilatation, since that phenomenon was postulated at the start in accordance with Eq. (1). The Moessbauer evidence cited by Sherwin [8] seems to confirm this clock-rate prediction experimentally for random (thermal) motions of radiation absorbers. It is to be observed how intimately EM theory and kinematics fit together: In Einstein's theory kinematic noninvariance attributed to length (Lorentz contraction) dovetails with a covariant (constant *c*) EM formalism; whereas here absorber convection (non-constant *c*) attributed to EM radiation dovetails with a length-invariant kinematics. Both theories predict much the same *measured* (detection) data, although the descriptive verbalisms are completely antagonistic, and each may well be considered "a tale told by an idiot" … albeit an idiot savant. That the tales are, however, not ultimately equivalent was argued in the Introduction with reference to the weak-field limit.

Finally, there is a sort of philosophical lesson: If you would simplify your understanding of physics, pay attention to what is *measured* and to the state of motion of the instrument that measures it. If, instead, you build your views on *inferences* about what is "really happening" out there, you will make of your intellectual life an endless apologia for paradoxes and enigmas. This, I aver, is the message that Bridgman and the operationalists sought to convey. It is illustrated well enough in quantum mechanics … better still in electromagnetism, the "classical field" physics that thinly veils the "most quantum" aspect of the quantum world.

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