Explicit Examples of Free-Space Non-Planar Electromagnetic Waves Containing Magnetic Scalar Potentials

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Electromagnetic waves (EMW) are formed by electric and magnetic fields, both together solution of Maxwell's equations. The magnetic field is solenoidal always, while the electric field is solenoidal in charge-neutral regions only. Hence, conventionally, free-space electromagnetic fields are transverse to the direction of propagation; also, there exists a electric scalar potential but not a magnetic companion. Contrarywise, for the same homogeneous case, we exhibit explicit examples to show that: (a) Longitudinal magnetic fields are compatible with linearly polarized non-planar EMW, and (b) Magnetic scalar potentials are compatible with EMW. The direction of propagation of non-planar EMW oscillates around the direction of propagation of the plane EMW.

Keywords: linearly polarized plane electromagnetic waves, non-planar electromagnetic waves, longitudinal magnetic fields, magnetic scalar potentials.

1. Introduction

It is well-known that electromagnetic waves (EMW) are a solution to the set of four Maxwell's eqs (1)-(4).[1,2] In charge-neutral regions (defined by $\rho = 0$), eq. (3) becomes a solenoidal condition, thus joining eq. (4) which always is. This is conventionally interpreted as implying that electric and magnetic fields **E** and **B** are transverse to the direction of propagation. Recently, we have obtained novel nonperiodic solutions of the homogeneous wave equation [3] that may be used to obtain new solutions of the field equation associated with Maxwell's equations (ME). While revisiting this well-known subject, we have surprisingly found that the class of solutions of ME may be larger than usually believed.[4] Some critics of our work have suggested that the set of new solutions of ME may be empty. To counter such argument, we exhibit here simple explicit examples of linearly polarized electromagnetic waves—solutions of ME—containing (a) a longitudinal component of the magnetic field along the average direction of propagation, and (b) a magnetic scalar potential.

This note is organized as follows. The standard material is collated in section 2 for easy reference. Section 3 exhibits the explicit examples and section 4 closes it.

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2. Summary Description of Electromagnetic Waves

In CGS units, Maxwell's equations (ME) are[1]

$$\nabla\times\mathbf{E} = -\,\partial\mathbf{B}/\partial w \tag{1}$$

$$\nabla\times\mathbf{B} = +\,\partial\mathbf{E}/\partial w + 4\pi\mathbf{J}/c \tag{2}$$

$$\nabla\bullet\mathbf{E} = 4\pi\rho \tag{3}$$

$$\nabla\bullet\mathbf{B} = 0 \tag{4}$$

where time is the geometric variable $w \equiv ct$. Other symbols and dimensions are: c, speed of light *in vacuo* [=] cm sec^{-1}; **E** / **B**, electric/magnetic field [=] dyne esu^{-1}; **J**, current density [=] esu $sec^{-1}cm^{-2}$; ρ, net charge density [=] esu cm^{-3}.

Charge conservation is assured by the standard continuity condition:

$$c\partial\rho/\partial w + \nabla\bullet\mathbf{J} = 0 \tag{5}$$

Two remarks are in order: (a) Eq. (5) is not an independent condition. It is automatically fulfilled by any fields **E** and **B** satisfying both eqs. (2) and (3). This claim may be immediately verified by substituting **J** from eq. (2) and ρ from eq. (3) into eq. (5). And, (b) In regions of time-independent charge density $\partial\rho/\partial w = 0$, charge conservation reduces to

$$\nabla\bullet\mathbf{J} = 0 \tag{6}$$

Eq. (6) applies, as a particular case, to charge-neutral problems $\rho = 0$. Note that in complete absence of charge there is no source to produce an electric field, so that there is a difference between a charge-free and a charge-neutral region. [5] The simplest solution of eq. (6) is $\mathbf{J} = \mathbf{0}$ (Examples 1 and 2), but there may exist solutions with $\mathbf{J} \neq \mathbf{0}$ too (Example 3).

As usual, direction of propagation of the EMW is parallel to vector **D** defined as

$$\mathbf{D} = \mathbf{E} \times \mathbf{B}. \tag{7}$$

An equation of continuity over energy and momentum density may be obtained by substituting eqs. (1) and (2) into $\nabla\bullet\mathbf{D}$ to get

$$c\partial\rho_E/\partial w + \nabla\bullet\mathbf{S} + \mathbf{E}\bullet\mathbf{J} = 0 \tag{8}$$

where the Poynting vector $\mathbf{S}=c\mathbf{D}/4\pi$ represents the flow of momentum per unit area and $\rho_E= (E^2 + B^2)/8\pi$ is the energy density of the EMW. The presence of the term $\mathbf{E}\bullet\mathbf{J}$ in eq. (8) means that not all energy of the EMW is contained in the Poynting vector when $\mathbf{J} \neq \mathbf{0}$. It is noted that eq. (8) is not new, and appears in older textbooks [6] (see Vol. 1, page 428). For the case of EMW propagating in dielectrics, Nelson [7] has recently obtained an expression similar to eq. (8). The last remark together with our findings on the difference between charge-neutral and charge-free regions [5] suggest that vacuum may be treated as a dielectric.

3. Explicit Examples

We will use variations of a simple linearly plane-polarized EMW in a charge-neutral region ρ=0.

Example 1. *Linearly plane-polarized EMW*. A particular solution of ME is $\mathbf{E} = E_x\mathbf{i} + E_y\mathbf{j} + E_z\mathbf{k}$ and $\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$ with $E_y=E_z=B_x=B_z=0$, $E_x = A\ \sin[K(z-w)]$, $B_y = A\ \sin[K(z-w)]$, where A and K are real constants. In this particular case, $\mathbf{J} = \mathbf{0}$ (the simplest solution of eq. 6).

In example 1, the components of direction are $D_x=D_y=0$ and $D_z=A^2\sin^2[K(z-w)]$ which means that propagation is along the Z-axis. The instantaneous Poynting vector is proportional to **D**. The average components of the Poynting vector along each coordinate axis are proportional to the time average of the individual directions Di (i = x,y,z) taken over a suitably defined time, say one cycle ($0 \le Kw \le 2\pi$): $\langle D_x \rangle = \langle D_y \rangle = 0$, $\langle D_z \rangle = A^2/2$.

Consider now a longitudinal magnetic perturbation of Example 1, obtained by adding a longitudinal magnetic field component B_z, dependent on the y coordinate (transversal to the direction of propagation):

Example 2. *Linearly polarized, nonplanar EM wave*. Another solution of ME for $\rho = 0$ is $B_y = A \sin[K(z-w)]$, $B_z = B \sin[K_L(y-w)]$, $B_x = 0$, and $E_y = E_z = 0$, $E_x = A \sin[K(z-w)] - B \sin[K_L(y-w)]$, where B and K_L are real constants. Again, in this particular case, **J** = 0.

Since the electric and magnetic fields depend of more than one coordinate, the wave described by Example 2 is *nonplanar*. [8] However, since the electric field lies completely on the X-axis the EMW is still linearly polarized along the X-direction. Indeed, from eq. (7), $D_x = 0$, $D_y = AB\sin[K(z-w)]\sin[K_L(y-w)] - B^2\sin^2[K_L(y-w)]$ and $D_z = A^2\sin^2[K(z-w)] - AB \sin[K(z-w)]\sin[K_L(y-w)]$ which means that the direction of propagation of the EMW oscillates now on the whole Y-Z plane (not only along the Z-axis as in the plane case), perpendicularly to the polarization X-axis.

The *instantaneous* direction of field **B** also lies on the Y-Z plane along the time-dependent direction δ given by

$$\tan \delta = B_y / B_z \tag{9}$$

where δ is measured relative to the Z-axis (direction of propagation of the unperturbed wave of Example 1). Clearly, there are no longitudinal magnetic components of magnetic field relative to the time-dependent plane defined by the X-axis and the direction δ. However, the presence of B_z leads to observable effects relative to a fixed frame of coordinates (the original system attached to the unperturbed wave of Example 1).

Let us introduce the following notation: amplitude ratio of the perturbation $R_a \equiv B/A$, and frequency ratio of perturbation $R_f \equiv K_L/K$. Fig. 1 illustrates the variation of δ for $R_f = 1$ and four values of R_a from 1 down to 0.001. Note that the *instantaneous* direction of propagation oscillates back and forth relative to the Z-axis, from π/4 in the counterclockwise direction to 3π/4 in the clockwise direction. It is remarkable that there is a small fraction of rays propagating perpendicular to the main direction of propagation (Z-axis), *i.e.* $\delta = \pi/2$. Since it does not appear in Example 1, such effect is entirely due to the presence of the longitudinal component of the magnetic field B_z in Example 2. As seen from Fig. 1, the frequency of the effect depends of the relative amplitude of the perturbation R_a.

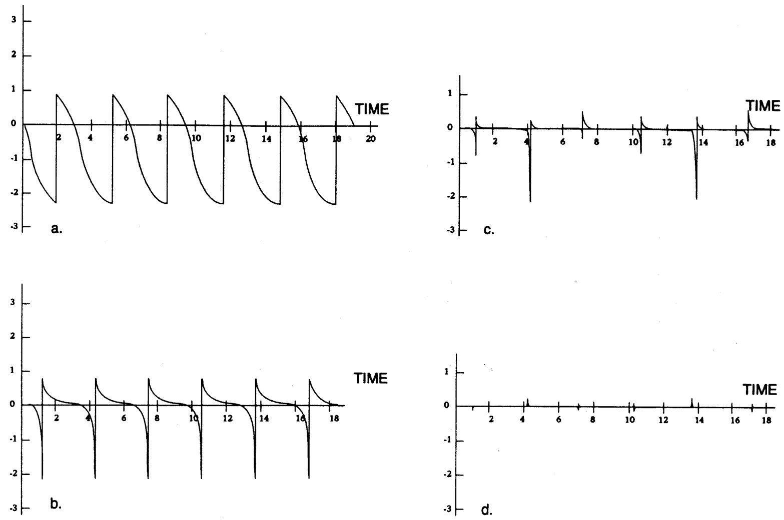
Since D_y and D_z are nonzero there is propagation of energy on the whole Y-Z plane, in contrast to the plane case where energy only propagates along the Z-axis. The average components of the Poynting vector are

$$\langle D_x \rangle = 0$$

$$\langle D_y \rangle = \left(AB\, F(w; K,K_L) - \int B^2 \sin^2[K_L(y-w)]\, dw\right)/w$$

$$\langle D_z \rangle = \left(\int A^2 \sin^2[K(z-w)]dw - AB\, F(w; K,K_L)\right)/w$$

where $F(w; K,K_L) = \int \sin[K(z-w)] \sin[K_L(y-w)]dw$. The value of this integral depends on the relative values of $R_f = K_L/K$. Two families arise:



OSCILATION RELATIVE TO PLANE-WAVE PROPAGATION. a. Ra=1 b. Ra=0.1 c. Ra=0.01 d. Ra= 0.001

Fig. 1. Oscillation in the direction of propagation of a linearly polarized electromagnetic wave containing longitudinal components of magnetic field. The vertical axis is angle δ in radians; it is positive for counter-clockwise oscillations, and negative for clockwise oscillations (this convention produces the discontinuities in the graph). The horizontal axis is time in dimensionless units of Kw (see Example 2). Angle δ was calculated with eq. (9) for the fields in Example 2 with R_f = 1 and four values of the amplitude of the perturbation: R_a = 1, 0.1, 0.01, and 0.001.
When the amplitude of the perturbing magnetic field equals the amplitude of the basic field (R_a = 1) the direction of propagation wiggles back and forth. As R_a decreases, the wave tends to propagate mostly along the direction of the plane wave (*i.e.* $\delta = 0$).

a) Wave numbers K and K_L are commensurable, then R_f is rational. It is possible to find two integer numbers n and m such that $R_f = K_L/K = m/n$. Then time integration may be taken over a time w such that $w = 2\pi n/K = 2\pi m/K_L$. In such cases, $F(w;K,K_L) = 0$, so that $\langle D_y \rangle = -B^2/2$, and $\langle D_z \rangle = A^2/2$. It is remarkable that the energy transported along the perturbed Z-direction is the same as in the unperturbed case (example 1). However, there is now a *negative* transfer of energy into the EMW from the perpendicular direction Y (*i.e.*, absorption from the surrounding medium). Note that pure (conventional) planar waves arise when $R_a \rightarrow 0$ (B=0). Hence, the *signature of a conventional planar polarized wave is the absence of absorption of energy from the surrounding medium,* or conversely, if an originally plane polarized EMW interacts with the medium via absorption perpendicularly to the direction of propagation, it becomes non-planar.

b) Wave numbers are incommensurable, then R_f is irrational. Hence, it is impossible to find two integer numbers n and m such that $R_f = n/m$. This implies that it is not possible to find an integration time making $F(K,K_L) = 0$. This finding has the additional implication that the EMW will never repeat itself along the Z-axis, so this EMW is *nonperiodic*. It is expected that such waves may be connected to our novel nonperiodic solutions. [3] Here, $\langle D_y \rangle = ABK_L F(w\,;K,K_L)/2\pi m - B^2/2$, $\langle D_z \rangle = A^2/2 - ABK\, F(w\,;K,K_L)/2\pi n$. Note that the energy carried along the Z-direction is lower than in the previous case, but it re-appears as energy along the perpendicular Y-direction.

From Malus law, [9,10] the intensity (*i.e.* total energy) of the main wave depends of $1 + R_a^2$. Hence, the presence of the longitudinal component is difficult to observe by measuring variations in the intensity of the EM beam, but it may be observed by studying phenomena associated with the temporal variations in the direction of propagation. For instance, the oscillatory effect in Fig. 1 may provide a classical straightforward explanation to the recently observed unexpected presence of a transversal light current in the absence of scatterers [11] (see Fig. 2 in Ref. 6, and the discussion in the paragraph previous to last).

Consider now a variation of Example 2, generated by the addition of a magnetic scalar potential.

Example 3. *Nonplanar EM wave in free-space with magnetic scalar potential*. Consider the scalar magnetic potential $\mu(\mathbf{r},w) = \mu_t(w)\mu(x,y,z) = \mu_0\, \mu(x,y,z)\, \exp(-H_0 w)$, where μ_0 and H_0 are constants with units of potential (erg/esu) and cm^{-1} respectively, and $\mu(\mathbf{r})$ is a solution to the dimensionless wave equation $\nabla^2 \mu(\mathbf{r}) = 0$ with suitable boundary conditions (located, for instance, at the edge of the observable universe).

Let the magnetic field in Example 2 be modified with the magnetic scalar potential above as $\mathbf{B} \to \mathbf{B} + \nabla\mu(\mathbf{r},w)$. This leads to the following solution of ME:

$$E_x = A \sin(K(z-w)) - B \sin(K_L(y-w)) + H_0\mu_0 \exp(-H_0 w) \int (\partial\mu/\partial y) dz - axF(w)$$

$$E_y = - H_0\mu_0 \exp(-H_0 w) \int (\partial\mu/\partial x) dz + ayF(w)$$

$$E_z = F_2(w)$$

$$B_x = \mu_0 \exp(-H_0 w)\, (\partial\mu/\partial x)$$

$$B_y = A \sin(K(z-w)) + \mu_0 \exp(-H_0 w)\, (\partial\mu/\partial y)$$

$$B_z = B \sin(K_L(y-w)) + \mu_0 \exp(-H_0 w)\, (\partial\mu/\partial z)$$

where a is a real constant and $F(w)$ and $F_2(w)$ are arbitrary functions of time.

It may be immediately verified that fields above are a solution of ME. Further, let $F_2(w) = \text{constant}$, and μ_0 be small, this still is an EMW propagating along the Z-axis, containing a longitudinal magnetic field B_z. It is noted that Ampere's law (eq. 2) leads to $\mathbf{J} \neq 0$ given by

$$J_x = (c/4\pi)[\, H_0^2 \mu_0 \exp(-H_0 w) \int (\partial\mu /\partial y) dz + ax\, dF(w)/dw]$$

$$J_y = (-c/4\pi)[H_0^2 \mu_0 \exp(-H_0 w) \int (\partial\mu /\partial x) dz + ay\, dF(w)/dw]$$

$$J_z = (-c/4\pi)[dF_2(w)/dw]$$

The presence of this current is, of course, consistent with continuity (eq. 6), and leads to a total energy density ρ_E that is not completely contained in the conventional Poynting vector (recall eq. 8).

It is noteworthy that the presence of the magnetic scalar potential $\mu(\mathbf{r},w)$ leads to a current density $\mathbf{J} \neq 0$, even when $\rho = 0$. Such current is both space and time dependent. Furthermore, if $dF_2(w)/dw = 0$, the current is *completely transversal* to the direction of propagation of the underlying plane wave (z-axis).

In this case, the direction of propagation has components along the three-dimensions. As expected, the X-component is completely due to the magnetic scalar potential (see Appendix). No new phenomena are apparent.

It is stressed that the EM fields exhibited in Examples 2 and 3 are completely independent of any interpretation in terms of potentials. The connection with potentials and gauges is deferred for future work.

4. Concluding Remarks

We exhibited a linearly polarized nonplanar wave containing a longitudinal component of magnetic field. Obviously, the longitudinal magnetic component disappears relative to a time-dependent plane defined by the instantaneous directions of **E** and **B**. However, there is a measurable effect consisting in an oscillation of the direction of the wave, relative to a system of coordinates attached to the observer (this is a system defined by the propagation of a linearly plane-polarized wave).

The presence of a longitudinal magnetic field component in a polarized wave may be difficult to observe by measuring the intensity of the beam, but it may manifest in several ways: (a) Studying phenomena associated with temporal-oscillations in the direction of propagation (recall Fig. 1). This may be a classical mechanism explaining some recent observations. [11] (b) Studying phenomena associated with the transversal dependence of the polarized electric field, the X-axis in Example 2. (c) Measuring the transfer of energy into a linearly (apparently-plane) polarized beam from the surrounding environment.

By analogy, the presence of a longitudinal component of magnetic field in a circularly polarized wave would produce an oscillation in the direction of propagation inside a cone of half-angle δ, rather than in a plane as in the linearly polarized wave discussed here.

We also exhibited a solution of Maxwell equations in free-space containing a magnetic scalar potential $\mu(\mathbf{r},w)$. This leads to the appearance of a space and time-dependent current density $\mathbf{J} = J_x\,\mathbf{i} + J_y\,\mathbf{j}$ *transversal to the direction of propagation* (z-axis).

The nonperiodic nonplanar waves associated with non-commensurable wave lengths (example 2) suggest that nonperiodic solutions of ME [3] are physically meaningful. Also, the presence of a magnetic scalar potential in Example 3 is compatible with the existence of a larger class of solutions of ME, discussed by us elsewhere. [4,12] All other interpretational matters are deferred for future work.

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Appendix. Direction of propagation and energy transport in a non-planar linearly polarized emw propagating in a vacuum with magnetic properties (Example 3)

$$
\begin{aligned}
D_x = &- B\, H_0\mu_0 \int (\partial\mu/\partial x)dz\, \exp(-H_0 w)\, \sin(K_L(y-w)) \\
&- H_0\mu_0^2\, \partial\mu/\partial z \int (\partial\mu/\partial x)dz\, \exp(-2H_0 w) \\
&+ ayF(w)\, B \sin(K_L(y-w)) \\
&+ ayF(w)\, \mu_0 (\partial\mu/\partial y)\exp(-H_0 w) \\
&- A\, F_2(w)\sin(K(z-w)) \\
&- \mu_0\, F_2(w)\, (\partial\mu/\partial x)\exp(-H_0 w)
\end{aligned}
$$

$$
\begin{aligned}
D_y = &\ AB \sin(K(z-w))\sin(K_L(y-w)) \\
&- B^2 \sin^2 (K_L(y-w)) \\
&+ B\, H_0\mu_0^2 \int (\partial\mu/\partial y)dz\, \exp(-H_0 w)\sin(K_L(y-w)) \\
&- ax\, BF(w)\sin(K_L(y-w)) \\
&+ A\, \mu_0 (\partial\mu/\partial z)\exp(-H_0 w)\sin(K(z-w)) \\
&- B\, \mu_0(\partial\mu/\partial x)\exp(-H_0 w)\sin(K_L(y-w)) \\
&+ H_0\mu_0^2(\partial\mu/\partial z) \int (\partial\mu/\partial y)dz\, \exp(-2H_0 w) \\
&- \mu_0 ax\, (\partial\mu/\partial z)F(w)\exp(-H_0 w)
\end{aligned}
$$

$$
\begin{aligned}
D_z = &\ A^2 \sin^2 (K(z-w)) \\
&- AB \sin(K(z-w))\sin(K_L(y-w)) \\
&+ A\, H_0\mu_0 \int (\partial\mu/\partial y)dz\, \sin(K(z-w))\exp(-H_0 w) \\
&- ax\, A\, F(w)\sin(K(z-w)) \\
&+ A\, \mu_0\, (\partial\mu/\partial y)\sin(K(z-w))\exp(-H_0 w) \\
&- B\, \mu_0\, (\partial\mu/\partial y)\sin(K_L(y-w))\exp(-H_0 w) \\
&+ H_0\mu_0^2\, (\partial\mu/\partial y) \int (\partial\mu/\partial y)dz\, \exp(-2H_0 w)
\end{aligned}
$$

$$- ax\, \mu_0 (\partial\mu/\partial y) F(w) \exp(-H_0 w)$$

$$+ H_0\mu_0^2 (\partial\mu/\partial x) \int(\partial\mu/\partial x)dz \exp(-2H_0 w)$$

$$- \mu_0\, ay(\partial\mu/\partial x) F(w) \exp(-H_0 w)$$

$$w\langle D_x\rangle = - B\, H_0\mu_0 \int(\partial\mu/\partial x)dz \int \exp(-H_0 w) \sin(K_L(y-w))\, dw$$

$$- H_0\mu_0^2\, \partial\mu/\partial z \int(\partial\mu/\partial x)dz \int \exp(-2H_0 w)\, dw$$

$$+ Bay \int F(w) \sin(K_L(y-w))\, dw$$

$$+ ay\mu_0 (\partial\mu/\partial y) \int F(w) \exp(-H_0 w)\, dw$$

$$- A \int F_2(w) \sin(K(z-w))\, dw$$

$$- \mu_0 (\partial\mu/\partial x) \int F_2(w) \exp(-H_0 w)\, dw$$

$$w\langle D_y\rangle = AB \int \sin(K(z-w)) \sin(K_L(y-w))\, dw$$

$$- B^2 \int \sin^2 (K_L(y-w))\, dw$$

$$+ B\, H_0\mu_0^2 \int(\partial\mu/\partial y)dz \int \exp(-H_0 w) \sin(K_L(y-w))\, dw$$

$$- ax\, B \int F(w) \sin(K_L(y-w))\, dw$$

$$+ A\, \mu_0 (\partial\mu/\partial z) \int \exp(-H_0 w) \sin(K(z-w))\, dw$$

$$- B\, \mu_0(\partial\mu/\partial x) \int \exp(-H_0 w) \sin(K_L(y-w))\, dw$$

$$+ H_0\mu_0^2(\partial\mu/\partial z) \int(\partial\mu/\partial y)dz \int \exp(-2H_0 w)\, dw$$

$$- \mu_0 ax\, (\partial\mu/\partial z) \int F(w)\exp(-H_0 w)\, dw$$

$$w\langle D_z\rangle = A^2 \int \sin^2 (K(z-w))\, dw$$

$$- AB \int \sin(K(z-w)) \sin(K_L(y-w))\, dw$$

$$+ A\, H_0\mu_0 \int(\partial\mu/\partial y)dz \int \sin(K(z-w)) \exp(-H_0 w)\, dw$$

$$- ax\, A \int F(w) \sin(K(z-w))\, dw$$

$$+ A\, \mu_0 (\partial\mu/\partial y) \int \sin(K(z-w)) \exp(-H_0 w)\, dw$$

$$- B\, \mu_0 (\partial\mu/\partial y) \int \sin(K_L(y-w))\exp(-H_0 w)\, dw$$

$$+ H_0\mu_0^2 (\partial\mu/\partial y) \int(\partial\mu/\partial y)dz \int \exp(-2H_0 w)\, dw$$

$$- ax\, \mu_0 (\partial\mu/\partial y) \int F(w) \exp(-H_0 w)\, dw$$

$$+ H_0\mu_0^2 (\partial\mu/\partial x) \int(\partial\mu/\partial x)dz \int \exp(-2H_0 w)\, dw$$

$$- \mu_0\, ay(\partial\mu/\partial x) \int F(w) \exp(-H_0 w)\, dw$$