O(3) Electrodynamics: A Second Reply to Hunter

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In O(3) symmetry electrodynamics the field tensor is governed by a non-Abelian Stokes Theorem, as in any non-Abelian gauge theory. The comments on the $\mathbf{B}^{(3)}$ component of this field tensor by Hunter in this Issue address $\mathbf{B}^{(3)}$ as if it were a U(1) symmetry field, governed by the ordinary Stokes Theorem, and are therefore sequentially erroneous, because there is a basic misunderstanding of the nature of O(3) electrodynamics inherent in the article.

Keywords: $\mathbf{B}^{(3)}$, Stokes Theorem

1. Introduction

The subject of O(3) electrodynamics has been developed recently [1-10] by the AIAS group, and its basic *ansatz* is that electrodynamics be governed by a vacuum topology described by gauge theory with internal gauge group O(3). A comment by Hunter [11] very similar to the present comment has been answered in full detail by the AIAS group [12]. It is shown in this reply that Hunter again makes the basic error of developing the $\mathbf{B}^{(3)}$ field as a component of U(1) gauge field theory applied to electrodynamics, the Maxwell Heaviside theory [13]. The argument given is based on those by Comay [14-17] which have been answered [18-21] already.

2. The Non-Abelian Stokes Theorem

The field tensor in O(3) electrodynamics is governed by the non Abelian Stokes Theorem [22]:

$$\oint D_\mu dx^\mu = -\tfrac{1}{2}\iint \left[D_\mu, D_\nu\right] d\sigma^{\mu\nu} \tag{1}$$

where $\left[D_\mu, D_\nu\right]$ is the commutator of O(3) covariant derivatives [23,24]. The integral over the closed loop on the left hand side is related to an integral over the hypersurface $\sigma^{\mu\nu}$ of the commutator. To reduce eqn. (1) to the ordinary Stokes Theorem used by Comay [14] and Hunter [11] the U(1) covariant derivative must be used:

$$D_\mu := \partial_\mu + igA_\mu \tag{2}$$

to give the result:

$$\oint A_\mu dx^\mu = -\tfrac{1}{2}\int F_{\mu\nu} d\sigma^{\mu\nu} \tag{3}$$

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The space part of this expression is the ordinary, or Abelian, Stokes Theorem

$$\oint \mathbf{A}\bullet d\mathbf{r} = \int \mathbf{B}\bullet d\mathbf{Ar} = \int \overline{\nabla} \times \mathbf{A}\bullet d\mathbf{Ar} \tag{4}$$

which relates the magnetic flux density **B** to the vector potential **A**. Faraday's Law of electromagnetic induction in U(1) electrodynamics and in S.I. units is, from eqn. (4):

$$\oint \mathbf{E}\bullet d\mathbf{r} + \frac{\partial}{\partial t}\int \mathbf{B}\bullet d\mathbf{Ar} = 0 \tag{5}$$

which is the integral form of:

$$\overline{\nabla} \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0} \tag{6}$$

However, the non-Abelian Stokes Theorem for the $\mathbf{B}^{(3)}$ field gives, from eqn. (1) [1-10], the phase factor:

$$\exp\left(i\oint \overline{\kappa}\bullet d\mathbf{r}\right) = \exp\left(ig\oint \mathbf{B}^{(3)}\bullet d\mathbf{Ar}\right) \tag{7}$$

of non-Abelian electrodynamics. Here the left hand side is a line integral over the dynamical phase, where κ is the wave-vector, and the right hand side is an area integral over the $\mathbf{B}^{(3)}$ field. The latter has been shown [1-10] to be responsible for interferometry. For example it accurately describes [1] the Sagnac effect, whereas U(1) electrodynamics fails completely to describe the Sagnac effect [25]. The correct differential form of the Faraday Law of induction in O(3) electrodynamics is, in the complex circular basis {(1), (2), (3)} [1-10].

$$\left.\begin{aligned} \overline{\nabla} \times \mathbf{E}^{(1)} + \frac{\partial \mathbf{B}^{(1)}}{\partial t} &= \mathbf{0} \\ \overline{\nabla} \times \mathbf{E}^{(2)} + \frac{\partial \mathbf{B}^{(2)}}{\partial t} &= \mathbf{0} \\ \frac{\partial \mathbf{B}^{(3)}}{\partial t} &= \mathbf{0} \end{aligned}\right\} \tag{8}$$

So the $\mathbf{B}^{(3)}$ component does not give rise to Faraday induction. This was proven experimentally by Raja *et al.* [26,27].

3. Misconceptions by Hunter

The basic misconception by Hunter [11], as pointed out in detail in ref. [12], is to confuse $\mathbf{B}^{(3)}$ with a field component of U(1) electrodynamics. The rest of his paper in this Issue is therefore sequentially erroneous. The same type of confusion exists in the literature cited by Hunter [11]. The latter again does not cite the replies [12] that clear up the confusion in [11]. In ref. [11] Hunter adopts the same method of citing criticisms, but not citing replies. Essentially therefore Hunter [11] (and Comay [14]) confuse eqn. (5) with eqn. (7), and attempt to apply eqn. (5) to the $\mathbf{B}^{(3)}$ component, a meaningless procedure. It was known from inception [28] that $\mathbf{B}^{(3)}$ is not a Maxwell-Heaviside field component. The reply to Comay [14] by Evans and Jeffers [18] is mathematically correct and simply uses the well known theorem that a necessary and sufficient condition that

$$\oint_C \mathbf{F}\bullet d\mathbf{r} = 0 \tag{9}$$

for every closed curve in the ordinary Stokes Theorem is that

$$\bar{\nabla} \times \mathbf{F} := \mathbf{0} \tag{10}$$

identically. The intent of the reply by Evans and Jeffers [18] is therefore to clarify the fact that if the ordinary Stokes Theorem is erroneously applied to $\mathbf{B}^{(3)}$, the result

$$\oint_C \mathbf{B}^{(3} \bullet d\mathbf{r} = 0 \tag{11}$$

is obtained for *any* closed curve C. This result is true in Cartesian, spherical polar or any other system of coordinates. The result (11), however, does not mean that $\mathbf{B}^{(3)}$ is zero. The correct type of Stokes Theorem to use for $\mathbf{B}^{(3)}$ is eqn. (1). In an accompanying paper in this Issue a non-Abelian Stokes Theorem of this type is derived from the definition [1-10] of $\mathbf{B}^{(3)}$. In a third paper in this Issue it is shown that the definition of $\mathbf{B}^{(3)}$ is Lorentz invariant in the vacuum.

Discussion

The paper by Hunter in this Issue is essentially a replica of ref. [11], which has been corrected in ref. [12]. In this reply we point out that the basic error being made is to apply the ordinary Stokes Theorem to a field component that is correctly described by a non-Abelian Stokes Theorem. By now it is well known that the $\mathbf{B}^{(3)}$ component of O(3) electrodynamics is responsible for and is a physical observable of the Sagnac effect [1] and Michelson interferometry [1-10]. These are major advances in understanding in optics and electrodynamics.

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