The Ephemeris

Focus and book reviews

Generalized Total Time Derivatives

Forms of generalized total time derivative proposed by Wesley and by Mocanu are compared. It is shown that in the important special case of a "test charge" q or small rigid-walled detector moving with instantaneous velocity $\mathbf{v} = \mathbf{v}(t)$ all forms agree on an electromagnetic force law that is the Lorentz law plus a nominally "unobservable" gradient term, $-q\nabla(\mathbf{A}\cdot\mathbf{v})$, which integrates to zero around closed circuits (A = vector potential).

Introduction

The total time derivative is of physical importance because—unlike the partial time derivative—it has the mathematical property of first-order (Galilean) invariance, and thus can be considered to describe results of physical measurement on which all inertial observers must agree. Since measurements can occur for many circumstances of motion and types of detection devices, there are numerous possible mathematical expressions for the total time derivative. Most physicists are acquainted only with the simplest form,

$$\frac{d\mathbf{A}}{dt} = \frac{\partial \mathbf{A}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{A}\text{ , (Traditional)} \tag{1}$$

where $\mathbf{A} = \mathbf{A}(x, y, z, t)$ is an arbitrary vector field that can for mathematical purposes be considered to describe a moving real or virtual "medium," t is laboratory or inertial frame time, and $\mathbf{v} = \mathbf{v}(t)$ is the instantaneous velocity in the laboratory of a detector responsive to some field property or manifestation of that medium. In physical applications the detector is commonly assumed to be very small compared to other dimensions of the problem and to move irrotationally. In the limit it may (as in the case of electromagnetism) be shrunk to a point "test particle."

Frequently v is assumed to be constant, but this is not a necessary restriction. Physically, detectors or test particles can move with velocities $\mathbf{v}(t)$ limited only by analyticity. It is sometimes thought to be of interest to treat $\mathbf{v}$ more generally as a "velocity field"; that is, to assume that $\mathbf{v} = \mathbf{v}(x, y, z, t)$. This covers the most general case in which the detector is a "mollusk" of variable size and shape, or subject to rotation. We shall pass lightly over that rather fanciful case here. It is to be emphasized throughout this discussion that the symbol "$\mathbf{v}$" always refers to detector velocity with respect to a particular inertial system, not to relative velocity of different inertial systems.

First, consider $\mathbf{v} = \mathbf{v}(x,y,z,t)$. For this case Wesley [1] has recently proposed the following generalization of (1):

$$\frac{d\mathbf{A}}{dt} = \frac{\partial \mathbf{A}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{A} + (\mathbf{A}\cdot\nabla)\mathbf{v}\text{ . (Wesley)} \tag{2}$$

Like (1), this has the property of Galilean invariance, but its full generality may be in doubt, since Wesley's derivation appears to limit detector motion to an "instantaneous circle." It is appropriate to mention here also an alternative expression derived by Mocanu [2] and stated by him to be one of several attributable to Helmholtz. This expression, likewise associated with $\mathbf{v} = \mathbf{v}(x,y,z,t)$, he terms "Helmholtz's total circulation derivative in intrinsic or topological form." It is

$$\frac{d\mathbf{A}}{dt} = \frac{\partial \mathbf{A}}{\partial t} + \nabla(\mathbf{A}\cdot\mathbf{v}) - \mathbf{v}\times(\nabla\times\mathbf{A})\text{ . (Mocanu-Helmholtz)} \tag{3}$$

This (as will presently be shown) is also Galilean invariant.

Alternative Representations

The standard vector identity [3]

$$\nabla(\mathbf{a}\cdot\mathbf{b})=(\mathbf{a}\cdot\nabla)\mathbf{b}+(\mathbf{b}\cdot\nabla)\mathbf{a}+\mathbf{a}\times(\nabla\times\mathbf{b})+\mathbf{b}\times(\nabla\times\mathbf{a}) \tag{4}$$

applies if both **a** and **b** are vector "fields," *i.e.*, $\mathbf{a}=\mathbf{a}(x,y,z,t)$, $\mathbf{b}=\mathbf{b}(x,y,z,t)$. If, instead, $\mathbf{b}=\mathbf{b}(t)$, then the first and third terms on the right drop out, since space derivatives of b vanish. Disregarding this special case and manipulating formally, we see that (1) can be rewritten as

$$\frac{d\mathbf{A}}{dt}=\frac{\partial\mathbf{A}}{\partial t}+\nabla(\mathbf{A}\cdot\mathbf{v})-(\mathbf{A}\cdot\nabla)\mathbf{v}-\mathbf{A}\times(\nabla\times\mathbf{v})-\mathbf{v}\times(\nabla\times\mathbf{A})\text{. (Traditional)} \tag{5}$$

In the same way (2) can be rewritten as

$$\frac{d\mathbf{A}}{dt}=\frac{\partial\mathbf{A}}{\partial t}+\nabla(\mathbf{A}\cdot\mathbf{v})-\mathbf{A}\times(\nabla\times\mathbf{v})-\mathbf{v}\times(\nabla\times\mathbf{A})\text{, (Wesley)} \tag{6}$$

and (3) as

$$\frac{d\mathbf{A}}{dt}=\frac{\partial\mathbf{A}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{A}+(\mathbf{A}\cdot\nabla)\mathbf{v}+\mathbf{A}\times(\nabla\times\mathbf{v})\text{. (M-H)} \tag{7}$$

Thus there is disagreement among the candidate forms for describing the mollusk detector [**v** = "velocity field" = $\mathbf{v}(x,y,z,t)$]. This we do not attempt to resolve here, but avoid the need by exploiting a remarkable simplification that occurs if we confine attention to the physically more interesting special case in which $\mathbf{v}=\mathbf{v}(t)$; that is, the case in which v is independent of spatial variables (point particle detector or rigid-walled irrotational field detector). In that case we see by inspection that all three forms [Eqs. (3), (5) and (6)] simplify to the "all-purpose" expression

$$\frac{d\mathbf{A}}{dt}=\frac{\partial\mathbf{A}}{\partial t}+\nabla(\mathbf{A}\cdot\mathbf{v})-\mathbf{v}\times(\nabla\times\mathbf{A})\text{, [All laws for }\mathbf{v}=\mathbf{v}(t)] \tag{8}$$

which appears formally identical to (3) but differs in that (3) assumes $\mathbf{v}=\mathbf{v}(x,y,z,t)$, whereas (8) assumes $\mathbf{v}=\mathbf{v}(t)$. Incidentally, Eq. (7) shows that in the latter special case the Mocanu-Helmholtz form simplifies to the traditional form (1), equivalent to (8). Note that this simplification does not require **v** = constant, merely that **v** be parametrized in a non-field-theoretical way, as a function of time only.

Hertzian Force Law

Arguments will now be given aimed at validating Eq. (8) or at least strengthening its plausibility. We consider the force law in the case of Hertzian electromagnetism [4-6]. It is one of the attractive features of Hertzian theory, in addition to the first-order (Galilean) invariance of its field equations, that the field equations suffice to yield a law of EM force on charged particles without need for postulation of a separate force law. This is of course not true of Maxwell's theory, which can be made covariant only by introducing second-order considerations and which must be supplemented by the Lorentz force law. (The latter is generally considered not deducible from Maxwell's field equations.) The Hertzian (Galilean invariant) force on a charge q (taking $c=1$) is simply

$$\mathbf{F}_{Hz}=q\mathbf{E}_{Hz}=q\left(-\nabla\phi-\frac{d\mathbf{A}}{dt}\right). \tag{9}$$

Note the substitution of the invariant total time derivative operator for the customary noninvariant partial time derivative operator. (This is necessary to ensure first-order invariance of the force and field vectors on the left.) It must be emphasized that the Hertzian field quantities differ from the corresponding Maxwellian ones. Using the all-purpose form (8) for d**A**/dt (which is formally the same as Eq. (3), the Mocanu-Helmholtz form), we find

$$\mathbf{F}_{Hz}=q\left(-\nabla\phi-\frac{\partial\mathbf{A}}{\partial t}+\mathbf{v}\times(\nabla\times\mathbf{A})-\nabla(\mathbf{A}\cdot\mathbf{v})\right)=\mathbf{F}_{Lor}-q\nabla(\mathbf{A}\cdot\mathbf{v})\text{, [All for }\mathbf{v}=\mathbf{v}(t)] \tag{10}$$

where $\mathbf{F}_{Lor}$ is recognized as the traditional Lorentz force in view of $\mathbf{B}=\nabla\times\mathbf{A}$, which holds for Hertzian as well as Maxwellian fields. The "magnetic" Lorentz force makes its appearance here automatically in consequence of the invariant formulation, despite our having started from a purely electric

Hertz field. (Here $\mathbf{v} = \mathbf{v}(t)$ is the velocity in the laboratory of the detector of Hertzian **E** and **B** fields.) The Hertz force is seen to differ from the Lorentz force only by a gradient term, $-q\nabla\left(\mathbf{A}\cdot\mathbf{v}\right)$, which would be difficult to observe in normal experimental conditions, since it vanishes when integrated around any closed circuit. Its effects might conceivably be inferred, for instance, from any observed anomalies of plasma diffusion rates or from other plasma anomalies—since "currents" in plasmas need not flow in closed circuits.

It might further be remarked that, in view of $\mathbf{v} = \mathbf{v}(t)$, the extra term just mentioned can be written $-q\nabla\left(\mathbf{A}\cdot\mathbf{v}_c\right)$, where subscript "*c*" denotes constancy under space differentiation. The identity (4) then yields $\nabla\left(\mathbf{a}\cdot\mathbf{b}_c\right) = \left(\mathbf{b}\cdot\nabla\right)\mathbf{a} + \mathbf{b}\times\left(\nabla\times\mathbf{a}\right)$. Application cancels the magnetic term from (10) and leaves

$$\mathbf{F}_{Hz} = q\left[-\nabla\phi - \frac{\partial\mathbf{A}}{\partial t} - \left(\mathbf{v}\cdot\nabla\right)\mathbf{A}\right] \text{ (All laws for } \mathbf{v} = \mathbf{v}(t)) \tag{11}$$

This [which is obvious directly from (1) and (9)] is the simplest form for computational purposes, but it hides the connection to established ways. Still, it is interesting that the **B**-field is apparently not needed in the force law, and that "magnetic force" can be replaced by the directional derivative of the vector potential in the direction of test charge motion. But then what is the "detector" detecting? An **A**-field? Shades of Aharonov-Bohm!

Galilean Invariance

Next we verify the invariance of (8) under the Galilean transformation; namely,

$$\mathbf{r}' = \mathbf{r} - \mathbf{V}t, \quad t' = t\ . \tag{12}$$

It will be taken as prior knowledge [4-6] that under this transformation

$$\nabla' = \nabla\ , \ \frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + (\mathbf{V}\cdot\nabla)\ , \ \ \mathbf{v}' = \mathbf{v} - \mathbf{V}\ . \tag{13}$$

A capital V is used here to emphasize the distinction between the relative velocity V of two systems of reference (generally assumed constant, as for inertial systems) and the arbitrary instantaneous detector velocity $\mathbf{v} = \mathbf{v}(t)$ with respect to an unprimed inertial system or $\mathbf{v}'$ with respect to a primed system. The first of the relations (13) expresses Galilean spatial gradient invariance, the second expresses Galilean noninvariance of the partial time derivative, and the third expresses Galilean velocity additivity.

Eq. (8) can be represented by the symbolic shorthand

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \nabla(\mathbf{v}\cdot \quad - \mathbf{v}\times(\nabla\times\ \ , \tag{14}$$

it being understood that a vector field operand **A** is to be supplied and that any open parentheses are to be closed. With this notation, using (13), we see that

$$\begin{aligned}\left(\frac{d}{dt}\right)' &= \frac{\partial}{\partial t'} + \nabla'(\mathbf{v}'\cdot \quad - \mathbf{v}'\times(\nabla'\times \quad = \frac{\partial}{\partial t} + (\mathbf{V}\cdot\nabla) + \nabla([\mathbf{v}-\mathbf{V}]\cdot \quad - [\mathbf{v}-\mathbf{V}]\times(\nabla\times \\ &= \frac{\partial}{\partial t} + \nabla(\mathbf{v}\cdot \quad - \mathbf{v}\times(\nabla\times \quad + (\mathbf{V}\cdot\nabla) - \nabla(\mathbf{V}\cdot \quad + \mathbf{V}\times(\nabla\times \\ &= \frac{d}{dt} + (\mathbf{V}\cdot\nabla) \ - \nabla(\mathbf{V}\cdot \quad + \mathbf{V}\times(\nabla\times\end{aligned} \ . \tag{15}$$

Invariance will be proved if we can show that in the last equality the three extra terms on the right in **V** vanish. This follows immediately from supplying an operand **A** and applying the identity (4). That is,

$$\left(\mathbf{V}\cdot\nabla\right)\mathbf{A} - \nabla\left(\mathbf{V}\cdot\mathbf{A}\right) + \mathbf{V}\times\left(\nabla\times\mathbf{A}\right) = -\left(\mathbf{A}\cdot\nabla\right)\mathbf{V} - \mathbf{A}\times\left(\nabla\times\mathbf{V}\right). \tag{16}$$

Both terms on the right vanish because for any Galilean transformation **V** is a constant. They would also vanish for more general rigid irrotational coordinate frame transformations specified by $\mathbf{V} = \mathbf{V}(t)$. Thus we have shown both Galilean and a more general type of invariance,

$$\left(\frac{d}{dt}\right)' = \frac{d}{dt}, \text{[All laws for } \mathbf{v} = \mathbf{v}(t), \mathbf{V} = \mathbf{V}(t)\text{]} \tag{17}$$

of the total time derivative (8), valid under the stated proviso that **v** and **V** are functions of time only.

Summary

We have proven the Galilean invariance (actually, a broader first-order invariance) of the all-purpose total time derivative (8), valid for $\mathbf{v} = \mathbf{v}(t)$. Under that proviso, (8) is actually equivalent to the traditional (1), but is for many purposes a more suggestive form, in that it exhibits explicitly the curl(**A**) term that yields the Lorentz "magnetic" force in Hertzian theory. Since the Wesley total time derivative (2) is readily seen to be that special case of the Mocanu-Helmholtz derivative for which curl(**v**) vanishes; and since such vanishing is a physically invariant property; it follows that the Wesley derivative is also Galilean invariant. Thus invariance offers no clue to guide choice in the more general problem of the mollusk detector. But the fact that the Mocanu-Helmholtz form is a covering theory of the Wesley form may suggest superiority of the former. Incidentally, some doubt about the algebraic sign of the last term of Wesley's law (2) has been expressed [7]. But it is easily seen that Eq. (8) holds independently of that sign, provided $\mathbf{v} = \mathbf{v}(t)$.

The importance of the EM force law in form (10) is: (a) that the Hertzian force law is seen to be a "covering law" of the Lorentz force law, and (b) that the former differs from the Lorentz law only by a gradient term that would be unobservable in casual experimentation with closed electrical circuits. Thus the Hertzian approach, although as yet unproven, is disproved by no observations made on closed circuits.

In summary, the weight of theoretical considerations seems to confirm the adequacy for electromagnetic applications of the all-purpose total time derivative form (8), which is formally identical to the Mocanu-Helmholtz total time derivative [Eq. (3) or (7)]—the only difference being that the Mocanu-Helmholtz equation claims validity, here unverified, for $\mathbf{v} = \mathbf{v}(x,y,z,t)$ (the "velocity field" or mollusk detector interpretation), whereas the all-purpose (8) is limited to $\mathbf{v} = \mathbf{v}(t)$ (the test-particle detector interpretation). From the physicist's standpoint this last is not a severe limitation.

Needless to say, empirical validation of the Hertzian alternative to Maxwellian field theory must await the devising of EM force law experiments sufficiently subtle to detect an extra force term $-q\nabla(\mathbf{A}\cdot\mathbf{v}) = -\nabla(\mathbf{A}\cdot\mathbf{I})$, where $q\mathbf{v}(t) = \mathbf{I}(t)$ is filamentary current. Less subtle experiments should be able to establish whether or not there is a further non-Lorentz force term—for example, of the form $\mathbf{A}\times(\nabla\times\mathbf{I})$, as required by the Wesley expression, Eq. (2). The present discussion has been limited solely to first-order physics. There are indications [5,6] that it can be pushed to higher orders by the simple expedient of formally replacing frame time *t* everywhere by the proper time τ_d of the detector.

References

[1] J.P. Wesley, "Theorem and proof for the total time derivative of a vector field as seen by a moving point," *Apeiron* **6**, 237-238 (1999).

[2] C.I. Mocanu, *Hertzian Relativistic Electrodynamics and its Associated Mechanics* (Hadronic Press, Palm Harbor, FL, 1991), vol. I, pages 34-38.

[3] J.A. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941), Appendix II.

[4] T.E. Phipps, Jr., "On Hertz's Invariant Form of Maxwell's Equations," *Phys. Essays* **6**, 249-256 (1993).

[5] T.E. Phipps, Jr., *Heretical Verities: Mathematical Themes in Physical Description* (Classic Non-fiction Library, Urbana, IL, 1986).

[6] T.E. Phipps, Jr., "Hertzian Invariant Forms of Electromagnetism" in: *Advanced Electromagnetism* (World Scientific, Singapore, 1995), ed. by T. W. Barrett and D. M. Grimes.

[7] D.P. Allen, Jr., private communication.7

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Editorial Comment

It is nice to have the proof of the Galilean covariance of the total time derivative. I think that the Phipps proof is wrong because he did not take into account that most of the vector operands, like the electromagnetic vector potential or momentum, under Galilean transformations get inhomogeneous terms proportional to the boost velocity. Only force is homogeneous under Galilean transformations. The author implicitly assumes that the Hertzian electromagnetic potentials are the same as the Lorentzian ones. Only on that basis his discussion on the relation between Hertzian and Loretzian forces has some sense. But then his proof proves the adopted assumption! In addition, the physically meaningful transformations are provided not by the Galilean group but by its one-parameter central extension. For the extended group of transformations his relations (13) are invalid and again it is not clear what his proof means.

Edward Kapuscik

Author's Reply

I appreciate Dr. Kapuscik's critique and the opportunity to respond to it. First, I prefer in describing my Eq. (17), $(d/dt)' = (d/dt)$, to speak of this as exhibiting Galilean invariance rather than "Galilean covariance," since no linear combinations of components are involved, as is normally the case for covariance. I discussed this distinction long ago [1]. I disagree with the assertion that I "did not take into account that most of the vector operands … under Galilean transformations get inhomogeneous terms proportional to the boost velocity." Such extra terms certainly do arise and I have taken account of them. Dr. Kapuscik refers to the fact that when Maxwell's field equations or their equivalent in terms of potentials are subjected to a Galilean transformation extra terms of first-order are generated involving the relative ("boost") velocity **V** of inertial frames, and these "invariance breaking" terms predict fringe shifts, etc., that are *not observed* and that contradict the relativity principle at first order. He overlooks the circumstance that Maxwell's equations and their associated potential equations employ *partial* time derivatives exclusively. (This is what gives rise to "spacetime symmetry," in view of the mathematical symmetry of partial space and time derivatives.) My discussion concerned *total* time derivatives. These differ from partial time derivatives in that they introduce a new velocity-dimensioned parameter **v**, a "convection parameter," that appears nowhere in Maxwell's theory. Therefore one cannot rely implicitly on the "learning" instilled by the latter. In this case when an inertial transformation, Eq. (13), is applied to the new parameter **v**, it transforms by Galilean velocity addition into $\mathbf{v}' = \mathbf{v} - \mathbf{V}$, thereby generating a new non-Maxwellian term in **V** that just happens in every case to cancel the extra invariance-spoiling **V**-term to which Dr. Kapuscik refers. This has been proven repeatedly [2-4]. In any case his observation does not show that my "proof is wrong," because that proof concerns a mathematical fact that stands independent of field or potential equations.

He says that "The author implicitly assumes that the Hertzian electromagnetic potentials are the same as the Lorentzian ones." No, I explicitly stated that Hertzian and Maxwellian quantities are different (as regards their operational definitions), but since the field equations [2-4] are of the *same form* in both cases (but differing by substitution of *d/dt* for $\partial/\partial t$), it follows that the equations relating fields to potential quantities are also of the same form, again differing by substitution of the invariant operator *d/dt* for the non-invariant operator $\partial/\partial t$. Hertzian EM potentials are formally related (through identical formal manipulations) to Maxwellian ones—so the "implicit assumption" of formal similarity that Dr. Kapuscik questions is a trivially proven fact. Finally, I am surprised by the claim that "the physically meaningful transformations are provided not by the Galilean group but by its one-parameter central extension." I don't know this terminology, but if "central extension" refers to the inclusion of spatial rotations I have to object that inertial (Galilean) transformations, without rotation, and the *physical property of inertiality*, are still "physically meaningful" at first order regardless of what discoveries may be made or claimed about physical meaning at higher orders. My Eq. (13) is certainly valid at first order, and that is what the proof refers to. It is my prejudice that one has to get first-order physics right before proceeding to higher-order approximations [4].

Finally, let me take this opportunity to correct a genuine error in my paper. The math is all right, but the physical interpretation of quantities I denoted $\mathbf{v}(t)$ and $\mathbf{V}(t)$ was faulty. What I said about those quantities in fact applies only in the one-dimensional case, where the motions of detector or reference frame are restricted to arbitrary time-variable rectilinear motions. But the vector notation I used suggests that my deductions can refer more generally to curvilinear motions … and this is false. Where the relative motion curves out of a single dimension, one gets $\mathbf{v}$ depending implicitly on spatial coordinates, so curl($\mathbf{v}$) does not vanish—contrary to my claims—and also in general the dyadic $\nabla\mathbf{v}$ {as in $\left(\mathbf{A}\cdot\nabla\right)\mathbf{v}$ } does not vanish. The upshot is that my Eq. (8) is not the panacea I thought it was, and we still have to face the problem—not only for the "mollusk detector" but even for the point detector or "test charge"—of choosing between Wesley and Mocanu-Helmholtz. Or else we must find some other generalization of the total time derivative when it operates on a general vector field or on a curvilinear path in space. At the moment I have switched my personal preference to Wesley's form on the basis of an alternative derivation he has provided [5].

References

[1] T. E. Phipps, Jr., "Covariance vs. Invariance," *Toth-Maatian Review* **1**, No. 3, 265-273 (1982).

[2] *Idem*, *Heretical Verities: Mathematical Themes in Physical Description* (Classic Non-fiction Library, Urbana, 1986), Chap. 4.

[3] *Idem*, "On Hertz's Invariant Form of Maxwell's Equations," *Phys. Essays* **6**, 249-256 (1993).

[4] *Idem*, "Hertzian Invariant Forms of Electromagnetism," pages 332-354 of *Advanced Electromagnetism: Foundations, Theory and Applications*, T. W. Barrett and D. M. Grimes eds. (World Scientific, Singapore, 1995).

[5] J. P. Wesley, private communication.

Thomas E. Phipps, Jr.

Further Comment

Dr Phipps wrote that he is not familiar with the terminology on the one parameter extension of the Galilean group. The extensions of groups is well-established theory and the one parameter extension of the Galilean group in classical physics, besides space-time coordinates, includes the action integral into the set of basic coordinates or in momentum space it acts in the five-dimensional space with energy, mass and momentum as coordinates. In quantum physics it is the phase of wave functions which provides the extra coordinate. Physically this means that instead of considering projective unitary representations of the Galilean group we may use ordinary unitary representations of the Galilean group.

Edward Kapuscik

Book Reviews

Statistical Geometry and Applications to Microphysics and Cosmology, Sisir Roy, Kluwer 1999.

In Statistical Geometry and Applications to Microphysics and Cosmology, physicist Sisir Roy reviews the many research directions involving probabilistic, fuzzyor fluctuating measures of distance and time intervals, and shows the importance of their consequences for quantum mechanics, particle physics and cosmology.

This is a significant and fascinating compendium, and provides in a single place summaries of dozens of important articles, with some extensions and a nice exposition of their importance, which would be accessible to the reader otherwise only by considerable acquisition and time. But because the publishing house seems to have completely abdicated its inherent editorial responsibilities, and as English is not the author's first language, most potential readers can be expected to return the book to the shelf and not buy it, or can expect often to be baffled by the text. As an estimate, there are about 10 minor language corrections per page, on average. But those accepting this inconvenience will probably recognize that in among these new mathematical models is to be found the likely answer to at least one of the most basic problems in the foundations of modern physics, namely the quantum measurement problem (see *e.g*. A. J. Leggett in *Quantum Implications*, ed. Hiley & Peat, Routledge 1987), and possibly the explanation for other anomalies as well. The various sources of metric uncertainty, from Planck length to measurement error, can lead to different consequences not fully distinguished, but this research area is still in progress; and there can be no simple conclusion about which sources dominate overall.

The reader may also wish to have certain clarifications, especially about a few undefined or multiply defined variables in some of the equations, *e.g*. the missing definition of the probabilistic phase factor S in eq. 3.20 for the effective potential the dimensionless constant a_o in the definition of the Modified Bessel function argument of eq. 3.28, a missing term in the driver of the wave equation 3.57, the dimensional error in the first line of eq. 3.58 for the fine element squared, confusion of n_μ with η^μ in the coordinate transformation eq 3.89, simple dimensional errors—*i.e*. missing factors—in the confinement potential of eq. 3.128, confusion of x and X and other problems in section 4.3, confusion of the stress tensor $t_{\mu\nu}$ with an undefined one-form t_ν, in eq. 4.72 for the angular momentum tensor, use of an undefined dimensionless x in the density equation 6.77 where x in 6.58 and 6.61 is a dimensional coordinate, typos in eq. 7.13 for the Hamiltonian and the unnumbered equation that follows it, two simple typos in the Compton effect formula 7.39, double usage of μ as an index and as a region of integration in eq. 7.58 for the expectation value of the stochastic metric tensor, *etc*. Eq. 2.85 for the relative phase uncertainty between two points needs to be corrected. Elsewhere there are missing or misplaced parentheses, typos in subscripts, *etc*., most of which are careless typesetting or text errors that readers will be able to correct on inspection.

While full of such careless errors, the overall synthetic coverage of different models for space-time here is important, and it is likely that some such scenario will correct errors subtly introduced at the root of mechanics, classical and quantum, in using functions of continuous infinitely-discriminable space and time points and which have propagated into paradoxical or unphysical results. The change of view of the ontology and framework of physical phenomena to something more string-like, described in the epilogue in terms from the ancient Upanishads, may lead to a qualitatively better model for dynamics.

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Relational Mechanics by André K.T. Assis, Apeiron, Montreal, 1999. ISBN 0-9683689-2-1. 285 pages, paperback, $25US, available from C. Roy Keys Inc., 4405 St. Dominique St., Montreal, Que. H2W 2B2 Canada.

The author of this remarkable book and of *Weber's Electrodynamics* (Kluwer, Dordrecht, 1994) has dedicated much of his professional life to reviving and exploiting the mathematical methods developed by Wilhelm Weber in the mid-1800's for treating electromagnetism and gravity. In a word, Weber adapted Newton's instant action-at-a-distance approach to the parallel treatment of both these topics. This was accomplished in a genuinely relativistic way—that is, by formulating point pair interaction potentials and forces as dependent only on the distance of separation of the points and on time derivatives of that separation. To define such a separation distance unambiguously requires an absolute simultaneity. For this reason the approach has had few followers since the advent of Maxwell and Einstein. But any reader of Assis's books must conclude that this eradication of pluralism in the foundations of physics constitutes a great loss to the discipline. It has inflicted a sort of tunnel vision upon the profession … but fortunately, it is never too late to reform. I can think of no better way of broadening one's physics perspective than the study of Assis's works. They bring up-to-date Weber's approach, and demonstrate that there exists an intellectually respectable alternative to the divergence-plagued methods of field theory, which have seemingly been explored to exhaustion.

In this respect the present book is particularly enlightening. In fact I feel obliged to use the overworked term "revolutionary." Here is indeed a new paradigm, for any who are willing to validate their claims to open-mindedness through a small investment of their time. The new paradigm is strongly Machian. As we know, Einstein began by claiming an implementation of Mach's approach in his general relativity theory … but soon realized the incompatibility of his own field-based causal ideas with the acausality of Mach's notion of an inertial "influence" of distant matter on present happenings.

Mach failed to quantify his conception. He made no attempt to show specifically how the "gravity" of distant matter might affect local inertial responses. This was because he did not challenge Newton's particular form of the law of gravity. Weber, however, did offer such a challenge. His idea, to modify the law of gravitational potential by a factor dependent on the relative motion of the interacting point masses—analogous to the velocity dependent potential by which he had successfully described Faraday induction and other effects in electromagnetism—was long ignored by physicists (owing to the advent and total dominance of field theory), but was reinvented by Schroedinger in a little-known paper [*Annalen der Physik* 77, 325-336 (1925)] of crucial importance. In this paper he integrated a Weber-type velocity-dependent gravitational interaction over a shell of distant matter and showed that local inertial effects could thereby be explicated. That demonstration provided a hint and gave the needed impetus to Assis's much more comprehensive exploitation in the present book of what emerges as a startlingly "new" descriptive theme … one that gives quantitative substance to "Mach's principle."

The book presents its subject matter in two parts, a review of classical Newtonian mechanics (which is instructive even for the reader well versed in conventional treatments) and a presentation of the new "relational" mechanics—so called because the Weber-type separation distance parameter on which it relies establishes an invariant "relation" between its two end-points. That is, observers in arbitrary states of motion must all agree on the spatial relationship between the paired particles so described. Hence the goal of Einstein's general relativity theory is implicitly realized through the Weber parametrization of the physical descriptive problem … without need for tensors, metrics, manifolds, curved space, *etc*. To gain access to this alternative there is only one price to be paid: Einstein's epiphany of the "relativity of simultaneity" has to be de-epiphanized. To me this seems a small price.

And for that price what do we buy? Assis shows that so-called "inertial forces" (centrifugal and Coriolis) can be attributed to the Machian influence of distant matter, with the consequence that "the sum of all forces of any nature acting on any body is always zero in all frames of reference." Thus "inertial forces" are not fictitious, but are in fact gravitational in origin. This is a great simplification of the foundations of mechanics. In its sequel, frames of reference have become a mere descriptive convenience, inasmuch as the basic force laws are formulated in relational terms independent of reference frames and invariant under their changes. This invariance bears no mathematical resemblance to "covariance," a device suited to field theory, not to instant-action theory. In starting down a new road, one has to rid one's mind of old "truths," even as to the meaning of invariance. To be sure, the physics on this new road is spooky … the distant matter cannot be ignored, as we are used to doing, but has to be included (in principle) via integration over an assumed uniform distribution of matter at great distances beyond our galaxy.

Applications to cosmology are discussed, and gratifying agreements with observed orders of magnitude are obtained … but as a physicist I find these less exciting than the possibility that future satellite experiments with gyroscopes, for instance, might probe small discrepancies between Newton's theory, Assis's theory, and general relativity. (During the whole of my mature lifetime I have heard promises of such an experiment … but it is never more than talk. Is it possible that the physics Establishment secretly shrinks from putting the general theory to the real test of a controlled experiment? Look at the gravy trains that might run off the track … and unnecessarily so, when fiddling of NASA priorities can easily prevent it. But such science-fear is foolish, since fiddling of adjustable functions in the general theory is as just easy and more Nobelworthy.)

Assis has made his point clearly and (I think) irrefutably, that Mach's principle suddenly makes quantifiable sense the moment we admit a Weberian velocity dependence of gravitational potential. But that is the lesser part of his story. The big news is that underlying Newtonian mechanics—which for three hundred years physicists have viewed as a completed intellectual creation—is a whole new discipline of Machian *pre-mechanics*. I have little doubt that what will come out of this challenge to field theoretical presuppositions will be a closer linkage between classical mechanics and quantum mechanics. The latter already displays spooky acausal features. Perhaps the creakiness at the joints between relativity theory and quantum theory, customarily seen as a shortcoming of the latter, will turn out to be curable by replacing relativistic mechanics with relational mechanics. I ask you, where can $25 buy you a better chance on a revolution?

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Relational Mechanics by André K.T. Assis, Apeiron, Montreal, 1999. ISBN 0-9683689-2-1. 285 pages, paperback, $25US, available from C. Roy Keys Inc., 4405 St. Dominique St., Montreal, Que. H2W 2B2 Canada.

It is well-known that in classical mechanics there were two distinct concepts of inertial mass: that of Newton (in which inertial mass is a property of a body with respect to absolute space), and that of Mach (in which inertial mass is the property determined by masses of distant stars). It is also known that when constructing General Relativity, Einstein started with the Mach principle, but had to reject it thereafter. However there still is no general agreement among scientists about the necessity of a total rejection of the Mach principle. As an example I recommend to read very interesting book *Relational Mechanics* by Andre Assis.

The author considers the following questions:

1) Can we still believe that Mach principle is correct?
2) Is there an absolute motion of any body relative to space or only relative motion between material bodies?
3) Can we prove experimentally that a body is accelerated relative to space or only relative to other bodies?
4) What is the meaning of inertia?
5) When Newton rotated the bucket and saw the water rising towards the sides of the bucket, what was responsible for this effect?
6) Was it due to the rotation of the water relative to some material body?
7) Is it the rotation of the earth relative to something?

The book shows that answers to these and other questions with *Relational Mechanics* are much simpler and more philosophically sound and appealing than in Einstein's theory of relativity. With an understanding of *Relational Mechanics*, a reader enters a new world, viewing the same phenomena with different eyes and from a new perspective.

Andrew E.Chubykalo

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