Inertial Mass in Mach-Weber-Assis Theory

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Using Dimensional Analysis we have improved a paper of Assis published in 1989, deriving, at first once, the right connection between gravitational mass and inertial mass for any body in the universe.

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1. Introduction

With the aid of a Weber-type law of force Assis was able to fully implement Mach's principle, explaining at once the origin of inertia [1-2-3-4] within an instantaneous far actions scenario [5]. A remarkable achievement of the whole theory is a relationship connecting three (up to now) independent magnitudes of physics,

$$G \approx \frac{H_o^2}{\rho_o} \tag{1}$$

where G is the Newtonian gravitational constant, H_o is the Hubble constant, and ρ_o is the mean matter density in the universe. Although not explicitly stated in the 1989 paper [1] the masses appearing in the work of Assis must be gravitational masses, as he recognized in his later work [2-3-4].

Strangely, Assis failed to get the right connection between the gravitational and inertial masses for a given test body. To overcome this minor omission we need to remember that the Newtonian theory of gravitation indeed rests upon two independent laws:

The force law which only includes gravitational mass (in our view an intrinsic property of matter, like electric charge) and the mutual distance between the involved particles, satisfying also the Newton's third law, $\mathbf{F}_{ji} = -\mathbf{F}_{ij}$.

$$\mathbf{F}_{ji} = -m_{g1} m_{g2} \left(\mathbf{r}_{ij}/\mathrm{r}_{ij}^3\right) \tag{2}$$

Being $\mathbf{F}_{ji}$ the force exerted by the point mass j on the point mass *i*. The equation 2 suffices to define the standard of gravitational mass [6,7] without the introduction of any (dimensional or dimensionless) constant.

The proportionality law between gravitational and inertial masses, valid for any test body in the universe,

$$m_g = k\, m \tag{3}$$

where m means inertial mass and k is a dimensional constant. We must not forget that equ.3 only enters in physics after the famous Newton's experiments, performed with pendulums filled with different substances.

As it is well known, the above proportionality lacks a theoretical explanation in Newtonian mechanics, as well as in Einstenian physics.

From (2) and (3) we get the familiar expression for Newton's law of gravitation, provided $k^2 \equiv G = 6.67 \times 10^{-11}$ N.m²/kg².

2. The Link between Gravitational and Inertial Masses

Our starting point is a Weberian modification of equ.2 which, when applied to the interaction between a body 1 of gravitational mass m_{g1} and a spherical shell with an isotropic matter density $\rho_G(R)$, spining with an angular velocity $\omega(t)$ relative to an arbitrary frame of reference S **(**1-3**)** reads:

$$d\mathbf{F} = (4\pi\xi/3c^2)m_{g1}\rho_g(R)\, R\, dR\, [\mathbf{a}_1 + \omega\mathbf{x}(\omega\mathbf{x}\mathbf{r}_1) + 2(v_1\mathbf{x}\omega) + \mathbf{r}_1\mathbf{x}(d\omega/dt)] \quad (4)$$

where ξ is a dimensionless constant that becomes equal to 6 in order to explain the planetary precession [1-3]. Here rests our main difference with Assis's formulation since we don't include *a priori* in (4) the gravitational constant which is *deduced* in our approach. Note also the sub-index g (absent in the Assis's first work) labeling the matter density ρ. The test body, of course, is located inside the spinning shell. Integrating along the known and observable universe we get [1-3],

$$\mathbf{F}_{I1} = -\phi m_{g1}(\mathbf{a}_1 + \text{non inertial terms})$$

where

$$\phi \equiv (4\pi\xi/3c^2)\int_0^{c/Ho} \rho_g(R)R\,dR = (2\pi\xi\rho_{go})/3H_o^2 \quad (5)$$

and we have assumed homogeneity in a very large scale ($\rho_g(R) = \rho_{g\,o}$).

Calling $\mathbf{F}_{A1} = \Sigma_j \mathbf{F}_{j1}$ the force on 1 due to local bodies and to anisotropic distributions of matter we get ($\mathbf{F}_{A1} + \mathbf{F}_{I1} = 0$) ,

$$\mathbf{F}_{A1} = \phi m_{g1}\,(\mathbf{a}_1 + \text{non inertial terms}) \quad (6)$$

Since (6) is Newton's second law of motion in a non inertial frame of reference, we define the inertial mass m_1 of the point 1 as

$$m_1 \equiv m_{g1}\phi \quad (7)$$

Since ρ (density of inertial mass) also scales as m, it will be

$$\rho = \rho_g \phi \quad (8)$$

On account of (8), (5) and (7) we get

$$m_1 = m_{g1}[(2\pi\xi\rho_0)/3H_o^2]^{1/2} \quad (9)$$

By comparison between (9) and (3) we get $G \equiv 3H_o^2/(2\pi\xi\rho_0)$, in accordance with (1).

Equation (9) is a remarkable consequence of the Assis's theory. Assuming H_o = constant (or, at least if ρ_0 goes to zero faster than H_o^2, allowing H_o to be dependent upon the distribution of matter in the universe) it shows that in a *diluted* universe ($\rho_0 \approx 0$) the inertial mass of a test particle would approach to zero, despite the fact that its gravitational mass would remain invariant.

Let M_{ge} be the gravitational mass of the earth and m_{g1} the gravitational mass of a freely falling test body labeled 1. On account of (2), (6) and (9) we get

$$M_{ge}m_{g1}/r^2 = m_{g1}[(2\pi\rho_0)/3H_o^2]^{1/2}\,\mathbf{a}_1$$

By doubling the matter of the universe, $\rho_0' = 2\rho_0$, we get $\mathbf{a}_1' = \mathbf{a}_1/\sqrt{2}$. This result is on conflict with that recently reported by Assis [8] in his book *Mecanica Relacional*. Assis gives $\mathbf{a}_1' = \mathbf{a}_1/2$ because inertial and gravitational masses are for him the same thing. The above flaw comes from the lack of a clear differentiation between gravitational and inertial mass.

The above analysis can also be applied, *mutatis mutandis*, to another (boundless, stationary) cosmological model able to take into account the absorption of gravity [2-3-4]. In such a case, the starting point will be a mutual gravitational energy given by [2-9-10]:

$$U_{ji} = -(m_{gj}m_{gi}/r_{ij})[1 - (v_r/c)^2]^3 \exp(-\alpha r)$$

where α is related to the characteristic length for the gravitational interaction and v_r means dr_{ij}/dt. With this approach, the gravitational constant becomes one half of the above calculated, amounting some 4/3 of the experimental value. The above occurs since the integration must now be extended to the whole universe. and there is no need to introduce a cutoff in the universe radius. If we assume G = constant, we get $\alpha \approx \rho_o^{1/2}$ [**2**].

Note

Dimensional analysis provides a good tool when we deal with theoretical physics, as we pointed out in a previous paper [11]**.** The relationship between gravitational and inertial mass resembles that occurring between mean kinetic energy per degree of freedom and absolute temperature : $<E> = kT/2$, where k means the Boltzmann constant. If energy is measured in erg and T in absolutes degrees, $k = 1.38 \times 10^{-16}$ erg/degree. We cannot identify energy with temperature (*i.e.* to make $k/2 = 1$) preserving the erg and the degree. For the above purpose we need to modify at least one of the primary standards if we put $<E> = T$.

The same remains valid for gravitational and inertial mass. In the international MKS system we get $m_g = \sqrt{G}\ m$, being $G = 6.67 \times 10^{-11} m^3/kg.s^2$ and m is the inertial mass of any particle having m_g units of gravitational mass. We cannot to take $G = 1$ preserving also the meter, the kilogram and the second as primary standards.

Our views are in full concordance with that due to Schrödinger in his pioneering work of 1925 devoted to the origin of inertia [6].

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