Relativistic Stars with Local Anisotropy: A Vindication of Einstein's Second Heresy

E. Santos Corchero
Departamento de Física. Universidad de Cantabria
Avenida de los Castros s/n
39005 Santander. Spain
FAX: 34-42-201402

Arguments are given in support of Einstein's claim that "the Schwarzschild singularities (black holes) do not exist in physical reality." Static solutions of relativistic stars with spherical symmetry and local anisotropy are studied. It is argued that local isotropy should not be imposed as an "a priori" constraint, but the condition of equilibrium (minimal energy for given total entropy) will determine whether the pressure is isotropic in the interior of the star. Using this condition, equilibrium configurations with local anisotropy are found for supermassive stars and white dwarfs with mass above the conventional limits.

Key words: relativistic stars - supermassive stars - white dwarfs - black holes

1. Introduction

It is well known that Einstein is considered heretical with respect to quantum mechanics because he never accepted it as a fundamental and complete theory of nature. What is not so well known is that he was also heretical with respect to general relativity, the theory which was the crowning achievement of his genius. In fact, Einstein never accepted the possible existence of singularities in space-time, today known as black holes. But his opinion is now viewed as a big mistake and even the man in the street has been instructed that Einstein's theory predicts the existence of black holes.

The name "black hole" was coined by J.A.Wheeler in 1968 (Misner *et al*. 1973) but, in connection with general relativity, the conclusion that some stars would unavoidably collapse to black holes followed from investigations of the early thirties about white dwarfs. These stars are a late stage in the life of ordinary stars like the Sun. Assuming that they are supported by the pressure of a degenerate electron gas, in 1930 Chandrasekhar (1939) proved that white dwarfs with mass above 1.5 M_o (solar masses) could not be stable against collapse, a discovery also made by Landau (1932). Today it is assumed that the collapse of a massive white dwarf does not lead primarily to a black hole but to a supernova explosion, a violent event which ejects the outer parts of the star whilst the core remains in the form of a neutron star (which is also supposed to collapse sometimes, see below). But immediately after the discovery of Chandrasekhar a controversy arose because some of the leading astrophysicists could not accept the collapse of a star towards a singularity and attempted to prevent it, even at the price of changing the known laws of physics. For instance Landau finished his 1932 article declaring that "we must conclude that all stars heavier than 1.5 M_o certainly posses regions in which the laws of quantum mechanics (and therefore quantum statistics) are violated." Eddington (1935) went further by actually modifying the equation of state of the degenerate relativistic electron gas in such a way that equilibrium states would exist for stars of arbitrary masses.

In spite of this initial opposition to gravitational collapse, the papers of Oppenheimer (1939) and others and, finally, the work of Wheeler and collaborators in the sixties convinced the community that collapse to black hole is the unavoidable fate of massive enough stars, and this is the established wisdom of present day astrophysicists. But the correct—in my opinion—solution to the problem came from Einstein, who obviously understood relativity theory better. He essentially proved that collapse is prevented by local anisotropy, the development of which is a consequence of relativity theory if correctly understood. In the present article I shall show how local anisotropy arises and prevents collapse in a simple but relevant case: supermassive stars with mass slightly above the conventional limit of stability.

Returning to Einstein, at the age 60 he published an article (Einstein 1939) where he solved the equations of general relativity for a system of spherical symmetry consisting of a cluster of masses, each moving independently except for the gravitational interaction. After several pages with calculations Einstein concluded:

> *The essential result of this investigation is a clear understanding as to why the "Schwarzschild singularities" do not exist in physical reality. Although the theory given here treats only clusters whose particles move along circular paths it does not seem to be subject to reasonable doubt that general cases will have analogous results. The Schwarzschild singularity does not appear for the reason that matter cannot be concentrated arbitrarily. And this is due to the fact that otherwise the constituting particles would reach the velocity of light.*
>
> *This investigation arose out of discussions the author conducted with Professor H.P. Robertson and with Drs. V. Bargmann and P. Bargmann on the mathematical and physical significance of the Schwarzschild singularity. The problem quite naturally leads to the question, answered by this paper in the negative, as to whether physical models are capable of exhibiting such a singularity.*

To anyone interested in the sociological aspects of science it should be certainly strange that these clinching statements of Einstein, the greatest scientist in centuries and the creator of the theory under discussion, are today considered as completely wrong. I may imagine two reasons for that. The first is that Einstein had already manifested a strong opposition to the established wisdom in quantum mechanics. The price paid for this opposition was that, within the community of physicists, the image of Einstein after 1927 (age 48!) was that of an old man unable to understand the new physics. The second reason is that few people had the powerful physical insight of Einstein who, just from the analysis of a particular quite idealized model, was able to derive "a clear understanding as to why [black holes] do not exist in physical reality." Ordinary people need more elaborate arguments.

In my opinion Einstein was right. In the present paper I will argue that the conclusion of the unavoidable collapse to black holes derives from a misunderstanding of the conditions of equilibrium of relativistic stars. There are two main types of star which are currently believed to collapse: neutron stars and supermassive stars. Both are related to objects already observed, pulsars and quasars respectively. The study of neutron stars requires numerical calculations, which were made for the first time by Oppenheimer and Volkoff (1939). They concluded that neutron stars with mass above 0.7 M_0 would collapse (more refined calculations including the effect of the nuclear strong interaction, performed in recent years, increase somewhat the mass limit). It is ironic that this paper was published in the same year as Einstein's. The calculations needed for the study of supermassive stars are much simpler and may be performed analytically, which makes the physics more transparent. For this reason these are the stars studied here.

The conventional treatments of both neutron and supermassive stars contain the same error. The equilibrium is calculated by imposing an undue constraint, namely that there is local isotropy everywhere, which is an isotropic distribution of particle momenta at all points inside the star. In contrast, the model of Einstein had an extreme local anisotropy: there was no radial motion at all, only transverse motion, because particles moved in concentric circles. This idealization was made for the sake of simplicity, but Einstein supposed that "more general cases will have analogous results." Stars with local anisotropy have been studied by several authors (for a recent review see Herrera and Santos 1997). But in all cases it is assumed that the anisotropy derives from the equation of state of stellar matter being essentially anisotropic, as is the case, *e.g.*, in crystalized outer layers of white dwarfs. In other words, the established wisdom excludes the possibility that the gravitational field could create local anisotropy in a fluid, for instance in the degenerate neutron gas which produces the pressure inside neutron stars. This contrasts with the widespread acceptance that the electric field does create momentum anisotropy (deformation of the Fermi sphere) in the electron gas of the conduction band of metals. But there is here a subtlety of relativity theory, the fact that the "gravitational force" depends on velocity, in addition to mass, but only on *radial* velocity (see below, eq.(1)). This fact makes possible the creation of local anisotropy in relativistic stars, while Newtonian stars in equilibrium seem to posses isotropic pressure everywhere (see end of section 3).

2. Supermassive Stars with Local Anisotropy

Supermassive stars are bodies with mass above $10^3\ M_0$, supposedly formed at the center of galaxies, consisting of a plasma of protons, helium nuclei and electrons plus electromagnetic radiation. The star is supported by radiation pressure and, to a smaller extent, by plasma pressure. The conventional theory of supermassive stars may be seen in Weinberg (1972) or Shapiro and Teukolsky (1983). The established wisdom is that these stars live for relatively short times (between 10 and 10^5 years), after which they collapse to black holes.

The theory of supermassive stars with local anisotropy may be stated as follows (Corchero 1998a). The configuration of the star will be completely determined if we know the statistical distribution functions in phase space, $f_j(\mathbf{r},\mathbf{k})$, of all their constituents (we label $\mathbf{k}$ the momentum in order to avoid confusion with the pressure, p, to be introduced later). To a good approximation the star may be considered as an isolated physical system (this is not strictly true because the star radiates, but the energy loss is slow). Consequently, the thermodynamic condition for equilibrium is that the entropy is a maximum for fixed total energy (or the energy is a minimum for fixed entropy). There are several constraints that I state explicitly in the following:

1. The baryon number is fixed. For supermassive stars this is equivalent to fixing the mass because these stars may be treated within Newtonian gravity, except for small corrections of general relativity.
2. The entropy per baryon is a fixed constant throughout the star. This hypothesis is consistent with convective equilibrium, a standard assumption for these stars which is stronger than just fixing the total entropy.
3. The gravitational equilibrium condition holds. This condition, usually called hydrostatic equilibrium, takes the following form in general relativity when we do not impose local isotropy "a priori" (we shall use units $G = c = 1$)

$$\frac{\mathrm{d}p_r}{\mathrm{d}r}+\frac{2\left(p_r-p_t\right)}{r}=-\frac{m\rho}{r^2}\left[1+\frac{p_r}{\rho}\right]\left[1+\frac{4\pi r^3 p_r}{m}\right]\left[1-\frac{2m}{r}\right]^{-1},$$
$$m\left(r\right)=4\pi\int_0^r \rho r^2 \mathrm{d}r. \tag{1}$$

where $\rho(r)$ is the mass density at a distance r from the center, and $p_r(p_t)$ is the radial (transverse) pressure (see, *e.g.*, Herrera and Santos 1997). The left hand side of eq.(1) is the radial component of the divergence of the stress tensor. Therefore the right side divided by the density may be taken as the mean local "gravitational force" per unit mass. At a difference with Newtonian gravity, where the force is just $-m/r^2$, we see that in general relativity the force on a particle depends on its mass and its *radial* velocity (we recall that p_r/ρ is the mean square radial component of the velocity of the particles at r).

But *I do not accept* the following constraint made in conventional theory:

4. There is local isotropy at every point inside the star (*i.e.* $p_r = p_t = p$).

The standard argument for local isotropy is that photon scattering and collisions among charged particles rapidly randomize the plasma. This is true, but randomization does not necessarily imply isotropization. The rigorous measure of randomness is the entropy, and the effect of collisions is just to raise the entropy. Therefore we should not assume *a priori* that the velocity distribution of particles or photons is isotropic at every point inside the star. We must find the functions $f_j(\mathbf{r},\mathbf{k})$ using the general condition of equilibrium (minimal energy for given entropy), and this condition is what will determine whether there is local isotropy or not inside the star.

In order to proceed further, we make several simplifying assumptions which are standard. Firstly, we assume that the star is nonrotating and possess spherical symmetry. As a consequence all phase space distribution functions will depend on just three coordinates, namely $r = |\mathbf{r}|$, $k = |\mathbf{k}|$ and θ, the latter being the angle between the vectors **r** and **k** (local isotropy would mean that nothing depends on θ). Secondly, we assume that the plasma forming the supermassive star behaves like an ideal gas, so that its (small) contributions to entropy, energy and pressure are known once the chemical composition (homogeneous throughout the star to a good approximation) and the baryon density in ordinary space, $n(r)$, is given.

Finally we shall assume that the dependence of radiation on the photon energy (or angular frequency, ω) is given by Planck's law. Then the most general form possible for the radiation phase space distribution function is

$$f\left(\mathbf{r},\mathbf{k}\right)\mathrm{d}^3\mathbf{k}=\left(2\pi^2\right)^{-1}\left\{\exp\left[\frac{h\omega}{k_B T\left(r,\theta\right)}\right]-1\right\}^{-1}\omega^2\mathrm{d}\omega\,\mathrm{d}\left(\cos\theta\right). \tag{2}$$

where k_B is Boltzmann's constant. All the dependence in r and θ goes on the quantity $T(r,\theta)$, which may be interpreted as a nonisotropic local temperature of the radiation. That is, we treat the radiation field inside the star as a mixture of noninteracting photon gases, characterized by the parameters r and θ each. The distribution of the energy amongst the different gases will be determined by the global equilibrium condition (maximum entropy for given total energy), but for each gas we maximize the entropy with fixed energy and this leads to Planck's law eq.(2).

For small local anisotropy we may approximate the θ dependence of $T(r,\theta)$ using the Legendre polynomial $P_2(\cos\theta)$ as follows

$$T(r,\theta) \approx T_o(r)\left[1-y(r)P_2(\cos\theta)\right],\ |y| \ll 1 \tag{3}$$

where y is a parameter measuring the local anisotropy. From eqs.(2) and (3) it is straightforward to get, *via* appropriate integrals in momentum space, the local properties of the radiation in terms of the mean local temperature, $T_o(r)$, and the anisotropy parameter, $y(r)$. Relevant for our purposes are the entropy per baryon, s, the energy per baryon, u, the mean pressure, p, and the radial pressure, p_r. We get

$$s = \frac{2a}{3n}\int T(r,\theta)^3\,\mathrm{d}(\cos\theta) = \frac{4a}{3n}T_o^3\left(1+\frac{3}{5}y^2\right)+O\left(y^4\right),$$
$$u = n^{-1}\int h\omega f(\mathbf{r},\mathbf{k})\mathrm{d}^3\mathbf{k} = \frac{a}{n}T_o^4\left(1+\frac{6}{5}y^2\right)+O\left(y^4\right),\quad p = n\frac{u}{3}, \tag{4}$$

$$p_{r4} = \int h\omega f(\mathbf{r},\mathbf{k})\cos^2\theta\mathrm{d}^3\mathbf{k} = \frac{1}{3}aT_o^4\left(1-\frac{8}{5}y+\frac{66}{35}y^2\right)+O\left(y^3\right), \tag{5}$$

where $a = 8\pi^5 k_B^4/15h^3$ is the Stefan-Boltzmann constant and we have introduced the standard expression for the entropy in terms of the temperature $T(r,\theta)$. The mass (or energy) density, ρ, is given by

$$\rho = (m_H + u)n, \tag{6}$$

where m_H is the mass of the hydrogen atom. Actually the expressions of the entropy, the energy and the pressure are somewhat more involved than eqs. (4) and (5) because we should include the contributions of the plasma (see Corchero 1998a), but here I shall ignore them for the sake of simplicity.

After that, the equilibrium configurations of supermassive stars may be obtained as follows. For a given entropy per baryon, s, eq.(4) enables us to obtain T_o as a function of n and y. Hence eqs.(1), (5) and (6) lead to two first order diferential equations with three unknown functions, namely $n(r)$, $y(r)$ and $m(r)$. If these functions were known we might find the total baryon number, N, and the mass of the star, M, by means of the integrals

$$N = 4\pi\int_0^R n\left[1-\frac{2m}{r}\right]^{-1/2} r^2\mathrm{d}r\ , \tag{7}$$

$$M = 4\pi\int_0^R \rho r^2\mathrm{d}r\ . \tag{8}$$

However, as there are more unknown functions than equations, the system is indeterminate and we may find infinitely many solutions for fixed baryon number, N, and entropy per baryon, s. The equilibrium configuration will correspond to the solution giving the minimal mass, M. The actual procedure to do that, in the case of small anisotropy (see eq.(3)), may be seen elsewhere (Corchero 1998a). Here we simply quote the results.

Supermassive stars radiate and therefore evolve losing energy (mass) and entropy (although the relative loss of mass is slow because $u \ll m_H$ in eq.(6)). For a given baryon number, N, if the entropy is high enough the star is locally isotropic and the conventional theory holds. The radius, R, of the star decreases the central density, ρ_C, increases until the limit

$$\rho_C = 2.0\times10^{18}\left(\frac{N_o}{N}\right)^{7/2}\text{ g cm}^{-3}, \tag{9}$$

N_o being the number of baryons of the Sun. In the conventional treatment it is predicted that the star collapses to a black hole at this moment (see *e.g.* Shapiro and Teukolsky 1983). In contrast I predict that a local anisotropy is developed, whilst the decrease of the radius and the increase of central density continue. I quote the laws of change of the surface red shift, M/R, the central density, ρ_C, and the central temperature, T_C, as functions of the total entropy, S:

$$\frac{M}{R}\propto S^{1/3}\left(S^{4/3}-S_o^{4/3}\right)^{-1},\quad \rho_C\propto S\left(S^{4/3}-S_o^{4/3}\right)^{-3},\ T_C\propto\left(\rho_C S\right)^{1/3}, \tag{10}$$

where S_0 is a constant. We see that all three quantities M/R, ρ_C, and T_C increase during the evolution (when S decreases). But these results are valid only for small local anisotropy and, therefore, I cannot make claims about the long-range evolution of the stars. However, eqs.(10) strongly suggest that, after some time, the central density and temperature will be high enough for the burning of hydrogen to start. It is likely that, after that time, the star enters in a stage of slow change of radius, luminosity and gravitational surface red shift. In any case our results strongly suggest that supermassive stars never collapse to black holes.

The main astrophysical application of these results is the study of quasars. These objects are currently assumed to be black holes originated in the collapse of supermassive stars. I conjecture that quasars are supermassive stars rather than black holes. Also, as I predict an increase of the surface redshift with time, a part of the quasar's redshift may be gravitational, which should modify our knowledge about the distribution of quasars and their evolution. A detailed study of these matters is beyond the scope of the present article.

It is interesting that a study of white dwarfs also leads to the prediction that a local anisotropy is developped, in the process of cooling, if the mass is above the Chandrasekhar limit (Corchero 1998b). However, in this case the local anisotropy does not prevent the supernova explosion, with the collapse of the core to a neutron star.

3. Einstein's Conjecture that Black Holes do not Exist

Using the general relativistic hydrostatic equilibrium eq.(1) it is easy to understand Einstein's model. In the model there is no radial pressure, p_r, only transverse pressure, p_t, because the particle velocities have no radial component. For a spherically symmetric system of this kind eq.(1) becomes

$$p_t = \frac{1}{2}\rho\left(\frac{r}{m}-2\right)^{-1}. \tag{11}$$

If we take into account that the density, the radial pressure and the transverse pressure are, for a system of particles,

$$\rho = \sum\delta\left(\mathbf{r}_j\right)E_j\, p_r = \sum\delta\left(\mathbf{r}_j\right)\frac{\left(\mathbf{k}_j\mathbf{u}\right)^2}{E_j\, p_t} = \frac{1}{2}\sum\delta\left(\mathbf{r}_j\right)\frac{\mathbf{k}_j^2-\left(\mathbf{k}_j\mathbf{u}\right)^2}{E_j}, \tag{12}$$

$\mathbf{k}_j(E_j)$ being the momentum (energy) of particle j, and $\mathbf{u}$ a unit vector in the radial direction. The obvious condition $\mathbf{k}_j^2 < \mathrm{E}_j^2$ leads to $p < \frac{1}{2}\rho$, whence eq.(11) gives (remember that $p_t = p$ in Einstein's model) $m/r < 1/3$ and the "Schwarzschild singularity" is never reached.

Einstein extrapolated this result to the general conjecture that “(black holes) do not exist in physical reality.” It is ironic that in the same year, probably without knowledge of Einstein’s article, Oppenheimer and Snyder (1939) gave what is today taken as a counterexample to Einstein’s conjecture. They studied a cloud of dust (*i.e.* noninteracting particles) with spherical symmetry, all the particles being at rest at the initial time, and calculated the evolution of the system using the equations of general relativity. The result is that the system collapses to a black hole whatever is the initial density. The authors even argue that, for any initial density, there is a mass such that the Schwarzschild singularity is crossed immediately, which makes the conclusion independent of the equation of state of the cloud (for low enough density, all kinds of particles behave as noninteracting).

The obvious objection to this counterexample is that it is highly idealized. A set of particles at rest is a system at the absolute zero of temperature, which is well known to be not accessible. Any real system of many particles has thermal and/or quantum fluctuations. Particles will collide, the temperature will increase and the Oppenheimer-Snyder “cloud of dust” will become a star if fluctuations are included. In particular, if the cloud has a density similar to that of air in our atmosphere (10^{-3} g/cm^3) the mass needed to cross the Schwarzschild singularity is of the order of that of a galaxy. A system of such mass and density will become a supermassive star if the idealizations are removed. And we have shown that supermassive stars do not collapse.

It is true that Einstein’s model is also highly idealized and, therefore, the extrapolation to real systems is not obvious. We may conclude that the question whether Einstein’s conjecture is true is still open, but certainly it is supported by our calculations of supermassive stars.

It is very interesting that the possibility of local anisotropy seems to be a specific property of general relativity. In Newtonian theory configurations with anisotropic pressure do not appear, either in supermassive stars (Corchero 1998a) or white dwarfs (Corchero 1998b).

4. Conclusion

Many stars have constant entropy per baryon, in particular this is the case if they are at zero Kelvin (which is a good approximation for cold neutron stars or white dwarfs). For such stars I propose that equilibrium requires that: 1) the general relativistic equation of hydrostatic equilibrium, eq.(1), is fulfilled, and 2) the total energy is stationary, for a given number of baryons, with respect to two possible variations. The first is the spatial distribution of matter, given in spherical symmetry by the function $\rho(r)$. The second is the distribution of linear momentum at every point in the interior of the star, given by the anisotropy parameter $y(r)$ (see eq.(3)). We remember that both the mass and the linear momentum contribute to gravity in the general theory of relativity. In the standard treatments the variations of the second kind are not allowed, that is local isotropy is imposed as an unjustified—in my opinion—constraint. The results reported in this article, obtained using this equilibrium condition, support Einstein’s conjecture that relativistic stars never collapse to black holes.

Acknowledgment

I thank Luis Herrera for useful comments. I acknowledge financial support from DGICYT, Project No. PB-95-0594 (Spain).

References

Chandrasekhar, S., 1939. *An Introduction to the Study of Stellar Structure,* University of Chicago Press, Chicago.

Corchero, E. S., 1998a, *Class. Quantum Gravity*. In press.

Corchero, E. S., 1998b, *Astrophys. Space Sci*. In press.

Eddington, A. S., 1935. In Minutes of a Meeting of the Royal Astronomical Society, *Observatory* **58**, 37.

Einstein, A., 1939. *Ann. Math*. **40**, 992.

Herrera, L. and Santos, N.O., 1997. *Physics Reports* **286**, 53-130.

Landau, L. D., 1932. *Phys. Z. Sowjetunion* **1**, 285.

Misner, C. W., Thorne, K. S and Wheeler, J. A., 1972. *Gravitation,* Freeman, San Francisco.

Oppenheimer, J. R. and Snyder, H., 1939. *Phys. Rev*. **56**, 455.

Oppenheimer, J. R. and Volkoff, G. M., 1939. *Phys. Rev.* **55**, 374.

Shapiro, S. L. and Teukolsky, S. A., 1983. *Black Holes, White Dwarfs and Neutron Stars. The Physics of Compact Objects*, John Wiley, New York.

Weinberg, S., 1972. *Gravitation and Cosmology,* John Wiley, New York.