Radius-Mass Scaling Laws for Celestial Bodies

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In this note we establish a connection between two-exponent *radius-mass* power laws for cosmic objects and previously proposed two-exponent Regge-like *spin-mass* relations. A new, simplest method for establishing the coordinates of Chandrasekhar and Eddington points is proposed.

Introduction

In previous papers (Muradian 1980, 1997) it has been suggested the two-exponent Regge-like relation ($n = 2{,}3$)

$$J = \hbar \left(\frac{m}{m_p} \right)^{1+1/n} \tag{1}$$

between the observed mass m and angular momentum J of celestial bodies. In this relation $\hbar$ and m_p stand, respectively, for the Planck constant and for the proton mass. The exponent $n = 3$ corresponds to star-like objects and $n = 2$ to multistellar ones, like galaxies and clusters of galaxies.

Relation (1), besides to fit reasonably well the observational data (see Figure 1), allows discovering two remarkable points: equating (1) to Kerr limit $J^{Kerr} = Gm^2 / c$ for the angular momentum of a rotating black hole, we obtain the following solution for m:

$$m = m_p \left(\frac{\hbar c}{G m_p^2} \right)^{\frac{n}{n-1}} \tag{2}$$

which, for $n = 3$, can be identified with the Chandrasekhar mass

$$m_{Ch} = m_p \left(\frac{\hbar c}{G m_p^2} \right)^{3/2} \tag{3}$$

and, for $n = 2$, with the Eddington mass

$$m_E = m_p \left(\frac{\hbar c}{G m_p^2} \right)^{2} \tag{4}$$

The Chandrasekhar limiting mass for stars (3) and Eddington formula (4) for the mass of the Universe are of special importance for astrophysics and cosmology (see, *e.g.*, Harrison 1972).

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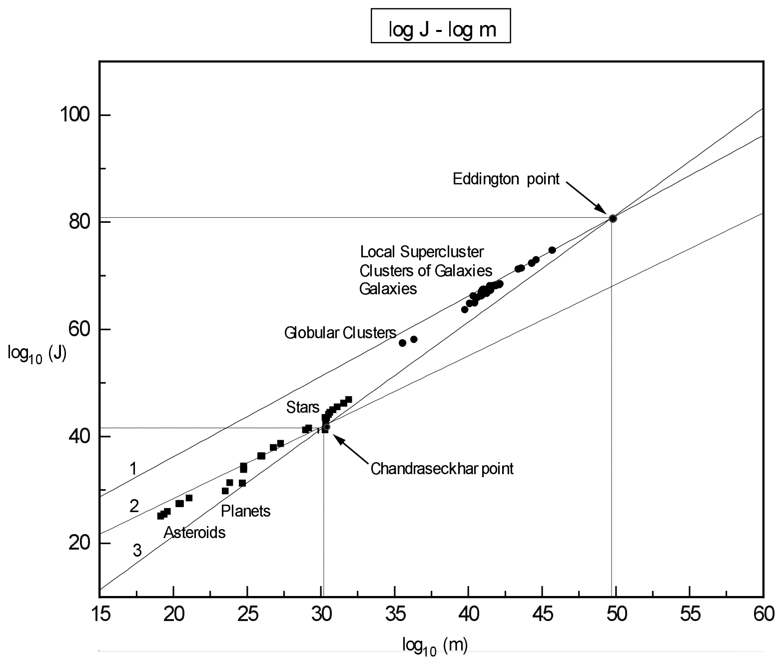

Figure 1. Spin-mass relation for celestial bodies. Observational data for masses and spins can be found in (Muradian 1980, 1997). Theoretical straight lines in this log-log plot correspond respectively to:

1) Regge relation for disk-like objects, $J = \hbar\left(\dfrac{m}{m_p}\right)^{3/2}$

2) Regge relation for ball-like objects, $J = \hbar\left(\dfrac{m}{m_p}\right)^{4/3}$

3) Kerr relation for rotating black-holes, $J = \dfrac{Gm^2}{c}$

The corresponding limiting angular momenta could be obtained by substitution of these expressions into (1), as it has been shown (Muradian 1980, 1997):

$$J_{Ch} = \hbar\left(\frac{\hbar c}{Gm_p^2}\right)^2 \qquad J_E = \hbar\left(\frac{\hbar c}{Gm_p^2}\right)^3 \tag{5}$$

Now we will try to establish the same formulae for coordinates of Chandrasekhar and Eddington points using *radius-mass* theoretical relations for dense objects, from which the observed celestial bodies originated according to Ambartsumian's cosmogony, based on concept that all cosmic bodies, including galaxies, stars and their systems, were formed due to the decay, fragmentation and subsequent evolution of primordial superdense matter with nearly nuclear density (Ambartsumian 1958, 1971).

First of all, let us note that a theoretical relation between m and r should be valid, in particular,

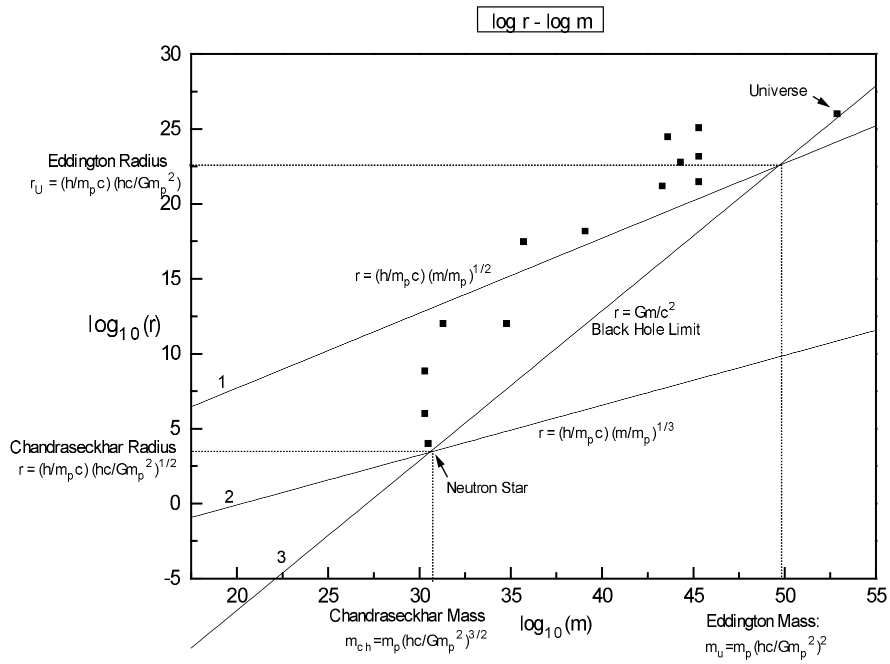

Figure 2. Radius-mass relation for celestial bodies. Observational data are taken from (Padmanabhan 1993). Theoretical lines correspond to

1) Disk-like objects, $r = r_p\left(\frac{m}{m_p}\right)^{1/2}$

2) Ball-like objects, $r = r_p\left(\frac{m}{m_p}\right)^{1/3}$

3) Black-hole limit, $r = \frac{Gm}{c^2}$

for Chandrasekhar and Eddington points, for which the following relation is valid (Carneiro 1998):

$$r = r_p\left(\frac{m}{m_p}\right)^{1/n} \tag{6}$$

where $r_p = \hbar / m_p c$ stands for the proton radius. Here, for $n = 3$ and $m = m_{Ch}$ we obtain the radius of neutron star

$$r_{NS} = \frac{\hbar}{m_p c}\left(\frac{\hbar c}{Gm_p}\right)^{1/} \tag{7}$$

while for $n = 2$ and $m = m_E$ the “Compton wavelength” of the Universe follows,

$$r_U = \frac{\hbar}{m_p c}\frac{\hbar c}{G m_p^2} \tag{8}$$

Indeed, from (4) and (5) we obtain

$$r_U = \frac{J_E}{m_E c} \tag{9}$$

In this last case relation (6) is just a possible expression for well known large number coincidences (Carneiro 1998, Frankel 1982).

In this way, (6) seems to be a good candidate for theoretical two-exponent (n = 2,3) law relating the radius and mass of primordial dense proto-objects, from which the present day cosmic bodies originated, in the sense of Ambartsumian's cosmogony (see Ambartsumian 1958, 1971, Muradian 1980 and references therein). Another observation in favor of this suggestion is connected to the fact that the same relations for r_{NS} and r_U follows from the expression for half of the gravitational Schwarzchild radius $r = Gm/c^2$ after substitution of Chandrasekhar m_{Ch} or Eddington m_E masses. This is consistent with the above mentioned fact that the Chandrasekhar and Eddington points correspond in the *J-m* Chew-Frautschi plane to maximally rotating black holes (see Figure 1).

Relation (6) is plotted in Figure 2 together with the observational data and the black hole limit line $r = Gm/c^2$. As expected, neutron star and Universe lie on this line. The theoretical line which follows from (6) with $n = 2$ fits crudely the data relating to clusters of galaxies, but in the case of star-like objects the theoretical relation following from (6) with $n = 3$ is completely uncorrelated with the data (except the point of neutron star).

The following reasoning can elucidate this disagreement. To (1) and (6) be consistent, one needs $J = mcr$, what means that (6) refers to maximally rotating objects. As we have seen, this is the case for the Chandrasekhar and Eddington points, for which the equation $mcr = Gm^2/c$ is equivalent to $r = Gm/c^2$. But, in general, celestial bodies are far away from this limit and, in consequence, their radii are systematically distributed above the lines representing (6) (see Figure 2).

But if (6) does not exactly represent the observational data, what does it represent? And why its partner, relation (1), fits well the data? A possible answer to these questions is that relations (1) and (6) represent an initial dense stage in the evolution of the bodies, when they have maximum, Regge-like, angular moments for some given radii. So, as bodies evolve, their radii change, diverging from the original values given by (6).

In a recent paper it has been suggested the existence of two-exponent scaling relation between mass and radius of cosmic objects. As indicated in (Pérez-Mercader 1996), the fact that there are two radically different power laws for two classes of objects could serve as indication that the objects within each class have a similar physical origin. A possible reason for exponents change is a different geometrical shape of the primordial objects: disk-like (n = 2) for multistellar objects and ball-like (n = 3) for stellar ones (Muradian 1980, 1997, Frankel 1982).

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