The Uncertainty Relations

J.R. Croca
Departamento de Física
Faculdade de Ciências, Universidade de Lisboa
Campo Grande, Ed. C1, 1700 Lisboa, Portugal
email: croca@fc.ul.pt

A discussion of the meanings and the limits of the usual Heisenberg uncertainty relations is presented. These relations, as stated by Bohr, are conceptually a natural consequence of the principle of complementarity and a direct mathematical result of the non-local Fourier analysis. The local wavelet analysis is mentioned and compared with the non-local Fourier analysis. It is shown, using the Morlet gaussian wavelet instead of infinite harmonic plain waves, that it is possible to derive a more general mathematical expression for the uncertainty relations. In order to test the validity of these more general uncertainty relations the new generation of microscopes, with a practical resolution that goes far beyond the usual Abbe's limit of half wave length, is briefly introduced. It is also shown that the working of a certain type of optical microscope, of this new generation, is not described by the complementarity principle, therefore the usual Heisenberg uncertainty relations do not apply to these type of measures. In this situation it is possible and natural to predict, before the actual measurement takes place, the uncertainties in position and momentum such that there product is much less than h.

1. Introduction

Since the fall of 1927[1] till now, the usual Heisenberg uncertainty relations have been accepted by the scientific community as the last word on our possibility to make predictions on the measurements. According to these view it is not possible to predict, before the actual measurement takes place, the values of two conjugate observables, *e.g.* the position and the momentum, such that the product of their uncertainties is less than h.

These relations follow naturally from Bohr's Principle of Complementarity and are a direct mathematical consequence of the non-local Fourier analysis. Since the Principle of Complementarity stands as the epistemological basis for the usual quantum mechanics which is intrinsically connected with the Fourier analysis, it is only natural to expect non-local consequences from the usual quantum mechanics.

Of course, Popper[2] and others[3] have questioned the interpretation of these relations. Nevertheless, they all accepted the final mathematical expression for the Heisenberg relations and therefore, implicitly, the non-local character of the usual quantum mechanics. As far as I know, only Paul Dirac[4] (in 1963) questioned the form of these relations. In his work *The Evolution of the Physicist's Picture of Nature* he wrote:

> *I think one can make a safe guess that the uncertainty relations in their present form will not survive in the physics of the future.*

Now, thanks to the recent development of the local wavelet analysis, it is possible to conceive a more general picture that may include Bohr's Principle of Complementarity, as a particular case, and therefore overcome its non-local implications, allowing a revitalization of causality on sound grounds. In such circumstances, it is looks quite natural to derive, from the local wavelet analysis, a new more general mathematical expression for the uncertainty relations. These new uncertainty relations, as expected, contain, in the asymptotic limit, the usual ones as a particular case.

Since a local causal alternative formal way is opened, the next problem is to ask which is the right model? The local, or the non-local approach? The answer to this question can only be given by concrete experiments. This problem of finding this experimental evidence to confirm or refute the causal local model has hunted physics for more than half a century. In order to solve the problem many, feasible concrete, experiments[5,6] have been proposed. A few of those experiments have been carried out, but so far till now, the results were not conclusive. So, they have to be remade in better circumstances in order to clarify the situation. The other proposed experiments, the great majority of them, have not yet been done, so the overall picture has not improved for some years. Now, as a consequence of the recent development of a new generation of microscopes, with a resolution going far beyond the usual limit for the Fourier microscopes, it is possible to show experimentally that indeed, Bohr's complementarity principle has reached its limits of applicability in nature. This consequently implies that the non-local model needs to be upgraded by a more general local causal theory.

2 – Non-Local Model

In March 1927 Heisenberg submitted for publication an article with the derivation of an inequality that later became known after him. Shortly after, Heisenberg presented his idea to Niels Bohr.

The situation of physics at that time was, in a bird eye view, something like this: Inspired by the particle nature of the quantum systems Heisenberg had developed his matrix mechanics; Schrödinger, using the de Broglie approach, based on the wave aspect of quantum entities, built his wave mechanics. As a result there were two theories: one based on the particle nature, the other on the wave aspect of the quantum systems. In order to overcome this strange situation, of having two different theories to describe the same phenomenon, Schrödinger was able to demonstrate the formal equivalence between the two mathematical descriptions. There was, however, the necessity of building a general conceptual framework that could include, at the same time, the particle and the wave properties experimentally shown by quantum entities.

This was precisely the problem faced by Niels Bohr. So when he saw the inequalities of Heisenberg he felt that there must be the starting point for the general synthesis. He developed those ideas in the coming months so that, at end of that summer, he arrived at his general conception. He presented his ideas, for the first time, to the scientific community, in his famous lecture [1] at Lake Como (Italy) just before the well-known Solvay Congress of October 1927.

Basically Bohr's idea is as follows: From the Fourier analysis it is known that when one wishes to represent a localized function, for instance, a gaussian function, the more localized the function the larger the bandwidth needed for the synthesis. This means that the smaller the function the more monochromatic plane waves must be added in order to build the initial function, and *vice-versa*. The next step is to associate, following Max Born, the probability of finding the particle with the squared modulus of the wave function. Therefore, the squared modulus of the wave function represents the region where the particle can be localized by a measurement. The smaller the width of the wave function the more precisely one knows its position, and consequently the more waves must be included for its synthesis. Since each wave corresponds to a well defined velocity, our knowledge of the velocity of the particle decreases. So at the end it is possible to establish an inverse relation between the two observables, position and velocity. This statement is one form of enunciating the principle of complementarity: The more precise the position of a particle the less one knows about its velocity, and *vice-versa*. This line of descriptive reasoning can be made more precise.

From the Fourier analysis, and following Niels Bohr as closely as possible, we can represent a well-behaved function as an infinite sum of monochromatic plane waves

$$f(x) = \int_{-\infty}^{+\infty} g(\sigma) e^{i\sigma x} d\sigma \tag{1}$$

where $x = 1/\ell$, represents the wave number. Choosing for the coefficient function a gaussian form

$$g(\sigma) = e^{-\frac{\sigma^2}{2(\Delta\sigma)^2}} \tag{2}$$

by substitution

$$f(x) = \int_{-\infty}^{+\infty} e^{-\frac{\sigma^2}{2(\Delta\sigma)^2}} e^{i\sigma x} d\sigma \tag{3}$$

and integration one has

$$f(x) = \sqrt{2\pi}\Delta\sigma \;\; e^{-\frac{x^2}{2/\frac{1}{(\Delta\sigma)^2}}} \tag{4}$$

which shows that

$$\Delta x \, \Delta\sigma = 1 \, . \tag{5}$$

In the case of temporal information on the particle one gets, following the same steps,

$$\Delta t \, \Delta \nu = 1 \, . \tag{6}$$

Remembering the fundamental relations of quantum mechanics, Planck-Einstein and de Broglie

$$E = h\nu \;\; ; \qquad p = \frac{h}{\lambda} = h\sigma$$

and by substitution in (5) and (6) one finally gets

$$\begin{aligned} \Delta x \Delta p_x &= h \\ \Delta t \Delta E &= h \end{aligned} \tag{7}$$

In this way, particularly simple and elegant, Niels Bohr was able to derive the Heisenberg uncertainty relations as a direct consequence of the Non-local Fourier analysis. In this context, from simple mathematical relations, these formulae assume the deep ontological meaning of translating, in an extremely condensed way, an aspect of his Principle of Complementarity. For this reason even though Heisenberg derived them for the first time, it was Bohr who understood their importance and gave them the status they have enjoyed in science during this century. Therefore, it would be quite correct to call them Heisenberg-Bohr relations, because while the former had the privilege of discovering them, it was Bohr who understood their deep meaning.

2.1 - Meaning of the Uncertainty Relations

The success of these relations was tremendous! The scientific community soon understood their great importance. Many works on the subject were published, not only in physics but also in philosophy, epistemology and even in plain literature. Perhaps for this reason, some authors, less learned in quantum mechanics, claim violations of the uncertainty relations in situations that, according to the usual non-local paradigm, are perfectly irrelevant. These erroneous conclusions are

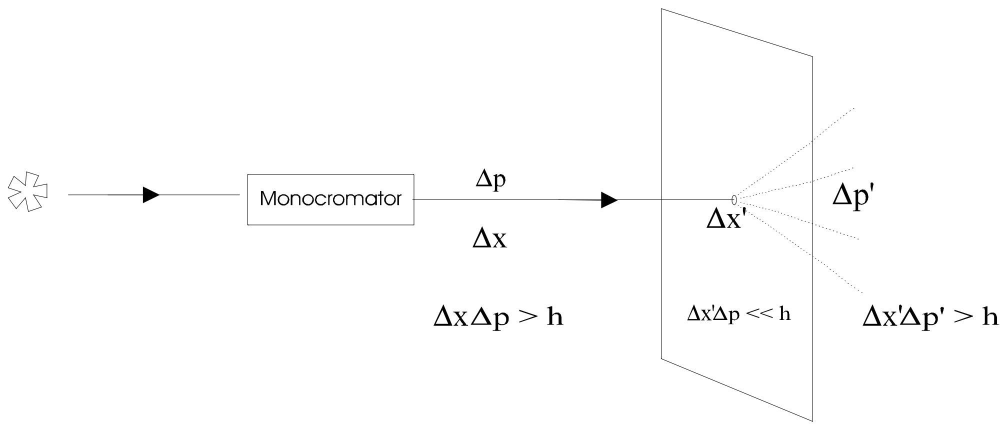


Fig.1 - Apparent violation of the uncertainty relations.

mainly consequences of a serious misunderstanding of the conceptual structure of the usual quantum mechanics, and consequently of the deep meaning of these relations.

The uncertainty relations are essentially connected with the problem of measurement. As a matter of fact, as we shall see, it is possible to make a measurement that violates, numerically, the limit imposed by the relations. This was recognized early by Heisenberg [7] himself, and by Bohr [1] ,and even Popper [2] wrote about it in the thirties. Since then it has been common lore in fundamental quantum mechanics. Still this possibility in no way challenges the true meaning of Heisenberg's uncertainty relations. Because after the measurement, after interaction, the product of their uncertainties does confirm the relations.

In order to illustrate this problem let us take a concrete example. This example, one among many possibilities, is based on work by Andrade e Silva [8].

Fig.1 is a sketch of an experiment that apparently violates the uncertainty relations just before the measurement takes place. An electron source emits electrons, one at a time. After the monochromator the uncertainty in the momentum of the electron is Δp and consequently its uncertainty in position is Δx so that $\Delta x \Delta p > h$.

In its trip the electron encounters a screen with a hole. Behind that hole there is an electron transfer detector that is, a detector that "sees" the electron without killing it. The hole and the detector are arranged in such a way that the uncertainty in position is $\Delta x'$. This uncertainty in position $\Delta x'$ is independent of the uncertainty in momentum Δp, because Δp depends only on the monochromator and $\Delta x'$ depends on the detecting device. The experimental set-up can be arranged in such a way that the product of the two uncertainties is much less than the limit imposed by Heisenberg, $\Delta x' \Delta p' << h$.

Still, during the interaction, with the detector, the uncertainty in momentum of the electron spreads in such a way that the smaller the uncertainty in position $\Delta x'$ the greater the uncertainty in momentum $\Delta p'$. Therefore, at the end, after measurement, after the interaction, the product of their uncertainties obeys Heisenberg's uncertainty relations $\Delta x \Delta p > h$.

This experiment clearly shows that the uncertainty relations only forbid the prediction of the uncertainties in the future, such that theirs product are less than h after measurement.

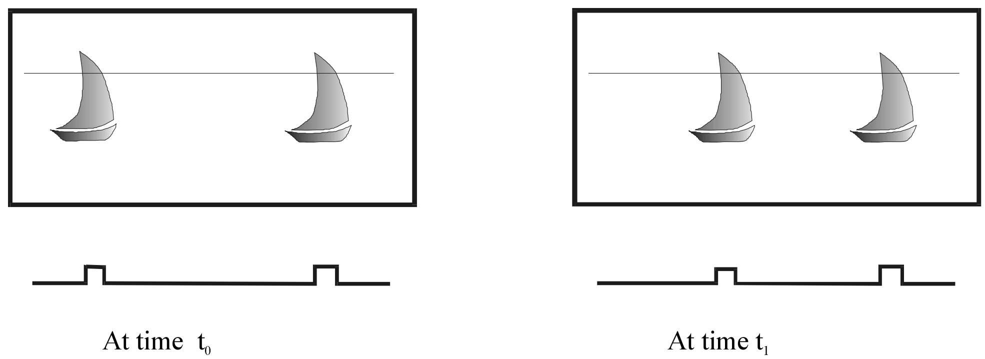


Fig.2 – Digitized images to Fourier analyze

3 – Local Model

The problem of the uncertainty relations can also be approached in a causal local way. Formally, the usual relations, as we have seen, are a mathematical consequence of the non-local Fourier analysis. In this situation the construction of a local model would require replacement of the non-local Fourier analysis by a local analysis. Now, thanks to the recent development of local wavelet analysis [9], we have the necessary mathematical tool that may allow us to build a causal local model.

The local wavelet analysis was developed in the early eighties in the domain of the Earth sciences, namely in the seismology were it is necessary to study signals changing rapidly in time. Still the explosive progress of the local wavelet analysis was mainly due to the necessity of information theory where compression and decompression of information in real time is the main goal.

In order to understand the deep meaning and importance of this new local analysis we shall make a brief comparison with the non-local Fourier analysis.

The Fourier analysis is considered non-local or global for two main reasons correlated in a certain sense. The first reason is because its basic elements, its constituting bricks, are monochromatic harmonic plane waves infinite in space and time. The second reason for its non-local character results from its global implications. In order to fully understand the non-local or global character of the Fourier analysis lets us consider the following situation.

Let us suppose that we want to record, line by line, the digitized image shown in Fig. 2, for instance, in compact disk, for later viewing. In order to simplify the problem let us consider only the line indicated in the Fig.2, where the intensity plot along that line is also represented. The treatment of the other lines, not shown, is precisely the same. In order to represent this variation in intensity it is necessary to find the monochromatic harmonic plane waves, with different frequencies, amplitudes and phases, such that their sum reproduces the initial function. This immensity of infinite waves interferes negatively in all points of space except in those two regions where the interference is constructive. Mathematically this is represented by the Fourier integral,

$$f(x) = \int_{-\infty}^{+\infty} \left[A(k)\cos kx + B(k)\sin kx \right] dk \qquad (8)$$

This analytic work, that is finding the coefficients $A(k)$ and $B(k)$, is in general automatically done with by the information processing equipment.

What happens now if, for instance, the first ship moves to the right, while the other remains in the same position? In this type of analysis both the first and the second ship are made from the same

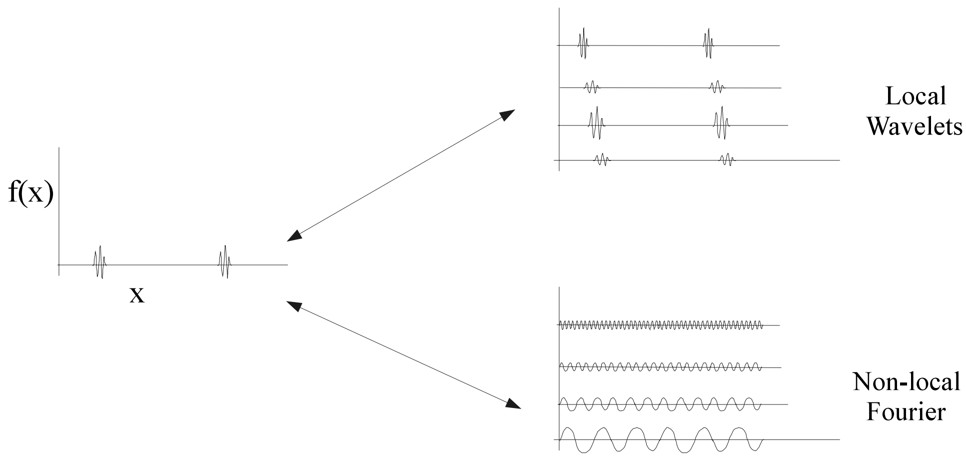


Fig. 3 – Local wavelet analysis and Non-local Fourier analysis

infinite harmonic plane waves. In this circumstances a modification in the position of the first object implies a modification in the amplitude frequency and phase of the waves, so that their interference is constructive only in the new position. This means that although, in this process of non-local Fourier analysis and reconstruction, the two ships may seem separate entities they are treated as a whole entity. So, a modification of one implies a simultaneous modification of the other. This fact has serious practical drawbacks, namely when, as is the case, one wants to register video images. In general, as is very well known, from video image to video image there are only slight variations. Nevertheless, in this type of non-local or global analysis, a minor modification on the image implies the need to analyze and rebuild the whole image.

If instead of the non-local Fourier analysis one uses local wavelet analysis, the modification of the position of one ship as nothing to do with the other, naturally if the two objects are sufficiently far one from another. In this case, the group of wavelets that allows the reconstruction of one object is completely different from the other. This is precisely why this type of analysis is called local analysis by finite waves.

Fig. 3 shows graphically the difference between the two types of analysis.

Another advantage of local wavelet analysis is related with the fact that to represent a given signal one is free to use different types of basic wavelets. So, a good choice of basic wavelet can further increase the rate of compression and the quality of the reconstruction.

From the above it seems natural to describe a local causal quantum particle not by a sum of infinite harmonic plane waves, non-local Fourier analysis, but instead with the help of local finite wavelets. In this situation one is able to avoid certain inconveniences of the usual quantum mechanics, such as instantaneous actions at a distance and retroaction in time.

With localized finite waves, it appears natural to represent the wave part of the of de Broglie real particle by a wavelet. It is worth recalling that de Broglie assumes [10] that a quantum particle is a real entity composed of an extended, yet finite localized wave, where the singularity is localized. This singularity carries practically all the energy of the particle and is responsible for the usual detection process. The finite wave guides the singularity through a non-linear interaction and is responsible for the interferometric properties of the quantum particle.

Since there is the possibility of using different types of basic wavelets to represent the wave part of the quantum particle we choose among the immense possibilities the gaussian wavelet. This basic

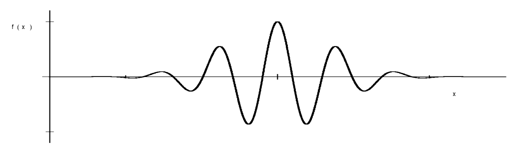


Fig. 4 - Real part of the gaussian or Malat wavelet, representing the wave part of a quantum particle.

wavelet is also called by some writers [11] the Malat wavelet because this author studied it thoroughly.

$$f_0(x) = e^{-\frac{x^2}{2\Delta x_0^2}+i\sigma x} \tag{9}$$

whose real part is shown in Fig. 4

The advantages of the choice of this particular wavelet to represent the quantum particle are many, namely:

1. The formal simplicity of this wavelet. Among the large variety of possible basic wavelets the gaussian is perhaps the simplest of them all. This property is very interesting because it allows the possibility of doing analytically the necessary calculations without recourse to approximate methods.
2. In a certain sense, it contains the usual kernel of the Fourier analysis as a particular case. As can be seen from (9) when the size of the wavelet Δx_0 goes to infinity it transforms itself into the kernel of the Fourier analysis.
3. With this representation the quantum particle has always a well defined energy, contrary to which happens in the usual Fourier representation, where a moderate localized particle has an infinite number of possible energies.
4. It allows the possibility of assigning to each particle a true dimension Δx_0 . Although it is convenient to mention that usually the quantum particle is not in the most single state. In general is formed by many waves of this type, giving rise to a final wave where the singularity is merged. On the other hand the usual sources usually emit bursts, and therefore only a relatively complex treatment [12] can isolate a single particle. In this way the usual normal pulses, emitted by the sources, must, just as in the usual analysis, be represented by the combination of waves. Only in this case they are finite waves instead of infinite waves.

Naturally the choice of this mathematical model, where the non-local Fourier analysis is replaced by local analysis by wavelets, for describing the quantum particles has deep implications at the very roots of the quantum mechanics. In particular, the Schrödinger equation needs to be replaced by a master non-linear equation as Einstein and de Broglie always have maintained.

3.1 – Derivation of a new form for the uncertainty relations

In order to make the derivation of the new more general form for the uncertainty relations more comprehensible, it will be done in parallel and step by step with the original of Niels Bohr presented previously. This process is shown in the following table, Fig.5

The new more general the dispersion relations

$$\Delta x^2 = \frac{1}{\Delta\sigma^2 + 1/\Delta x_0^2} \tag{11'}$$

can also be written in a more convenient form

$$\Delta x^2 = \frac{h^2}{\Delta p^2 + h^2 / \Delta x_0^2} . \tag{11}$$

As suspected, by using local finite wavelets instead infinite plane waves of the non-local Fourier analysis, we were led to a new form for the uncertainty relations. It is easily seen that when the basic wavelet Δx_0 is sufficiently large the new relation is transformed into the old usual Heisenberg relations, which is a very satisfactory result.

Fig. 6 represents the plot of the new relations, as a solid line, for three values of the size of the basic wavelet, together with the usual Heisenberg relations as a dashed line.

From the plot one sees that only in a small region, near the origin, the two relations lead to different results. In the great majority of the cases the two relations predict exactly the same results.

This first derivation of the new form for the uncertainty relations was made by assuming, as a first approximation, that the width of the particle does not depend on the wave number. For quasi-monochromatic beams this approximation must be very near to the truth, but for beams with a larger dispersion in *k* it no longer holds. In this case it is necessary to derive the expression that gives the length of the particle as a function of the wavelength

$$\sigma = \sigma(\lambda) \tag{12}$$

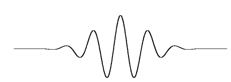
Non-local Fourier analysis	Local wavelet analysis
Kernel – Sinus and cosinus	Kernel – Gaussian wavelet
	
$f_0(x) = e^{i\sigma x}$	$f_0(x) = e^{-\frac{x^2}{2\Delta x_0^2} + i\sigma x}$
Representation of the particle	
$f(x) = \int_{-\infty}^{+\infty} g(\sigma) e^{i\sigma x} dx$	$f(x) = \int_{-\infty}^{+\infty} g(\sigma) e^{-\frac{x^2}{2\Delta x_0^2} + i\sigma x} d\sigma$
Coefficient function gaussian $g(\sigma) = e^{-\frac{\sigma^2}{2\Delta\sigma^2}}$ by substitution and integration $f(x) \propto e^{\frac{x^2}{2\Delta x^2}}$ which gives	
$\Delta x = \frac{1}{\Delta\sigma}$	$\Delta x^2 = \frac{1}{\Delta\sigma^2 + 1/\Delta x_0^2}$
by substitution for the quantum value: $p = h_{\leftarrow}$	
$\Delta x \ \ \Delta p_x = h$	$\Delta x \ \ \Delta p_x = \sqrt{1 - \frac{\Delta x^2}{\Delta x_0^2}} \ \ h$

Fig. 5 – Derivation of the new form for the uncertainty relations in parallel with the usual Bohr's derivation.

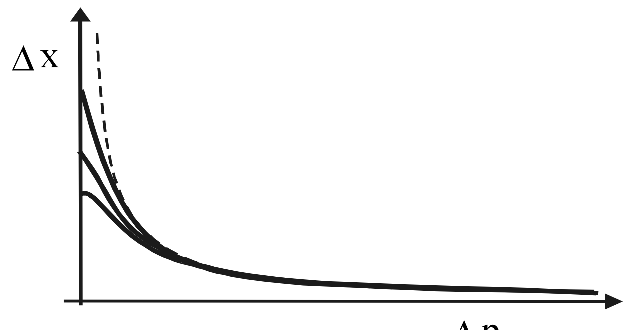


Fig.6 - Plot for the uncertainty relations: solid line new relations for different values of the basic wavelet; dashed line the usual old relations.

In order to do that, one assumes that the basic mathematical tool to describe a real quantum particle is a finite local wavelet, unlike the usual theory where the primacy goes to non-local infinite, in time and space, monochromatic plane waves. Furthermore it is assumed that this basic wavelet, for simplicity, can as before be a gaussian Malat wavelet undergoing an affine transformation:

$$\psi(\xi) \to \psi\left(\tfrac{x-b}{a}\right).$$

That is

$$\begin{aligned}&\exp\left(-\tfrac{(x-b)^2}{2\sigma^2}+ik(x-b)\right)=\\&\exp\left(-\beta_1\left(\tfrac{x-b}{\sqrt{\beta_1}\sqrt{2}\sigma}\right)^2+i\beta_2\left(\tfrac{x-b}{\beta_2/k}\right)\right)=\psi\left(\tfrac{x-b}{a}\right)\end{aligned} \tag{13}$$

meaning that

$$\sqrt{2\beta_1}\,\sigma=\tfrac{\beta_2}{k}=\tfrac{\lambda\beta_2}{2\pi},$$

which leads to

$$\sigma=\frac{\sqrt{2}\beta_2}{4\pi\sqrt{\beta_1}}\lambda \tag{14'}$$

or finally to

$$\sigma=M\lambda, \tag{14}$$

where

$$M=\frac{\sqrt{2}\beta_2}{4\pi\sqrt{\beta_1}}, \tag{15}$$

is an universal constant relating the true size of a quantum particle with its wavelength. This constant *M*, can experimentally be determined in special interferometric measures [12] relating the true size of the particle with its wavelength.

Taking in consideration these assumptions the wavelet (9) must now be written

$$\psi(x,k)=Exp(-\beta^2k^2x^2)\;Exp(ikx), \tag{16}$$

where

$$\tfrac{1}{2\sigma_p^2}=\beta^2k^2,$$

with

$$\beta = \frac{1}{2\pi M}. \tag{17}$$

In such circumstances, and for the previous conditions, the total pulse emitted by the source is given by

$$f(x) = \int_{-\infty}^{+\infty} Exp(-\beta^2 k^2 x^2)\, Exp(ikx)\, Exp(-\tfrac{(k-k_0)^2}{2\sigma_k^2})dk, \tag{18}$$

which upon integration, that can be made directly with the help of the *Mathematica*, leads to

$$f(x) = \sqrt{2\pi}\frac{\sigma_k}{\sqrt{2\sigma_k^2\beta^2x^2+1}} Exp\left\{-x^2/\left[2\left(\tfrac{2\sigma_k^2\beta^2x^2+1}{2\beta^2k_0^2+\sigma_k^2}\right)\right]\right\} Exp\left(i\tfrac{\sigma 2_k}{2\sigma_k^2\beta^2x^2+1}x\right) \tag{19}$$

meaning that

$$\sigma_x^2 = \frac{2\sigma_k^2\beta^2x^2+1}{2\beta^2k_0^2+\sigma_k^2}. \tag{20}$$

Let us know look more carefully at the expression (19). For $x \to \infty$, *f*(*x*) goes to zero relatively quickly. So we may consider the situation

$$2\sigma_k^2\beta^2x^2 << 1, \tag{19}$$

since

$$\beta = \frac{1}{2\pi\, M} << 1.$$

In this approximation

$$\sigma_x^2 = \frac{1}{\sigma_k^2+2\beta^2k_0^2}, \tag{20´}$$

or

$$\sigma_x^2 = \frac{1}{\sigma_k^2+\gamma^2}, \tag{20}$$

which is precisely equal to (11') for $\gamma = 1/\Delta x_0$

Expressing the constants as functions of *M*, one gets:

$$\sigma_x^2 = \frac{1}{\sigma_k^2+\frac{1}{2\pi^2}\frac{k_0^2}{M^2}} \tag{21'}$$

or, making the usual quantum substitutions,

$$\Delta x^2 = \frac{h^2}{\Delta p_x^2+\frac{k_0^2}{2\pi^2}\frac{h^2}{M^2}} \tag{21}$$

This last derivation leads, within a good approximation, to the same value deduced in the first case, where the size of the extended region of the particle is assumed to be constant.

3.2 – The physical meaning of the new relations

It was shown, using wavelets instead of the infinite harmonic plane waves of the non-local Fourier analysis, that it is possible to derive, formally, a different more general expression for the uncertainty relations. Although the new relations contain the usual old relations as a particular case, the problem we face is to know what is the physical meaning and the interest of these relations. Are they mere formal relations, resulting from more or less clever mathematical manipulations? Or, in fact, they have some deep physical meaning?

In order to answer these questions we must recall that Physics is a natural science, an experimental science, and thus the answer has to be found in the experiments. In order to let Nature speak we shall make a brief reference to a new generation of microscopes.

3.3 – New generation of microscopes

Until recently the only way to look at the minute world was based on the Fourier type microscopes. The maximum theoretical resolution limit of a half-wavelength, for these apparatus, was established by Abbe, based on the Rayleigh the diffraction criteria. The basic principle underlying the working of these microscopes is a textbook example of Heisenberg uncertainty relations and consequently of the Fourier analysis.

In the middle of the 80's this picture changed drastically with the development of a new generation of microscopes that in practice violate Abbe's theoretical barrier. This new generation of microscopes is typified by the scanning tunneling electron microscope, developed by Bennig and Roher, [13] who received the Nobel prize for the discovery. The force field scanning apertureless microscope immediately followed, opening a new realm for micro-world imaging. These ideas were also extended to the optical domain. In 1984 Pohl *et al*.[14] were able to demonstrate the feasibility of the scanning apertureless optical microscope with a spatial resolution of $\ell/20$. Ten years later [15] it was possible, for them, to attain resolutions of $\ell/50$ or even better.

There are many types of scanning optical microscopes, as can be seen in reference [15]. Here we are interested only in the normal type not based in the tunneling effect. In these microscopes the light emitted by the sample is simply collected by the sensing probe as can be seen in Fig.7.

Basically this microscope is composed of a sensor, or light detector, a scanning system, not shown in the sketch, that controls the position of the probe, and a computer with a display device.

The light detector is in general made of a very thin optical fiber with the area of the tip much smaller than that of a section of a human hair. The light collected by the sensor is directed to a large electronic detector that converts the light intensity into an electrical pulse. In some cases the sensor extremity can be a simple very small solid state detector converting directly the light into an electric pulse. In any case the light that impinges on sensing area of the detector is converted into an electric pulse that feeds the computer.

The scanning system, not shown in the figure, is commonly composed of a cantilever which arms are made of piezo-electric quartz crystal. The electrical field applied to the arms of the scanning device controls the position of the sensor allowing a complete scanning of the whole sample.

The computer receives the electrical pulse from the sensor and, after a suitable processing of the

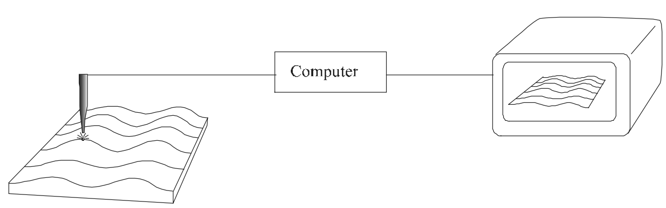


Fig. 7 – Scanning apertureless optical microscope.

information, produces a final amplified image of the sample that appears on the display. This image is the result of the following process: The sample is illuminated and its points diffuse light in all directions. The sensor, positioned over one point of the sample, collects some of the diffused light, and transforms it into an electric pulse proportional to the light intensity. The light intensity captured depends on the distance between the sensor tip and the surface of the sample, and also on the collecting area of the sensor tip. Thus, during the scanning the computer records the variation of light intensity along a scanning line. By scanning successive lines over the whole sample we finally obtain the desired amplified image. For example, if only one point of the sample diffuses light, in the image shown on the display one observes a continuous uniform background surface with only one discontinuity. This discontinuity represents the enlarged image of that single point.

The experimental resolution of the apparatus depends on the area of the sensor tip, the accuracy of the scanning device and the minimum distance between the sample and the sensor's extremity.

It may be argued that these kinds of microscopes only work with a large number of photons therefore are useless when dealing with a single photon. If this claim had any grounds, then it ought to be also applied to the usual Fourier microscope. Nevertheless it can easily be shown that, in principle, these two types of microscopes can operate with one single photon. When the single photon strikes a small particle (see Fig.7) and is diffused, it may happen that the small detector is not triggered. Then no measurement is made. If the diffused photon is seen by the detector, then its energy is converted into an electric impulse, and feeds the computer, which produces a high peak on the background. In the displayed image there appears a continuous background with a small spot, representing the particle. The dimension of the spot depend naturally on the overall experimental resolution of the apparatus.

For this kind of scanning apertureless optical microscope, experiments [15] have shown that it is possible to obtain spatial resolutions of $\ell/50$. The authors believe that the technique can be improved to attain resolutions of over $\ell/100$.

It is worth mentioning that in the entire process the photon always behaves like a particle, either when it strikes the particle or when it is caught at the detector. Therefore, this type of measurement cannot be described by the complementarity principle, and naturally, the usual rules, wave or particle, do not apply.

4. A Measurement Process that goes beyond Heisenberg's Uncertainty Relations

In order to see if there are some very special experimental situations not described by the complementarity principle thus violating Heisenberg's uncertainty relations, let us consider the well-known Heisenberg microscope. For the sake of methodical clarity we are going to study this classical example both with the common Fourier microscope and with the microscope of the new generation developed by Pohl.

In Fig. 8, the two examples are shown side by side, first for the prediction of the uncertainty in the momentum, then for the uncertainty in position, after the interaction of the photon with the small particle.

The uncertainty in momentum of the particle *M*, can be predicted in many ways, as can bee seen in the different textbooks on quantum mechanics [16,17,18]. Each author tries a slightly different approach, taking in account more or less factors, using horizontal, vertical or other illumination, but, at the end, they all arrive at the same formula. This is so because, as we shall see, the uncertainty in

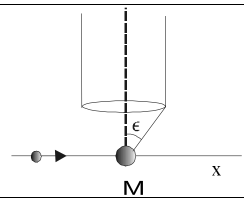
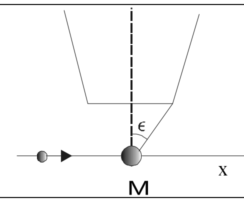
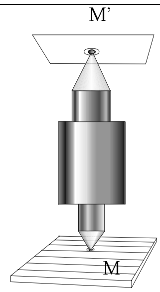
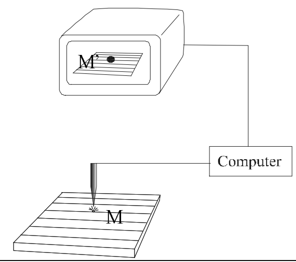
Fourier microscope	Scanning optical microscope
Uncertainty in momentum	
	
$\Delta p_x = 2\frac{h}{\lambda}$	
Uncertainty in position	
	
$\Delta x = \frac{\lambda}{2}$	$\Delta x = \frac{\lambda}{50}$
Predictions for the product of the uncertainties	
$\Delta x \Delta p_x = h$	$\Delta x \Delta p_x = \frac{1}{25} h$

Fig. 8 – Prediction for the product of the uncertainties for the Fourier microscope and for the scanning apertureless optical microscope

position for all the usual Fourier microscopes is fixed by Abbe's criterion. Therefore, since all the authors are eager to arrive at the Heisenberg's uncertainty relations they have to derive the same value for the uncertainty for the momentum, otherwise they could arrive at unwanted surprising results. Fig. 8 shows the region of detection of the two microscopes for a horizontal illumination side by side. The reasoning that leads to the prediction for the final uncertainty for the momentum, of the small microparticle particle M after the interaction with the impinging photon, is identical both for the Fourier microscope and for the scanning optical microscope. This is so, because the physical situation is precisely the same in the two cases. The photon strikes the small particle M and is diffused. After that is eventually caught, in the first case by the objective lens of the microscope and in the second case by the light sensor.

The predictions for the uncertainties for the position of the particle *M*, after the interaction, are completely different for the two microscopes:

In the case of the Fourier microscope the minimum separation with which it is possible to separate two points is given by Abbe's resolution rule, derived from the diffraction theory. For the best possible case, when the diffusing angle is near $\prec/2$, it corresponds to half wavelength $\ell/2$, naturally, as expected, the practical resolution of the usual microscopes is generally poorer.

For the non-Fourier microscope there is no mathematical formula, derived from first principles, for its resolution limit. Nevertheless one knows the practical experimental resolution, which certainly will be greater than the theoretical limit. For the microscope under consideration experiments have shown that the practical resolution is of the order of $\ell/50$.

The product of the two uncertainties for the Fourier microscope gives the usual Heisenberg relations. For the microscope of the new generation this product gives a violation of the usual uncertainty relations by a significant factor of 1/25.

This violation of the usual Heisenberg uncertainty relation, in this very special experimental situation, with the new generation scanning optical microscope, need not surprise us. A careful look at the experiment shows that this apparently surprising result follows naturally.

As can be easily seen the principle of complementarity describes perfectly well the behavior of the Fourier microscope. First, when the photon strikes the small particle M and is diffused by, it behaves like a particle, after which it goes through the microscope, producing a diffraction spot that behaves like a wave. In fact this experiment is, in the eyes of Bohr, one of the purest manifestations in Nature of the principle of complementarity.

In the second case, with the microscope of the new generation, the photon behaves like a particle in both cases, when it strikes the particle or when it is caught at the small light detector. Therefore, since in the entire measurement process the photon behaves as a particle, the dual nature of quantum systems, wave or particle, is not manifest. In such conditions, it is perfectly natural to expect that the complementary principle does not apply to this very particular kind of measurement. Since this experiment is not described by the complementarity principle it follows necessarily that Heisenberg's uncertainty relations also do not also apply.

6. Conclusion

It has been shown, in a manner similar to the one initially presented by Bohr, and since then used in most textbooks on quantum mechanics, that the uncertainty principle is not so general as claimed. The working of Pohl's non-Fourier microscope is not described by Bohr's complementarity principle since, throughout the measurement process, the photon behaves like a particle. Therefore it is natural to expect that Heisenberg's uncertainty relation, being a direct consequence, does not apply. Thus, the experimental discrepancy by a significant factor of 1/25 is perfectly natural. This shows that, in certain very special experimental settings, it is possible to predict, before an actual measurement process (interaction) takes place, the future uncertainties of position and momentum of a particle in a way that their product is less than h.

The stated facts allow us to maintain that, finally, after a so long struggle, experimental evidence clearly shows that the usual quantum mechanics has reached its limits of applicability and therefore must be replaced by a new more general theory. This new theory must, of course, include formally the old theory as a particular case, giving nonetheless a new meaning to the mathematical formalism.

References

[1] N. Bohr, (1928) – Como Lectures, *Collected Works*, Vol. 6, North-Holand, Amsterdam, 1985.

[2] Carl Popper, *The Quantum Mechanics and the Schism*, (Hutchinson, London, 1982)

[3] J. Scheer and M. Schmidt, Phys. Lett. A, 115, 322(1986); T. Götch, T. Coch, G. Luning, J. Scheer and M. Schmidt, in: *Problems in Quantum Physics*, Gdansk, eds. L. Kostro, A. Posiewnik, J. Pykacz and M. Zukowski, (World Scientific, 1987).

[4] P.A.M. Dirac, *Scientific Americam*, May 1963, Volume 208 Number 5 Pag. 49

[5] J.R. Croca, *Found. Phys*. 17(1987)971; In Quest of the Broglie Waves, in: *Waves and Particles in Light and Matter*, Ed. Alwyn van der Merwe and A. Garuccio, Plenum, 1994

[6] J.R. Croca, *Physics Essays*, 3(1990)71; On the Uncertainty Relations, in: *Problems in Quantum Physics*, Ed. Alwyn van der Merwe and M. Ferrero, Plenum, 1995.

[7] W. Heisenberg, *The Physical Principles of Quantum Theory*, (Dover, New York, 1930)

[8] J. Andrade e Silva, *Portugalie Physica*, 4(1996)257.

[9] C.K. Chui, *An Introduction to Wavelets*, Academic Press, New York, 1992.

[10] L. de Broglie, *The Current Interpretation of Wave Mechanics: A Critical Study*, (Elsevier, Amsterdam, 1969).

[11] P. Kumar, Wavelet Analysis for Geophysical Applications, *Rev. Geophysics*, 35(1997)385.

[12] J.R. Croca, Laboratory-Scale Test of de Broglie's Tired -Photon Model, *Apeiron*, 4(1997)41, (IJ)

[13] G. Benning and H. Roher, *Reviews of Modern Physics*, 59(1987)615.

[14] D.W. Pohl, W. Denk and M. Lanz, *Appl. Phys. Lett*. 44(1984)651.

[15] H. Heiselmann, D.W. Pohl, *Appl. Phys*. A, 59(1994)89.

[16] L. de Broglie, *Les Incertitudes d'Heisenberg el l'Interprétation Probabilistique de la Mécanique Ondulatoire*, (Gauthier-Villars, Paris, 1982)

[17] D. Bohm, *Quantum Theory*, Prentice-Hall, New York, 1955.

[18] L.I. Schiff, *Quantum Mechanics*, McGraw-Hill, New York, 1968.