Gravitational Charge in Newton's and Einstein's Theories

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Two existing representations of the nature of the gravitational charge are analysed. The mass plays this role in Newton's theory, the energy - in the general theory of relativity. Arguments in favor of the first representation are presented.

The questions, towards which the conversation leads below, have already been touched upon by the authors [1,2]. However, the fundamental importance of the problem makes necessary their special consideration.

Newton's theory

As known, the potentials of Newton (Φ) and Coulomb (ϕ) have an analogous form. One can say the same about the equations for the corresponding energies

$$E_e = e\phi, \tag{1}$$

$$E_g = m\Phi. \tag{2}$$

the mass m is named the gravitational charge for the indicated similarity (see, e.g.,[3]). This analogy becomes particularly significant if one takes into account that the mass (as an electric charge) is a 4-scalar (Lorentzian invariant) according to the relativity theory. Remind that this conclusion directly follows from the known Minkowski equation [4]

$$p^i = mu^i. \tag{3}$$

Here the 4-vector of energy-momentum figures at left $p^i = (E/c, \vec{p})$, and the 4- velocity (kinetic potential) at right, $i = 0,1,2,3$. Returning to eq. (2) and taking into account that the time component figures at left there, we conclude that the time component of a 4-vector (Φ^i).

The density of the gravitational charge (mass) is a source of the gravitational field for a continuous distribution of matter. We have also the density vector $\vec{J}$ of mass current and, in total—the 4-vector $\vec{J}$ of mass current (*i.e.*, with the 4-vector of electric current).

Example. A body of mass m resting at the point with gravitational potential Φ_1 has the total energy

$$E_1 = mc^2 + m\Phi_1. \tag{4}$$

As a result of a free fall to the point with potential Φ_2, where its energy is described by the equation,

$$E_2 = mc^2 + mv^2/2 + m\Phi_2 \tag{5}$$

it acquires the velocity

$$v = \sqrt{2(\Phi_1 - \Phi_2)} = \sqrt{2gh}. \tag{6}$$

Here g is the acceleration of gravity force and h, the difference of the corresponding heights.

Einstein's theory

The energy undertakes the role of the gravitational charge in the general theory of relativity (GTR) based on the representation of mass increase with velocity: "...the potential energy, which is equal to the potential energy of the 'gravity' mass E/c^2, corresponds to every quantity of energy E in the gravitational field"[5]. It should be emphasized that the statement of energy gravity is consequence of the mentioned 'pre-covariant' convictions "that the inert mass of a physical system must grow with increasing the total energy (in particular, with increasing the kinetic energy)"[6]. This statement is a direct consequence of the use of the non-covariant $E = mc^2$ (valid in the rest frame only) to moving bodies. As the representation of growing mass with velocity contradicts the Lorentz covariance, this, as far as one can judge, led Einstein to the conclusion "that there is no place for a satisfactory theory of gravity in the special theory of relativity"[6].

The increase of the gravitational charge in GTR connected with the rise of its rank (in the sense of tensor calculus: scalar $\to$ 4-vector) leads to the corresponding rise of the gravitational potential rank. We have a tensor of rank 2:

$$h_{ik}/2 = (g_{ik} - \eta_{ik})/2 \tag{7}$$

instead of a 4-vector, where g_{ik} is the metric tensor, η_{ik} is the "constant" Minkowski tensor. For this, in particular,

$$h_{00} = 2\Phi/c^2,\ h_{\alpha\alpha} \cong -2\Phi/c^2, \tag{8}$$

where $\alpha = 1,2,3$. As a result, for the total (covariant) energy of a body in the gravitational field, we have (see, e.g.,[7])

$$E_0 = mc^2 g_{00}\frac{dx^0}{d(c\tau)} = mc^2 \frac{dx^0}{\sqrt{g_{00}(dx^0)^2 - g_{\alpha\alpha}(dx^\alpha)^2}} \tag{9}$$

Using (8), we obtain

$$E_E \cong mc^2 + mv^2/2 + m(1 - 3v^2/2c^2)\Phi\,. \tag{10}$$

As seen, unlike relativized Newton equation (5), the third term depends not only on the body position but also on its velocity, *i.e.* it loses the properties of "classical" potential energy and becomes "potential-kinetic".

Example. In accordance with (10), the term $-3mv^2\Phi_2/2c^2$ should be added in equation (5). As a consequence of this, we have

$$v \cong \sqrt{2gh(1 + 3\Phi_2/c^2)} \tag{6'}$$

instead of known expression (6), *i.e.* the body velocity in a free fall turns out depend on the absolute value of the potential.

But as Einstein remarks himself [8]: "The observable laws of nature must not depend on the absolute values of the gravitational potential (or gravitational potentials)". Furthermore, the energetic factor $m(1 + v^2/2c^2)$ must, it would seem, figure before the potential according to the logic of Einstein's remark [5]. Similar difficulties do not arise in the Lorentz covariant theory of gravity; the gravitational charge - mass and the relativistic (4-vector) Newton potential lie at the basis of this theory[9].

References

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