The Twins, the Mesons, and the Paradox

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Einstein s special theory of relativity has left us with an enduring topic of concern in its prediction of time dilation. This prediction appears well validated in the apparently slow decay of rapidly moving unstable particles. But the prediction also apparently leads to the well-known Twin Paradox , which confounds ordinary logic. The present paper attempts to shed new light on this subject.

Key Words: Special Relativity Theory, Time Dilation, Twin Paradox

1. Introduction

Among the predictions of Special Relativity Theory (SRT; Einstein, 1905 and 1907) is the phenomenon of time dilation: a rapidly moving clock appears to run slow in the perception of stationary observers holding stationary clocks that they have synchronized by propagating light signals.

Time dilation is unique among SRT predictions, in actually having been observed directly in experiment. It is definitely known that cosmically generated high-speed mesons colliding with earth arrive less decayed than locally generated lower-speed mesons that decay for the same earth-judged time interval.

The concept of time dilation comes up in the context of uniform motion, as is appropriate to SRT. If we generalize only slightly, to allow a turn-around, we immediately generate a troubling paradox, known as the Twin Paradox.

The essence of the Twin Paradox is this: a pair of twins is separated, one of them staying at home and the other being sent on a journey at high positive velocity outbound, followed by high negative velocity inbound, ultimately to reunite with his brother. SRT seems to say that at every moment during this trip, clocks synchronized with the traveler can be observed to run slow as compared to clocks synchronized with the home-stayer. So SRT seems to say that at the reunion, the traveler should be younger than his brother. This conclusion confounds ordinary logic.

Any such paradox demands resolution. Many thinkers have supposed that a resolution of the Twin Paradox would involve the acceleration at turn-around, and would therefore lie beyond the scope of SRT. See, for example, Bridgman (1983), Møller (1952), Pauli (1958), Born (1962).

Bridgman argues that we have a paradox solely as the result of applying SRT in a situation that is inappropriate because it involves acceleration. He recognizes that denying the applicability of SRT in situations with acceleration essentially vitiates the practical utility of SRT. But then he says it is not necessary to pursue the topic further!

Møller actually carries out an analysis with acceleration, and shows that with infinite acceleration he gets to the same paradoxical clock discrepancy. Although he claims that this exercise solves the paradox, in fact it only *perpetuates* it, showing that in fact consideration of acceleration does *not* reconcile the clocks.

Pauli carries out an analysis with gravitation, and focuses on the slowing of clocks in a gravitational field. He quotes Einstein to the effect that this is the basis of the explanation for the Twin Paradox, although he does not show in detail how.

Born applies GRT to resolve the paradox, and shows that no paradox actually occurs in the case of straight-line journey out and return.

That result would seem to be decisive, except for one thing. In recent times, it has become possible to generate high-speed charged mesons and expose them to a magnetic field, so that they traverse circular paths. And the circulating mesons really *do* seem to age slowly.

Now if there is no Twin Paradox for a journey with an abrupt turn around, then there ought not be a Twin Paradox for a journey with a sinusoidal velocity profile, and then there ought not be a Twin Paradox for a journey with sinusoidal velocity profile in two dimensions, so there ought not be a Twin Paradox for a circular journey.

So what actually is true? That is the question for the present author, and for a number of other current authors discussed later.

The present author believes that the Twin Paradox results from insufficiently detailed and careful application of SRT. Just for example, the predicted clock discrepancy requires similar clock slowing on both the outbound and the inbound legs of the trip. This in turn requires Lorentz transformations from rest to outbound and from rest to inbound coordinate frames that differ only in the sign of velocity. This in turn requires that the three coordinate frames of interest, namely rest, outbound and inbound, all have their spatial origins coincident at the same time, namely at the beginning of the trip.

But this is *not* the actual case. Although the rest and outbound frames have their spatial origins coincident at the beginning of the trip, the rest and *inbound* frames should have their spatial origins coincident at the *end* of the trip.

There is no proof whatever that this offset of coordinate frames is not the whole cause of the Twin Paradox. Indeed, the present author believes that it is.

The present paper attempts to offer a clear and detailed analysis that removes the Twin Paradox. In place of a nasty paradox, we end up with a nice parable. The twins make perfect sense after all.

And so do the high-speed mesons in circular paths. There is a simple reason why they decay less than we expect.

The basis for explaining for both phenomena is a general mathematical model (Whitney, 1996). There it is shown that SRT is one member from a larger family of coordinate-transformation theories that are characterized by a single parameter describing the results of clock synchronization in different coordinate frames. The model is summarized in the next Section. Then its implications in regard to the twins and the mesons are developed.

2. Desynchronization Model

We postulate that, between two frames in relative motion, mutual synchronization is, for whatever reason, *not* generally achieved. So between clocks in different coordinate frames, there is generally a *desynchronization; i.e.* a discrepancy in displayed time at momentarily coincident positions.

For purposes of model development, let us assume that we have two coordinate frames with constant relative velocity in the *x* direction. Let us further assume that two clocks located at the two coordinate origins can maintain common Galilean time T.

To some, this second assumption, that relatively moving clocks can maintain common Galilean time, sounds contradictory to SRT. But we shall see later that it is not; the model developed will include SRT as a special case, as shown in the Appendix.

Let us further allow that other clocks located away from coordinate origins, although synchronized within their own respective coordinate frames, nevertheless exhibit frame-to-frame desynchronization.

If we follow Einstein just to the extent of assuming space-time linearity and homogeneity, we can model the desynchronization between the two coordinate frames by a single parameter, *i.e.* a slope. Let

$$\delta = \frac{\Delta t}{\Delta x} \tag{1a}$$

where Δt is time desynchronization between a clock at distance Δx from the origin of, say, Frame 1 and a clock at the origin of Frame 2.

Our overall model is that the two frames have mutual desynchronizations that vary oppositely with their respective x coordinates. Clocks at x_1 in Frame 1 and x_2 in Frame 2 read

$$t_1 = T + x_1\delta \quad \text{and} \quad t_2 = T - x_2\delta \tag{1b}$$

Figure 1 illustrates this situation. Frame 2 moves with respect to Frame 1 at Galilean speed V, defined in terms of the common Galilean time T.

We shall show below that any such desynchronization model exhibits all the qualitative features of SRT: finite observable velocity, length contraction, time dilation, *etc*.

Firstly, an observer in Frame 1 cannot directly observe that an object fixed in Frame 2 moves with Galilean speed V in Frame 1. Figure 2 shows how the time Δt required to go a standard distance Δx is reported erroneously as

$$\Delta t_1 = \Delta t + \Delta x\delta \tag{2a}$$

So the observer in Frame 1 sees speed

$$v = \Delta x / \Delta t_1 = \frac{\Delta x}{\Delta t + \Delta x\delta} = \frac{V}{1 + V\delta} \tag{2b}$$

This expression for observable speed v is quite interesting, because even though the Galilean V may range through indefinitely large values, the observable v will have a finite limit. This behavior is seen in SRT. Indeed, if δ is chosen to match SRT, then v is exactly the Einstinian speed, limited to c.

Similarly, a rod of length L_2 fixed in Frame 2 measures up short according to observers in Frame 1. After the front end of the rod passes a clock in Frame 1, its tail end continues to move forward while the rearward clocks in Frame 1 catch up. Figure 3 shows how the length L_1 measured in Frame 1 determines a flight time

$$\Delta T = L_1\delta \tag{3a}$$

and the flight time fits into the length relationship

$$L_2 = L_1 + V\Delta T = L_1 + L_1 V\delta \tag{3b}$$

So the value of L_1 is determined as

$$L_1 = \frac{L_2}{1 + V\delta} \tag{3c}$$

That is, the Frame-2 rod appears shortened by the factor $(1 + V\delta)$ when viewed in Frame 1. This effect is recognizably similar to the so-called length contraction that occurs in SRT. Note that it is a purely *kinematic* effect; the rod simply *appears* contracted.

Similarly again, a clock fixed in Frame 2 appears to run slow when tracked by observers in Frame 1. Figure 4 shows how the Frame-1 clocks that the Frame-2 clock passes are successively more and more advanced. The clock that reads t_1 when being passed is located at

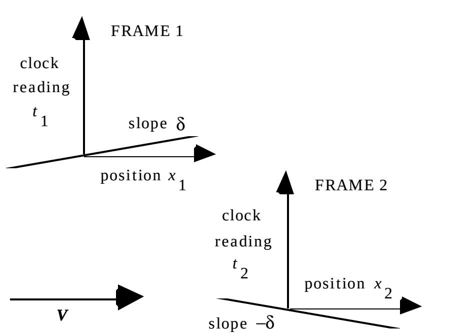

Figure 1. Two coordinate frames with mutual desynchronization parameter δ. Frame 2 moves at speed *V* with respect to Frame 1.

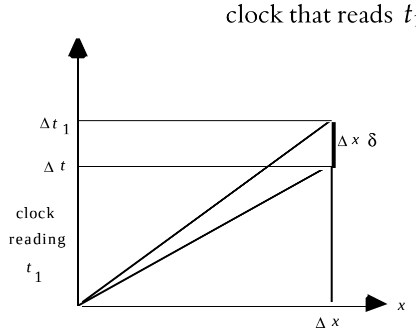

Figure 2. Galilean speed V is observed as speed $v = V / (1 + V\delta)$ because the time to travel Δx is seen as Δt_1 instead of Δt.

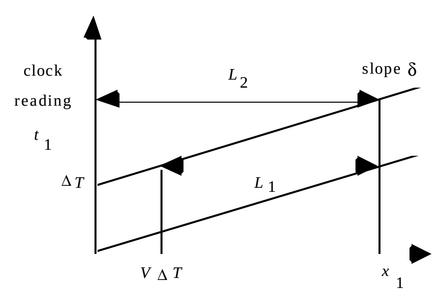

Figure 3. A rod of length L_2 fixed in Frame 2 appears shortened to length L_1 in Frame 1 where it moves.

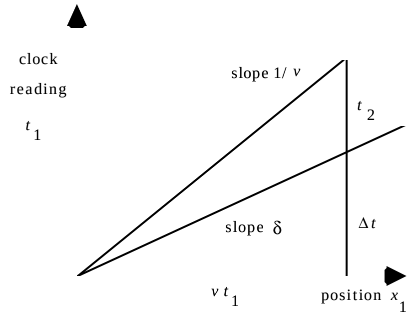

Figure 4. A clock fixed in Frame 2 appears to run slow when observed in Frame 1 where it moves.

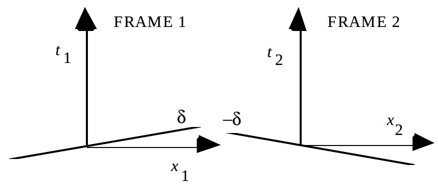

Figure 5a. Situation during outbound journey.

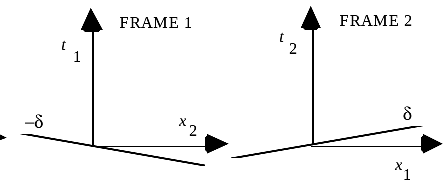

Figure 5b. Situation during return journey.

$$x_1 = vt_1 \quad (4a)$$

and so is advanced over t_2 by

$$\Delta t = x_1 \mathsf{d} = vt_1 \mathsf{d} \quad (4b)$$

So t_2 satisfies

$$t_2 = t_1 - \Delta t = t_1 - vt_1 \mathsf{d} = t_1(1 - v\mathsf{d}) \quad (4c)$$

That is, the Frame-2 clock appears to run slow by the factor $(1 - v\mathsf{d})$ when viewed in Frame 1. This effect is recognizably similar to the so-called time dilation that occurs in SRT. Note that it is a purely *kinematic* phenomenon; the Frame-2 clock does not *really* run slow.

We see now that a theory in which clock desynchronization is expressed by a single parameter d captures all the familiar qualities of SRT. The details of how d can be chosen to reproduce SRT exactly are included in the Appendix. But for understanding the Twin Paradox, it is sufficient just to focus on d as a concept rather than a specific formula.

3. Resolving the Twin Paradox

Any theory with a non-zero d parameter seems to present a Twin Paradox, because any such theory presents apparent slowing for a clock in motion. The argument is: if the traveler s clock runs slow on the outbound leg of a journey, and it also runs slow on the inbound leg of a journey, then when twins reunite, the traveler must be younger than his twin. But this is not so.

There is a clue about the twins in Section 2: even though mutual synchronization between Frames is not possible, and clocks compared to other clocks in passing appear to run slow, nevertheless the clocks at the two coordinate-frame *origins* never deviate from the common time T. This clue leads to the following parable.

The twins are born to wealthy parents who equip them at birth with numerous silver rods and clocks. As children, the twins construct their own personal coordinate frames using the rods and clocks. Twin 1 has Frame 1 and Twin 2 has Frame 2. Each Frame consists of a spatial grid of rods with clocks attached to the rod-ends and synchronized to a master clock that the Twin in charge keeps on his person. Servants being no problem, each Twin commandeers some of them from the household to form a cadre of observers that he posts throughout his coordinate frame.

The twins grow to young adulthood. They have quite different personalities. Twin 1 only wants to make good in the family business, while Twin 2 wants to experience adventure. Twin 2 embarks on a journey, leaving home at velocity V . That means he *changes his state of motion*. Having done so, he finds that he has *spoiled the synchronization* of the clocks he has deployed throughout his coordinate frame. He has to *re-do* it right away. After that, his clocks *no longer match* the clocks his brother is maintaining.

After some time goes by, there is some distance accumulated between Twin 1 and Twin 2. The situation between the two Twins is described by Fig. 5a. Like Fig. 1, Fig. 5a is symmetric in regard to the two Frames, even though the histories of the two Twins are *not* symmetric. This is because the Figure depends only on their relative velocity, not how they came to *have* that relative velocity.

Observe that the Frame-2 clock currently passing Twin 1 is way ahead of Twin 1 s master clock, and the Frame-1 clock currently being passed by Twin 2 is way ahead of Twin 2 s master clock. Each Twin, relying only on his own distant observers, thinks the other Twin is aging slowly. But of course it is only an illusion, caused by the reliance on distant observers and by the synchronization procedures.

Twin 1 of the placid disposition doesn t care, but Twin 2 feels alarmed. He reads the situation the other way around: that he himself is aging fast while his brother is aging normally. He wonders if he is ill. He decides to go home and seek medical attention. So he reverses direction. That means he changes his state of motion *again*. That means he spoils the synchronization of his clocks *again*. That means he has to re-do the synchronization *again*.

Again Twin 2 ends up with clocks that do not match those of his brother. But now the discrepancies are reversed. The situation between the two brothers is described by Fig. 5b.

Note that in this new situation, both V and d have reversed, so the product $V\mathsf{d}$ has *not* changed. So for this portion of the journey, Twin 2 continues to think Twin 1 ages slowly (and of course Twin 1 thinks the same about Twin 2).

There is one thing peculiar though. Now, the Frame-2 clocks passing near Twin 1 are way *behind* Twin 1 s master clock, and the Frame-1 clocks passing Twin 2 and way *behind* Twin 2 s master clock. So each brother has distant observers telling him that the other brother has suddenly aged a lot. How does this happen? Does anybody *really* age like this?

To track the answers down, we need to look at the situation during *transition*, after Twin 2 has started to turn around, but before he has finished resynchronizing all his clocks. We suppose that as he accelerates continuously, he sends out resynchronization signals continuously, at each instant corresponding to his current state of motion, and that these signals propagate outward from him at some finite speed, say c . This means that his input to the desynchronization between Frames 1 and 2 is described not by a single line, but rather by line segments, as illustrated by Figure 5c for Frame 2. (The complementing situation exists in Frame 1.)

Two waves of resynchronization are propagating from the origin of Frame 2, one to the right for positive x_2, and one to the left for negative x_2.

Now in this situation, where the resynchronization waves have not yet made their way through the system, each brother temporarily sees the master clock of the other brother appear to run fast. At first it just seems to run the same amount fast as it previously seemed to run slow. That is, its apparent rate is fast by the factor $(1+v\mathrm{d})$ instead of slow by the factor $(1-v\mathrm{d})$. This enhanced clock rate persists over fraction b of the return trip. Suppose L is the separation at turn-around; then the clock reading accumulates correection $\mathrm{b}L\mathrm{d}$.

Then, as the steep part of the resynchronization wave propagates by, the clock seems to run *really* fast. Indeed, for infinite acceleration at turn-around, it would seem to run infinitely fast. But quantifying the rate is unimportant; only the accumulated jump really matters. Ultimately, the speeding clock seems to end up just as much ahead as it had previously seemed behind. That is, the clock jumps by $2x\mathrm{d}$, where x is the remaining separation, $(1-\mathrm{b})L$.

But all this is only an illusion. It is all caused by reliance on distant observers and by the synchronization procedures.

Nevertheless, Twin 2 believes his distant observers, that his brother has first gradually and then suddenly aged, and he thinks the whole family is in trouble. It upsets him terribly, and through his distant observers, he anxiously monitors the further aging of his brother. He is relieved to see that subsequent aging looks slow, but again he wonders if that just means that he himself is aging fast.

Finally, the anxious prodigal Twin makes it home. (He comes to a stop yet *again* changing his state of motion and *again* resynchronizing his clocks.) The brothers embrace, and then compare master clocks. Guess what: they read the *same*. There has been *no* differential aging at all!

A full tabulation of all clock discrepacies accumulated through the adventure runs as follows:

1) On the outbound path, from start to turn-around:

$$\mathrm{d}t_o = -L\mathrm{d} \tag{5a}$$

2) On the beginning of the inbound path, from turn-around to resynchronization:

$$\mathrm{d}t_{i1} = +\mathrm{b}L\mathrm{d} \tag{5b}$$

3) Jump as resynchronization is accomplished:

$$\mathrm{d}t_j = +2(1-\mathrm{b})L\mathrm{d} \tag{5c}$$

4) On the end of the inbound path, from resynchronization to reunion:

$$\mathrm{d}t_{i2} = -(1-\mathrm{b})L\mathrm{d} \quad (5c)$$

The sum of all accumulated clock discrepancies is *zero*. *Clearly*, they were all only illusions!

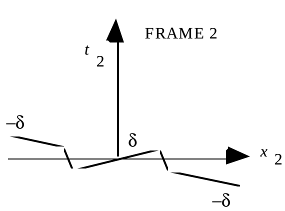

Figure 5c. Situation during transition.

4. What About The Mesons?

Recall that when unstable, fast-moving charged mesons are exposed to a magnetic field, and so made to traverse a circular path, they return to their point of origin having decayed less than would be predicted for an orbit time of $t = 2\mathrm{p}R/v$. This discrepancey has been taken as illustration of a real-life Twins Paradox. But in fact it is not.

The explanation for the apparent clock discrepancy lies in the distinction between observable velocity v (max limited by Eq. (2b)) and Galilean velocity V (not limited). Observable v is a ratio $\Delta x/\Delta t$ where Δt is the difference between readings on two different clocks separated by Δx. By contrast, Galilean V is a ratio $\Delta x/\Delta T$ where ΔT is the difference between two readings on *one* clock.

This Galilean V is not observationally accessible in straight-line motion, but it becomes *the* accessible thing in circular motion. One has only to wait for the mesons to return to a given point, and check the time. By contrast, measuring v involves multiple observers and multiple clocks, a lot more trouble.

However, *neither* v nor V is actually measured. Decay is measured. The mesons truly go around the circle at speed V, larger than speed v. So they take a shorter time than predicted based on speed v, and decay less than expected based on speed v. Hence we have the observed data concerning decay.

5. Discussion

We have analyzed the Twin Paradox in terms of a simple parameterized model describing the mutual desynchronization between clocks belonging to and synchronized within different coordinate frames. We have claimed that this model is valid inasmuch as all of SRT can be captured by it. The main text supports this claim with results pertaining to finite observable velocity, time dilation and length contraction. The Appendix supports this claim further with detailed specialization of the model to SRT.

With the desynchronization model, it seemed easy to articulate a resolution of the Twin Paradox. Now we have a parable instead. While Twin 2 travels, each twin thinks the other ages slowly. When Twin 2 turns around, it takes a while for him to complete a resynchronization of clocks in his coordinate frame. While this process goes on, each brother appears to the other to age quickly, first making up and then reversing the discrepancies due to previous apparent slow aging. In the end, all apparent slowings and speedings cancel out. There is no differential aging. It was all an illusion. Even the charged mesons in circular motion present no contradiction to this conclusion.

The present analysis alters the classical view that the Twin Paradox can be resolved only by going beyond the context of SRT, inasmuch as acceleration is involved. Here, the resolution for the Twin Paradox is shown to lie, not in the acceleration *per se*, but rather in the finite signal propagation speed that governs the *accommodation* to that acceleration. Where focusing on acceleration would put the explanation beyond the scope of SRT, focusing instead on finite propagation speed puts it well within the scope of SRT.

The situation prompting this analysis, as described in the Introduction, was so unsatisfactory that a number of other authors have also been struggling with it in recent times. Some of their ideas can be compared and contrasted with those of the present author.

Phipps (1995) has speculated that Twin Paradox implies a relative velocity asymmetry, and so constitutes a real contradiction to the fundamental assumptions of SRT. By contrast, the present analysis says the Twin Paradox does *not* in fact occur, so it can t very well challenge SRT. The reader should note, however, that the present author (1996) *does* believe that SRT is indeed vulnerable to *other* challenges.

Martin (1994) has argued for a regrounding of SRT in Galilean concepts as a way of resolving the Twin Paradox. Much of his theory is entirely consistent with that of the present author. The only difference is that the present theory dispenses with the specific Lorentz factor $(1-\beta^2)^{1/2}$ and uses the general shrink factor $(1-v\delta)$, and has no distinction between Galilean velocity and proper velocity.

Percival (1995) has demolished a full catalog of fragile arguments purporting to reconcile the Twins Paradox. He concludes that a real resolution requires an extension to SRT. His extension is along the lines of Lorentzian absolute space and velocity-dependent physical effects. By contrast, the present author identifies observational illusions as the cause of any apparent paradox. The argument detailed here has, however, never been addressed within the context of the standard descriptions of SRT. So to that extent, it does represent an extension of standard theory, although not the same sort of extension as Percival speaks of.

Appendix I: Specialization to SRT

Let us revisit Eq. (2b) as simplified to:

$$v=\frac{V}{1+V\delta} \tag{6a}$$

This rearranges to

$$v+vV\delta=V \tag{6b}$$

or

$$V-Vv\delta=v \tag{6c}$$

or

$$V=\frac{v}{1-v\delta} \tag{6d}$$

This is an expression recognizably similar to the definition of covariant velocity in SRT.

Furthermore, the combination of (6a) and (6d) implies that generally

$$(1+V\delta)(1-v\delta)\equiv 1 \tag{7a}$$

For convenience we can define the Spread factor

$$S=1+V\delta \tag{7b}$$

for the length-contraction denominator in (3c). The corresponding shrink factor

$$s=1/S=1-v\delta \tag{7c}$$

captures the time-dilation slowing in (4c). The Spread factor S is recognizably similar to the factor

$$\gamma=1/\sqrt{1-\beta^2} \tag{7d}$$

with $\beta=v/c$, which occurs in SRT, and of course the shrink factor s is similar to $1/\gamma$.

The relationships between V, v, S and s suggest a variety of equivalent expressions for δ. We have

$$\delta=\frac{V-v}{Vv} \tag{8a}$$

or

$$\delta=\frac{S-1}{V} \tag{8b}$$

or

$$\delta=\frac{1-s}{v} \tag{8c}$$

Using any of these expressions, we can see that SRT corresponds to the special case

$$\delta_{SRT}=\frac{\gamma-1}{V}=\frac{1-\gamma^{-1}}{v} \tag{8d}$$

Thus SRT is just one member of a family of possible theories whose members are characterized by the desynchronization slope parameter δ. The larger family includes SRT, with $\delta=\delta_{SRT}$, and it includes classical Galilean theory, with $\delta=0$, and it includes possibly other members with δ equal to neither of these values.

Suppose we are given Frame-1 coordinates x_1 and t_1, and wish to obtain corresponding Frame-2 coordinates x_2 and t_2. For this we need a coordinate transformation method.

Let us consider a simple Galilean transformation expressed in terms of the coordinate-origin time T that occurs in the desynchronization model:

$$T=t_1-x_1\delta=t_2+x_2\delta \tag{9a}$$

The Galilean transformation for space is simply

$$x_2=x_1-VT \tag{9b}$$

The corresponding transformation for the desynchronized time away from the coordinate origin is

$$t_2=t_1-(x_1+x_2)\delta \tag{9c}$$

These three equations can be reduced to two equations:

$$\begin{aligned} x_2 &= x_1-V(t_1-x_1\delta)=(1+V\delta)x_1-Vt_1 \\ &= Sx_1-Vt_1 \end{aligned} \tag{10a}$$

and

$$\begin{aligned} t_2 &= t_1-x_1\delta-(1+V\delta)x_1\delta+Vt_1\delta \\ &= (1+V\delta)t_1-\delta(2+V\delta)x_1 \\ &= St_1-\delta(S+1)x_1 \end{aligned} \tag{10b}$$

We can in general define a vector x_1, ct_1 which is transformed to a vector x_2, ct_2 by a matrix

$$M=\begin{bmatrix} \partial x_2/\partial x_1 & \partial x_2/\partial ct_1 \\ \partial ct_2/\partial x_1 & \partial ct_2/\partial ct_1 \end{bmatrix} \tag{11a}$$

Equations (9a) and (9b) imply

$$M = \begin{bmatrix} S & -V/c \\ -c\delta(S+1) & S \end{bmatrix} \quad (11b)$$

Compare this to the matrix for the Lorentz transformation (LT) that occurs in SRT:

$$M_{\mathrm{LT}} = \begin{bmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{bmatrix} \quad (11c)$$

It seems quite interesting that, despite the fact that we started with a Galilean concept for coordinate transformation, matrix M of (11b) has most of the properties of matrix M_{LT} of (11c). Note that:

1) The diagonal elements in M are equal:

$$\partial x_2 / \partial x_1 = \partial ct_2 / \partial ct_1 = S = 1 + V\delta \quad (12a)$$

2) Matrix M has unit determinant, and inverse formed by reversing the sign of the off diagonal elements:

$$\begin{aligned} Det(M) &= \tfrac{1}{2} Trace(MM^{-1}) = S^2 - V\delta(1+S) \\ &= (1+V\delta)^2 - V\delta(2+V\delta) \equiv 1 \end{aligned} \quad (12b)$$

3) The upper right element of M divided by the lower right element of M is the observable velocity normalized by c:

$$\frac{\partial x_2 / \partial t_1}{\partial t_2 / \partial t_1} = \frac{\partial x_2}{\partial t_2} = \frac{V}{cS} = \frac{V}{c(1+V\delta)} = \frac{v}{C} \quad (12c)$$

In fact, the only departure from M_{LT} is that the off-diagonal elements of M are not manifestly equal. Only if we insert the δ_{SRT} from (8d) can we establish the equality:

lower left matrix element

$$\begin{aligned} &= -c\delta(S+1) \\ &= -\frac{\gamma - 1}{\gamma\beta}(\gamma + 1) \quad (12d) \\ &\equiv -\gamma\beta = -V/c \end{aligned}$$

≡ upper right matrix element

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