Extended Electromagnetic Theory, Angular Momentum and the $B^{(3)}$ Field

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An extended electromagnetic theory with space charge in vacuo *has been applied to axisymmetric wave modes. The solutions predict that the photon should have a magnetic field component in the direction of propagation, a small magnetic moment, no net electric charge, a small but non-zero rest mass and the angular momentum of a boson. These results are reconciled with the approach of Maxwell equations with non-zero conductivity* in vacuo*. Here, the axial magnetic field component becomes reconcilable with the* $\mathbf{B}^{(3)}$ *field as proposed by Evans and Vigier.*

Introduction

In a monograph by Evans and Vigier [1],as well as in related investigations [2-4] new discoveries in photon physics have been presented, leading to an associated magnetic field component, $\mathbf{B}^{(3)}$, in the direction of propagation, and to a small but non-zero rest mass of the photon. These predicted properties of the photon can be shown to be connected with the earlier proposed Lorentz invariant extension of Maxwell s equations, which is based on the hypothesis of a non-vanishing field divergence *in vacuo* [5-7], and with non-zero conductivity *in vacuo* [8]. As one of the consequences of this extended theory, electromagnetic space-charge (EMS) waves should exist *in vacuo*. Consequently, the axisymmetric nature of this wave mode should be of special interest to photon physics [7], and will thus be considered here, and be shown to be related to the theory of Evans and Vigier [1].

Longitudinal and Nontransverse Electromagnetic Waves

Here we shall concentrate on two modified forms of Maxwell s equations *in vacuo*, based on a nonzero electric field divergence and on nonzero conductivity.

2.1 Maxwell Equations Modified by Non-zero Electric Field Divergence

The extended field equations *in vacuo* become [5-7,9]

$$curl\mathbf{B} = \mathbf{j} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (1)$$

$$curl\mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} \quad (2)$$

$$\mathbf{j} = \bar{\rho}\mathbf{C} \quad (3)$$

in S.I. units. Here $\mathbf{B}$ and $\mathbf{E}$ are the magnetic and electric fields, $\mathbf{j}$ is the current density and $\bar{\rho}$ is the charge density arising from a non-zero electric field divergence *in vacuo*. Then

$$div\mathbf{E} = \frac{\bar{\rho}}{\varepsilon_0} \quad (4)$$

$$div\mathbf{B} = 0\ ,\ \mathbf{B} = curl\mathbf{A} \quad (5)$$

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t} \quad (6)$$

Now,

$$\Box \cdot (\mathbf{j}, ic\bar{\rho}) = 0$$

when

$$\Box = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, \frac{\partial}{ic\partial t}\right)$$

where $(\mathbf{j}, ic\bar{\rho})$ is a four vector. The potentials $\mathbf{A}$ and ϕ are derived from the sources $\mathbf{j}$ and ρ through the above equations which yield

$$\Box^2\left(\mathbf{A}, \frac{i\phi}{c}\right) = -\mu_0(\mathbf{j}, ic\bar{\rho})\,. \quad (7)$$

When combined with Lorentz condition, this gives

$$div\mathbf{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t} = 0 \quad (8)$$

It should be observed that equation (7) is a Proca type equation. Again, Lorentz invariance leads to

$$\mathbf{j}^2 - c^2\bar{\rho}^2 = const = 0 \quad (9)$$

where

$$j^2 = |\mathbf{j}|^2 and\ \mathbf{C}^2 = c^2$$

The following limiting cases now may arise:

(a) When $div\mathbf{E} = 0$ and $curl\mathbf{E} \neq 0$, the result is a conventional transverse electromagnetic wave, henceforth denoted as the EM wave .
(b) When $div\mathbf{E} \neq 0$ and $curl\mathbf{E} = 0$, a longitudinal purely electric space-charge wave arises, here denoted as the S wave .
(c) When both $div\mathbf{E} \neq 0$ and $curl\mathbf{E} \neq 0$, a hybrid nontransverse electromagnetic space charge wave appears,

here denoted as an EMS wave . The S wave can be considered as a special degenerate form of EMS wave. It can be shown that both the EM and S waves are separate modes which cannot be derived from the EMS mode by simple asymptotic processes.

A general form of electromagnetic field can be obtained from a superposition of various EM, S and EMS modes. It should be observed that the EMS modes can have different field vectors $\mathbf{C}$.

2.2 Maxwell Equations and Non-zero Conductivity *in vacuo*

The Maxwell equations *in vacuo* have also been modified by assigning a small non-zero conductivity $(\sigma \neq 0)$. This gives rise to a displacement current *in vacuo* as already observed by Bartlet *et al.* [10]. This type of displacement current is conceivable in the extended Maxwell equations with a non-zero space-charge *in vacuo*, as pointed out above. In the latter case, however, the vacuum seems to be dissipative in nature in contrast to the usual Maxwell vacuum. Maxwell equations with nonzero conductivity can be written as

$$div\mathbf{E} = 0\,,\; curl\mathbf{H} = \sigma\,\mathbf{E} + \varepsilon_0 \chi_e \frac{\partial \mathbf{E}}{\partial t} \tag{10}$$

$$div\mathbf{H} = 0\,,\; curl\mathbf{E} = -\mu_0 \chi_m \frac{\partial \mathbf{H}}{\partial t} \tag{11}$$

with

$\mu_0 =$ vacuum permeability constant,

$\chi_e =$ relative dielectric constant and

$\chi_m =$ relative permeability constant.

Here the four current

$$j = (\mathbf{j}, j_0) \text{ where } \mathbf{j} = \sigma\,\mathbf{E}\,,\; j_0 = 0 \tag{12}$$

Again

$$curlcurl(\mathbf{E}) = -\nabla^2 \mathbf{E} \tag{13}$$

which together with Maxwell equations gives

$$\nabla^2 \mathbf{E} = -\varepsilon_0 \chi_e \chi_m \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} + \sigma \mu_0 \chi_m \frac{\partial \mathbf{E}}{\partial t} \tag{14}$$

This equation is not time reversal invariant. The second term on the right hand side indicates that there will be a dissipation of energy during the propagation of a photon. It is to be noted that in this approach

$$div\mathbf{E} = 0$$

and

$$curl\vec{\mathbf{E}} \neq 0$$

appears to be the limiting case (**a**) of the former approach. But here we can obtain a non-zero photon mass associated with the longitudinal wave. On the other hand, in the former approach, it implies only a transverse electromagnetic wave denoted as the EM wave. But if we try to formulate the extended E-B-P theory in the Maxwell vacuum with $\sigma \neq 0$ in a fully relativistic manner, and make it gauge invariant, it is necessary to introduce the concept of space-charge *in vacuo*, as $j = (\mathbf{j}, j_0)$ where $j_0 \neq 0$ and j_0 is related to $\bar{\rho}^{(2)}$. Then we have both $div\mathbf{E} = 0$ and $curl\mathbf{E} \neq 0$. This is nothing but the EMS wave as indicated in the former approach, *i.e.*, in the framework of electromagnetic theory with non-zero electric field divergence.

Axisymmetric Wave Modes, Angular Momentum and Nonzero Photon Mass

We now study axisymmetric wave modes varying as $\exp[i(-\omega t + kz)]$ in a cylindrical frame (r, ϕ, z) of reference when ϕ is an ignorable coordinate [7,9]. In this frame the velocity vector is assumed to have the form

$$\mathbf{C} = c(0, \cos\alpha, \sin\alpha) \tag{15}$$

with a constant α . We further define the operators

$$D_1 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \tag{16}$$

$$D_2 = \frac{\partial}{\partial t} + c(\sin\alpha)\frac{\partial}{\partial z} \tag{17}$$

$$D_3 = \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \tag{18}$$

The basic equations then reduce to

$$\begin{aligned} & D_2(div\mathbf{E}) = 0 \\ & D_2\left(D_1 - \frac{1}{r^2}\right)E_r = 0 \\ & D_3\left(D_1 - \frac{1}{r^2}\right)E_\phi = \frac{1}{c}(\cos\alpha)\frac{\partial^2}{\partial z \partial t} D_1 E_z \end{aligned} \tag{19}$$

This set of equations corresponds to two branches of solutions:

1. When $D_2 E_r$ and $D_2 E_z$ are different from zero, this represents a classical electromagnetic (EM) mode with vanishing electric field divergence.
2. When $D_2 E_r = D_2 E_z = 0$, this branch represents an electromagnetic space-charge (EMS) mode with non-zero electric field divergence *in vacuo*.

For axisymmetric EMS modes of the second branch, we have $div\mathbf{E} \neq 0$; the dispersion relation becomes

$$\omega = kc\sin\alpha \tag{20}$$

and the above equation (19) takes the form

$$\begin{aligned} & \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - k^2(\cos\alpha)^2\right]E_\phi \\ & = -(tg\alpha)\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - k^2(\cos\alpha)^2\right]E_z \end{aligned} \tag{21}$$

We now introduce the function

$$\begin{aligned} G_0 \cdot G &\equiv E_z + (\cot\alpha)E_\phi \\ G &\equiv R(\rho)\exp[i(-\omega t + kz)] \end{aligned} \tag{22}$$

where G_0 is an amplitude factor and $R(\rho)$ is dimensionless. The operator

$$D = \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} - \xi^2(\cos\alpha)^2, \; \xi = kr_0 \tag{23}$$

is further defined. The electric field components then become

$$\begin{aligned} E_r &= -iG_0\xi\frac{\partial}{\partial\rho}\left[\left(1-\rho^2 D\right)\right] \\ E_\phi &= G_0(tg\alpha)\rho^2 DG \\ E_z &= G_0\left(1-\rho^2 D\right)G \end{aligned} \tag{24}$$

Similarly the magnetic field components can be written as

$$\begin{aligned} B_r &= -\left[\frac{1}{c\sin\alpha}\right]E_\phi \\ B_\phi &= \left[\frac{1}{c\sin\alpha}\right]E_r + i\left[\frac{1}{\xi c\sin\alpha}\right]\frac{\partial E_z}{\partial\rho} \\ B_z &= \left[\frac{1}{\xi c\sin\alpha}\right]\left(\frac{\partial}{\partial\rho}+\frac{1}{\rho}\right)E_\phi \end{aligned} \tag{25}$$

Consequently, the function G can be regarded as a generating function from which the entire electromagnetic field of an elementary axisymmetric EMS mode can be determined. It is possible to superimpose elementary EMS modes of different wave numbers k in the z direction, to form a travelling wave packet by means of Fourier analysis, and for such a packet to have extensions also in the positive and negative z directions. Here this is assumed to be the case, and $\mathbf{E}$ and $\mathbf{B}$ to be the resulting electric and magnetic fields which then become finite at $r = 0$ and zero at $r \to \infty$ and $z = \pm\infty$. For a packet of this kind to represent a limited spectral line width and a well-defined frequency $\omega = kc($ $)$, the spatial exte - sion of the packet in the -direction then has to be much larger than its wave length $2\pi/k$. With these assumptions we first turn to the total integrated charge. With $\mathrm{d}V$ and $\mathrm{d}S$ as volume and surface elements, it becomes

$$q_0 = \varepsilon_0\int_0^\infty (\nabla\cdot\mathbf{E})\,\mathrm{d}V = \varepsilon_0\int_0^\infty \mathbf{n}\cdot\mathbf{E}\,\mathrm{d}S = 0 \tag{26}$$

when the electric field vanishes at large distances from the packet. We next consider the integrated magnetic moment which becomes

$$M_0 = \frac{1}{2}\varepsilon_0 c(\cos\alpha)\int r(\nabla\cdot\mathbf{E})\mathrm{d}V \tag{27}$$

and does not generally disappear. Turning further to the energy density of the electromagnetic field [7]

$$w = \frac{1}{2}\left(\bar{\rho}\phi + \mathbf{j}\cdot\mathbf{A}\right) = \frac{1}{2}\bar{\rho}\left(\phi + \mathbf{C}\cdot\mathbf{A}\right) \tag{28}$$

where $\mathbf{A}$ and ϕ are the magnetic and electric potentials, the equivalent mass in the frame which allows the propagation of the wave packet becomes

$$m_0 = \frac{2\pi}{c^2}\int_0^\infty\int_{-\infty}^{+\infty} rw\,\mathrm{d}r\,\mathrm{d}z \tag{29}$$

Likewise the integrated angular momentum becomes

$$s_0 = \left[2\pi\frac{\cos\alpha}{c}\right]\int_0^\infty\int_{+\infty}^{-\infty} r^2 w\,\mathrm{d}r\,\mathrm{d}z \tag{30}$$

For a wave packet with finite characteristic dimensions r_0 and z_0 in the r and z directions, and with an energy density w proportional to $\overline{G}_0^2$ as obtained from the fields $(\mathbf{E},\mathbf{B})$ and the generating function G, we introduce the normalized quantities $\rho = r/r_0$, $\zeta = z/z_0$ and $w(r,z) = \overline{G}_0^2 W(\rho,\zeta)$. Then

$$\begin{aligned} m_0 &= \left(\frac{2\pi}{c^2}\right)r_0^2 z_0\overline{G}_0^2 J_m \\ J_m &= \int_0^\infty\int_{-\infty}^{+\infty}\rho W\,\mathrm{d}\rho\,\mathrm{d}\zeta \\ s_0 &= \left[2\pi\frac{\cos\alpha}{c}\right]r_0^3 z_0\overline{G}_0^2 J_s \\ J_s &= \int_0^\infty\int_{-\infty}^{+\infty}\rho^2 W\,\mathrm{d}\rho\,\mathrm{d}\zeta \end{aligned} \tag{31}$$

where $\overline{G}_0$ thus stands for an equivalent amplitude factor. Assuming the present system to behave as a boson, we have

$$s_0 = \frac{h}{2\pi} \tag{32}$$

with h denoting the Planck constant and

$$m_0 r_0 = \frac{hJ_m}{2\pi c\cos\alpha}J_s \tag{33}$$

When requiring m_0 and r_0 to become small, forms of W have to be found which make the ratio J_m/J_s small. That such functions are likely to exist is illustrated by the simple form $\rho^\gamma\exp(-\rho)$. Now let us consider the energy content of a wave packet of narrow line width as well as the asymptotic limit of a monochromatic wave of infinite axial extension and a discrete frequency ω. With the phase and group velocities $v = c\sin\alpha$, the energy relations due to Planck and Einstein then yield

$$\frac{hw}{2\pi} = m_0 c^2\left[1-\frac{v^2}{c^2}\right]^{-\frac{1}{2}} \tag{34}$$

This leads to

$$\omega = \frac{cJ_m}{J_s r_0(\cos\alpha)^2} \tag{35}$$

With $\cos\alpha \ll 1$ and $J_m/J_s \ll 1$, the frequency ω thus becomes relevant to the present concept of a propagating wave packet with a defined frequency and a small radius r_0. It is to be noted that the above conditions imply m_o and $m_0 r_0$ become small, and at the same time the condition of a quantized angular momentum is satisfied. Again it should be observed that the introduction of a non-zero but small conductivity *in vacuo*, as described above, introduces a small but finite photon rest mass. The phase and group velocities have been calculated in this framework [4]. In this approach the phase velocity is also less than the speed of light. Now the approach of a non-zero electric field divergence *in vacuo* can be reconciled with this concept if we choose the angle α very close to $\pi/2$, *i.e.*, where $0 < \cos\alpha \ll 1$. Then the phase velocity $c\sin\alpha$ also becomes smaller than c.

Summary and Conclusions

When applied as a model for the photon, the axisymmetric EMS wave packet and related elementary modes demonstrate the following features:

(1) The axisymmetric EMS modes allow for physically acceptable solutions which are finite at the axis and vanish at infinity. This is not the case for conventional axisymmetric electromagnetic modes.

(2) The total integrated electric charge of the wave packet vanishes. It is then not necessary to rely on the idea that the photon is its own antiparticle.

(3) There is a non-zero magnetic moment and an associated magnetic field component in the direction of propagation, obtained from the introduction of the space-charge current density in the basic equations. For a wave packet the corresponding axial magnetic field component, which results from the superposition of the elementary modes of the Fourier spectrum, becomes reconcilable with the $\mathbf{B}^{(3)}$ field of Evans and Vigier. Due to the small expected dimensions of the photon, this local axial magnetic field can become strong even when the magnetic moment turns out to be quite small [7,9].

(4) The present axisymmetric EMS solutions require the phase and group velocity to become slightly less than c. In this way these solutions allow for a very small but non-zero rest mass of the photon. This small rest mass, and the related electromagnetic energy density in a frame following the motion of the wave packet, generates an angular momentum (spin) when combined with ϕ component of the velocity vector $\mathbf{C}$ of the space-charge current density. Consequently, a physically plausible picture is obtained of the origin of spin for the photon. The previous discussion further shows how the spin quantum condition can be satisfied even when the rest mass and the radial dimensions are small, at least for a class of spatial distribution functions of the energy density.

(5) A novel gauge condition has been developed for the non-zero rest mass of photon. This theory will be valid in the limit $A_\mu A_\mu \to 0$ for all practical purposes (FAPP). This is known as the Roy-Evans gauge condition [11].

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