The Role of Action-at-a-Distance in the Electro-Magnetic Field Radiation Produced by an Accelerated Charge

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An inadequacy in the traditional description of the phenomenon of electro-magnetic field radiation created by a point charge moving along a straight line with an acceleration is identified and discussed. The possibility of the simultaneous coexistence of Newton instantaneous long-range interaction and Faraday-Maxwell short-range interaction is pointed out.
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Introduction

The problem of *interaction at a distance* was raised for the first time more than 300 years ago by Newton in the first edition of his book, *Mathematical Principles of Natural Philosophy*, and has not lost its relevance today (see *e.g.* [1]). The question as to the choice of one or another conception of interaction at a distance, namely *Newton instantaneous long-rang interaction* (NILI) or *Faraday-Maxwell short-rang interaction* (FMSI) seems finally to have been resolved in favour of the latter. But lately, many authors (see *e.g.* [2-4,6,7-12]) have repeatedly resorted to NILI, a concept which was given up by contemporary physics long ago.

The necessity of introducing NILI (or, at least, making allowance for it) is based either on the possible incompleteness of Maxwell theory [3,12] or possible inaccuracies of the main theses of Special Relativity Theory (SRT) [2,6]. At the same time, a number of authors have indicated an incompleteness in Maxwell theory without referring to the NILI problem [13-18].

Sometimes, the conclusions are connected with the experiments: for example, the results of the Graneau experiments [5] are interpreted in [3] as an indication of the existence of a difference between electrical lepton-lepton and hadron-hadron interactions. The author [3] explains this difference by means of NILI, although in this case the energy transfer occurs with a certain delay, because it is accomplished by exchange with the zero-point quantum-mechanical background. Thus, we see that the author, to avoid coming in conflict with SRT, has to resort to quantum mechanics when discussing a completely non-quantum problem.

In this short note we shall use a simple thought experiment to show that in the case of rectilinear accelerated motion of a charge, Maxwell theory cannot give a completely correct description of the process *until* allowance has been made for NILI.

A Thought Experiment

Let a charge q move in a reference laboratory system with a constant velocity $\mathbf{V}$ along the positive direction of the X-axis. Then, let us consider the electric field $\mathbf{E}(\mathbf{R})$ in a general point $\mathbf{R} = (x, y, z)$. Applying the STR transformations, it is straightforward to show that the electric field $\mathbf{E}(\mathbf{R})$ is directed radially along the vector $\mathbf{R}$, since the delay effect is absent in our case of constant speed $\mathbf{V}$. (Note that in the case of an accelerated motion the field $\mathbf{E}$ is not directed radially everywhere, only in the direction of motion of the charge [19]). It can easily be shown [19] that the modulus $E(R) = |\mathbf{E}(\mathbf{R})|$ of the electric field at the point $\mathbf{R}$ of the reference system is given by:

$$E(R) = \frac{q\left(1-\beta^2\right)}{R^2\left(1-\beta^2\sin^2\alpha\right)^{3/2}}, \tag{1}$$

where $R(t) = |\mathbf{R}| = \left[(x-Vt)^2 + y^2 + z^2\right]^{1/2}$ is the distance between the charge and a point of observation P lying on the X-axis, $\beta = V/c$, c being the velocity of light, α is the angle between the vectors $\mathbf{V}$ and $\mathbf{R}$, $V = |\mathbf{V}|$ and t is the time in the reference system. In the case under consideration the coordinates y and z are equal to zero, and $|x - Vt|$ represent the distance between the charge and the point P in the reference laboratory system.

Now let us apply the concepts of momentum and energy densities to our moving field. The momentum density is given by

$$\mathbf{P} = \frac{1}{c^2}\mathbf{S} \qquad \mathbf{S} = \frac{c}{4\pi}[\mathbf{E},\mathbf{H}], \tag{2}$$

where $\mathbf{S}$ is the Poynting-Umov (energy-flux) vector. The energy density is

$$W = \frac{E^2 + H^2}{8\pi}, \tag{3}$$

and the energy conservation condition for the electro-magnetic field, in the differential form, is:

$$\frac{\partial W}{\partial t} = -\nabla \cdot \mathbf{S}\,. \tag{4}$$

In the case of the moving charge considered here the change of W with time on the left-hand side of (4) is:

$$\frac{\partial W}{\partial t} = \frac{q^2V^2\left(1-\beta^2\right)}{2\pi\,(x-Vt)^5}\,. \tag{5}$$

Since $\mathbf{H} \equiv 0$ along the direction of the charge motion, which follows from the Maxwell equations, the vector $\mathbf{S}$, as well as the momentum density (2), also turn out to be *zero* along the same axis.

But what will happen if we suddenly accelerate the charge in the direction of the X axis? In this case expressions (2), (3) and (4) must be true as previously everywhere, including the X axis. In classical electrodynamics an electric field created by an arbitrarily moving charge is given by the following expression:

$$\mathbf{E} = q\frac{\left(\mathbf{R} - R\frac{\mathbf{V}}{c}\right)\left(1-\frac{V^2}{c^2}\right)}{\left(R - \mathbf{R}\frac{\mathbf{V}}{c}\right)^3} + q\frac{\left[\mathbf{R},\left[\left(\mathbf{R} - R\frac{\mathbf{V}}{c}\right),\frac{\dot{\mathbf{V}}}{c^2}\right]\right]}{\left(R - \mathbf{R}\frac{\mathbf{V}}{c}\right)^3}\,. \tag{6}$$

We recall that here the value of $\mathbf{E}$ is taken at an instant t and the values of $\mathbf{R}$, $\mathbf{V}$ and $\dot{\mathbf{V}}$ are taken at an earlier instant $t_0 = t - \tau$, where τ is retarded time . In our approach, since all the vectors are collinear, the second term in (6) cancels out, and we obtain

$$\mathbf{E}(t) = q\frac{\left(1-\frac{V^2(t_0)}{c^2}\right)}{x^2(t_0)\left(1-\frac{V(t_0)}{c}\right)^2}\mathbf{i}\,, \tag{7}$$

where $\mathbf{i}$ is a unit vector along the X-axis. In the case V = const. it is easy to prove that (7) can be reduced to (1). But the vectors $\mathbf{S}$, and consequently $\mathbf{p}$, are identically *zero* along the whole X axis. On the other hand, from (3) and (4) we see that W and $\partial W/\partial t$ must differ from *zero* everywhere along X, and there is a linear connection between W and E^2, *i.e.*, a conflict arises: if, for example, the charge is vibrating in some mechanical way along the X axis, then the value of W (which is a point function like E) on the same axis will *also* be oscillating. Then the question arises: *how does the point of observation, lying at some fixed distance from the charge on the continuation of the X axis, know about the vibration of the charge?* The fact of *knowing* is obvious.

The presence of retarded time τ in (7) indicates that a longitudinal perturbation along the X-axis should propagate with energy transfer (contrary to Eq. (2)). Since the energy-flux vector $\mathbf{S}$ is the product of the energy density and its propagation velocity

$$\mathbf{S} = W\mathbf{v} \tag{8}$$

(here $\mathbf{v}$ is the velocity of propagation of the perturbation), we can assume, for instance, that this velocity equals *zero* everywhere along X except the region where the charge is localized, *i.e.*, *the energy transfer or radiation transfer does not occur along X*! It is known that Maxwell s equations forbid the propagation of *any* longitudinal electro-magnetic perturbation in vacuum. But P.A.M. Dirac writes ([20], p.32):

> *As long as we are dealing only with transverse waves, we cannot bring in the Coulomb interactions between particles. To bring them in, we have to introduce longitudinal electromagnetic waves... The longitudinal waves can be eliminated by means of mathematical transformation. ...Now, when we do make this transformation which results in eliminating the longitudinal electromagnetic waves, we get a new term appearing in the Hamiltonian. This new term is just the Coulomb energy of interaction between all the charged particles:*
>
> $$\sum_{(1,2)} \frac{e_1e_2}{r_{12}}$$
>
> *...This term appears automatically when we make the transformation of the elimination of the longitudinal waves.*

But in this term the delay effect is not taken into account! So if we place a test charge q_0 on the X axis at some fixed distance from the vibrating charge q, then the test charge will feel the influence of the charge q in *an unknown way*! Dirac writes [20]: ...but it also means a rather big departure from relativistic ideas . Now if W in (8) is supposed to be *zero*, then the question of the meaning of $\mathbf{v}$ loses sense. We have to assume that energy is not stored in the field along X. Moreover, calculations made in the book [21] (see also [19] Ch. IV, § 33) can give us some indirect proof that the own field of charged particles *does not directly contain* energy. Indeed, it is possible to show that the total 4-momentum of the system of charged particles interacting with the electro-magnetic field

$$P_i = \sum_{\alpha} \rho_i^{\alpha} + \int_V \Theta_{i4}\mathrm{d}V\,, \tag{9}$$

where Θ_{i4} is the symmetrical 4-momentum tensor of the electro-magnetic field [19,21], is represented by the sum of the 4-momenta of *free* particles and *free* field. It is important to note that such a field is always transverse in vacuum. The analogous statement is true for the 4-angular-momentum, *i.e.*, it is just the sum of the 4-angular-momenta of *free* particles and *free* field [21].

Discussion

In one of the most recent works [22], the authors also discuss the paradox which is considered in our paper. They correctly note that if one decomposes the total electric field in terms of its transverse and longitudinal components, one must deal with the fact that the longitudinal component is propagated *instantaneously*. Then, *imposing* the same condition on the longitudinal component as on the transverse, concerning the limit of propagation velocity of the interaction, they were able to demonstrate that a space-time transverse electric field appears which contains a term that exactly cancels the instantaneous longitudinal electric field. However, in their speculations, the authors made the obvious logical error: the absence of instantaneous action-at-a-distance was derived from the *hypothesis* of its non-existence (see Eq.(8) in [22]).

It follows from the thought experiment considered above that an *instantaneous long-range interaction* must exist as *a direct consequence* of the Maxwell theory. Indeed, we found that the energy (or radiation) transfer is not carried out along the *X*-axis. Nevertheless, if we place a test charge on the axis at some fixed point away from the vibrating charge *q*, we must observe *an influence* of the latter which *cannot be explained satisfactorily* on the basis of FMSI.

Of course, it is quite desirable to save *both* the STR and the Maxwell theory. On the other hand, an instantaneous long-range interaction must also exist. Consequently, it seems reasonable to introduce a certain *principle of electrodynamic supplementarity*. From this standpoint, both pictures, the NILI and the FMSI, should be considered as two *supplementary* descriptions of one and the same reality. Each of the descriptions is only *partly* true. In other words, both Faraday and Newton in their external argument about the nature of interaction at a distance turned out to be right: instantaneous long-range interaction takes place not *instead of*, but *along with* the short-range interaction in the classic field theory.

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