Elementary Particles as Micro-Universes: A Geometric Approach to "Strong Gravity"*

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We present here an overview of our unified, bi-scale theory of gravitational and strong interactions [which is mathematically analogous to the last version of N. Rosen's bi-metric theory, and yields physical results similar to those of strong gravity*]. This theory, developed during the last 20 years, is purely geometrical in nature, adopting the methods of General Relativity for the description of hadron structure and strong interactions. In particular, hadrons are associated with "strong black holes", from the external point of view, and with "micro-universes", from the internal point of view. Results presented here include derivations of (i) confinement and (ii) asymptotic freedom for hadron constituents; (iii) Yukawa behaviour for the strong potential at the static limit; (iv) the strong coupling "constant", and (v) meson mass spectra.*

Premise

Probably all of us, at least when we were young, have sometimes imagined that every small particle of matter could be, at a suitably reduced scale, a whole cosmos. This idea has very ancient origins. It is already present, for example, in some works by Democritus of Abdera (about 400 B.C.). Democritus, simply inverting that analogy, spoke about huge atoms, as big as our cosmos. And, to be clearer, he added: if one of those super-atoms (which build up super-cosmoses) abandoned his "giant universe" to fall down on our world, our world would be destroyed...

Considerations like these are linked to fantasies about the physical effects of a dilation or contraction of all the objects which surround us, or of the whole "world". Such fantasies have also been exploited by fiction writers: from J. Swift, the narrator of Samuel Gulliver's travels (1727), to I. Asimov. It is probably because of the great diffusion of such ideas that, when the planetary model of the atom was proposed, it achieved a great popular success.

Actually, we encounter such intuitive ideas in the scientific arena too. Apart from Democritus, cited above, let us recall the old conception of a *hierarchy* of universes—or rather of cosmoses—each endowed with a particular *scale factor* (let us think, for instance, of a series of Russian dolls). Nowadays, we can really recognize that the microscopic analysis of matter has revealed *grosso modo* a series of "Chinese boxes": so that we are entitled to suppose that something similar may be met also when studying the universe on a large scale, *i.e.*, in the direction of the *macro* as well as the *micro*. Hierarchical theories were formulated for example by J.H. Lambert in 1761 and, later on, by V.L. Charlier in 1908 and 1922, and by F. Selety in 1922-24; followed more recently by O. Klein, H. Alfvén and G. de Vaucouleurs, up to the works of Salam and co-workers (Salam & Strathdee 1977, 1978; Salam 1978, 1977), Sinha & Sivaram (1979), Markov (1966), Recami and colleagues (Recami & Castorina 1976; Mignani 1976; Caldirola, Pavsic & Recami 1978a, 1978b; Caldirola & Recami 1979; Recami 1979), Ivanenko and collaborators (Ivanenko 1979), Sachs (1982), J.E. Charon, H. Treder, P. Roman, Oldershaw (1896), and others (see also Rosen 1980).

Very recently, we discovered such issues frequently discussed in this Journal (Kokus 1994; Jaakkola 1987; Browne 1994; Broberg 1987; *Cf.* also Huber 1992; Pesteil 1991; Pecker 1988: Broberg 1993), and we would, therefore, like to make our own approach known to this Journal's readers. In particular, it is interesting enough that some of the starting points or results in Broberg (1987), Browne (1994), Jaakkola (1987) and Kokus (1994) are analogous to—even if independent of—the ones presented by Caldirola, Pavsic & Recami (1978a).

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Introduction

In this paper we confine ourselves to examine the possibility of considering elementary particles as micro universes (see *e.g.* Recami 1983a, 1983b, 1979; *Cf.* also Ammiraju, Recami & Rodrigues 1983): that is to say, the possibility that they be similar—in a sense to be specified—to our cosmos. More precisely, we shall refer to the thread followed by P. Caldirola, P. Castorina, A. Italiano, G.D. Maccarrone, M. Pavsic, V.T. Zanchin and ourselves (for an extended summary of that theory, see *e.g.* Recami 1982, and refs. therein; Recami, Martínez & Zanchin 1986; and Recami & Zanchin, 1992; see also Recami & Zanchin 1986).

Let us recall that Riemann, as well as Clifford and later on Einstein (see *e.g.* Einstein 1919) believed that the fundamental particles of matter were the perceptible evidence of a strong local space curvature. A theory which stresses the role of space (or, rather, space-time) curvature already does exist for our whole cosmos: General Relativity, based on Einstein's gravitational field equations, which are probably the most important equations of classical physical theories together with Maxwell's electromagnetic field equations. While much effort has already been made to generalize Maxwell's equations, passing for example from the electromagnetic field to Yang-Mills fields (so that almost all modern gauge theories are modeled on Maxwell's equations), on the contrary, the Einstein equations have never been applied to domains other than gravitation. Nevertheless, like any differential equations, they *do not* contain any in-built fundamental length, so they can be used *a priori* to describe cosmoses of any size.

Our first purpose is to explore whether it is possible to apply successfully the methods of general relativity (GR) to the domain of the so-called nuclear, or *strong*, interactions, in addition to the world of gravitational interactions (for a similar approach, see also Sachs 1981, 1982); namely, to the world of the elementary particles called hadrons. A second purpose is linked to the fact that the standard theory (QCD) of strong interactions has not yet fully explained why the hadron constituents (quarks) seem to be permanently *confined* in the interior of those particles; in the sense that nobody has yet seen an isolated "free" quark, outside a hadron. To explain that confinement, it has been necessary to invoke phenomenological models, such as the so-called "bag" models, in their MIT and SLAC versions, for instance. The "confinement" could be explained, on the contrary, in a natural way and on the basis of a well-grounded theory like GR, if we associated with each hadron (proton, neutron, pion, *etc.*) a particular "cosmological model".

The Model with Micro-Universes

Let us now try to justify the idea of considering the strongly interacting particles (*i.e.*, hadrons) as micro-universes. We find a first motivation if we think of the so-called "large number coincidences", already known since several decades and stressed by H. Weyl, A.I. Eddington, O. Klein, P. Jordan, P.A.M. Dirac, and others.

The most famous among those empirical observations is that the ratio R/r between the radius $R \approx 10^{26}$ m of our cosmos (gravitational universe) and the typical radius $r \approx 10^{-15}$ m of elementary particles is *grosso modo* equal to the ratio S/s between the strength S of the nuclear ("strong") field and the strength s of the gravitational field (we will give later a definition of S, s):

$$r \equiv \frac{R}{r} \approx \frac{S}{s}. \quad (1)$$

This immediately suggests the existence of a *similarity*, in a geometrico-physical sense, between cosmos and hadrons. As a consequence of such similarity, the "theory of models" tells us—by exploiting simple dimensional considerations—that, if we contract our cosmos by the quantity

$$\rho = R/r \approx 10^{41}$$

(*i.e.*, if we transform it into a hadronic micro-cosmos *similar* to the previous one), the field strength would increase in the same ratio, such that the gravitational field would be transformed into the strong field.

If we observe, further, that the typical duration of a decay is inversely proportional to the strength of the interaction itself, we are also able to explain why the mean-life of our gravitational cosmos ($\Delta t \approx 10^{18}$ s: duration—for example—of a complete expansion/contraction cycle, if we accept the theory of the cyclic *big bang*) is a multiple, with the same ratio, of the typical mean-life ($\Delta t \approx 10^{-23}$ s) of the "strong micro-universes", or hadrons:

$$\Delta t \approx r\, \Delta t. \quad (2)$$

It is also interesting that, from the self-consistency of these deductions—as we shall show later—the mass M of our cosmos can be deduced to be equal to $r^2 \approx (10^{41})^2$ times the typical mass M of a hadron: a fact that seems to agree with reality, and constitutes a further "numerical coincidence", the so-called Eddington relation. Another numerical coincidence is shown and explained in Italiano & Recami (1984).

By making use of Mandelbrot's (1983) language and of his general equation for self-similar structures, what precedes can be mathematically translated into the claim that cosmos and hadrons are systems, with scales N and $N-1$, respectively, whose "fractal dimension" is $D = 2$, where D is the self-similarity exponent that characterizes the hierarchy. As a consequence of all this, we shall assume that cosmos and hadrons (both treated, of course, as finite objects) be *similar* systems: *i.e.* that they be governed by similar laws, differing only for a "global" scale transformation which transforms R into r and gravitational field into strong field. [To fix our ideas, we may *temporarily* adopt the naïve model of a "Newtonian ball" in three-dimensional space for both cosmos and hadrons. Later on, we shall adopt more sensible models, for example Friedmann's]. We might, incidentally, add that we should be ready *a priori* to accept the existence of other cosmoses besides ours: recall that mankind in every epoch has successively called "*universe*" his valley, the whole Earth, the solar system, the Milky Way and today (with the same simple-minded view) our cosmos, as we know it on the basis of our observational and theoretical instruments.**

Thus, we arrive at a *second* motivation for our theoretical approach: That physical laws should be covariant (= form invariant) under *global* dilations or contractions of space-time. We can easily realize this if we notice that: (i) when we dilate (or contract) our units of space and time, physical laws, of course, should *not* change their form; (ii) a dilation of units is totally equivalent to a contraction (leaving now "meter" and "second" unaltered) of the observed world.

Actually, Maxwell's equations of electromagnetism—the most important equations of classical physics, together with Einstein's equations, as already noted—are by themselves also covariant under conformal transformations and, in particular, under dilations. In the case when electric charges are present, this covariance holds provided that charges themselves are suitably "scaled". Analogously, also Einstein's *gravitational* equations are covariant†† under dilations: provided that, again, in the presence of matter and of a cosmological term Λ, they too are scaled according to correct dimensional considerations. The importance of this fact was realized by Einstein himself, who in connection with his last unified theory wrote, two weeks before his death: "From the form of the field [*gravitational + electromagnetic*] equations it follows immediately that: if $g_{ik}(x)$ is a solution of the field equations, then also $g_{ik}(x/a)$, where a is a positive constant, is a solution ("similar solutions"). Let us suppose, for example, that g_{ik} represents a finite crystal embedded in a flat space. It is then possible that a second 'universe' exists with another crystal, identical with the first one, but dilated a times with respect to the former. As long as we confine ourselves to considering a universe containing only one crystal, there are no difficulties: we just realize that the size of such a crystal (standard of length) is not determined by the field equations…" These lines are taken from Einstein's preface to the Italian book *Cinquant'anni di Relatività* (Pantaleo 1955). They were written in Princeton on April 4, 1955, and stress the fact, already mentioned by us, that differential equations—like all the fundamental equations of physics—do not contain any in-built "fundamental length". In fact, Einstein equations can describe the internal dynamics of our cosmos, as well as of much bigger super-cosmoses, or of much smaller micro-cosmoses (suitably "scaled").

A Hierarchy of "Universes"

As a first step for better exploiting the symmetries of the fundamental equations of classical physics, let us therefore fix our attention on the space-time *dilations*

$$x'_m = r\, x_m \tag{3}$$

** This clarifies that our geometrico-physical similarity holds between two classes of objects of different scale (hadrons and cosmoses), in the sense that the factor r will vary according to the particular cosmos and hadron considered. This will be important for practical applications. Finally, let us recall that in Mandelbrot's philosophy, analogous objects do exist at every hierarchical level, so that we can conceive a particular type of cosmos for each particular type of hadron, and *vice versa*. As a consequence, we should expect r to change a little in each case (for example, depending on the type of hadron considered).

†† Notice that we do not refer here to the usual "general covariance" of the Einstein equations (which are supposed to hold in our cosmos), but to their covariance with respect to transformations (dilations or contractions) between—for example—our cosmos and the hadronic micro-cosmos.

with $x_m \equiv (t;x,y,z)$ and $m = 0,1,2,3$, and explicitly require physical laws to be covariant with respect to them: under the hypothesis, however, that only *discrete* values of r are realized in nature. As before, we are moreover supposing that r is constant as the space or time position varies (global, besides *discrete*, dilations).

Let us recall that natural objects interact essentially through four (at least) fundamental forces, or interactions: the gravitational, the "weak", the electromagnetic and the "strong" ones; here listed according to increasing strength. We can express the strengths by pure numbers, in order to compare them with one another. For instance, if we choose to define each strength as the dimensionless *square* of a "vertex coupling constant", the electromagnetic strength is measured by the (dimensionless) coefficient $Ke^2/\hbar c \equiv \alpha \approx 1/137$, where e is the electron charge, $\hbar$ the reduced Planck constant, c is the light speed in vacuum and K is the electromagnetic interaction universal constant (in the International System of units, $K = (4\pi \varepsilon_o)^{-1}$, with ε_o = vacuum dielectric constant). Here we are interested in particular in the gravitational and strong interaction strengths:

$$s \equiv Gm^2/\hbar c; \quad S \equiv Ng^2/\hbar c,$$

where G and N are the gravitational and strong universal constants, respectively; quantities m and g representing the gravitational charge (= mass) and the strong charge (see note ‡‡ and Figure 1; and *cf.*, *e.g.*, Recami 1979), respectively, of one and the same hadron: for example of a nucleon $\mathcal{N}$ or of a pion π. More precisely, we shall often adopt in the following the convention of calling M and g "gravitational mass" and "strong mass", respectively.

Let us consider, therefore, two identical particles endowed with both gravitational (m) and strong (g) mass, *i.e.*, two identical hadrons, and the ratio between the strengths S and s of the corresponding strong and gravitational interactions. We find $S/s \equiv Ng^2/Gm^2 \approx 10^{40 \div 41}$, so that one verifies that $\rho \equiv R/r \approx S/s$. For example for $m = m_p$ one gets $Gm^2/\hbar c \approx 1.3 \times 10^{-40}$, while the $pp\pi$ or $pp\rho$ (or quark-quark-gluon: see below) coupling constant squares are $Ng^2/\hbar c \approx 14$ or 3 (or 0.2), respectively.

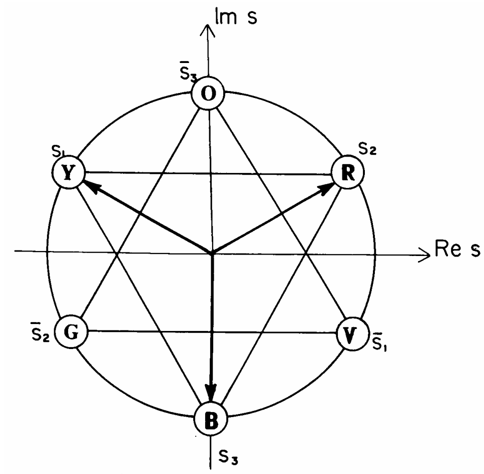

Figure 1 – *"Coloured" quarks and their strong charge* – This scheme represents the complex plane (see note ‡‡; and *cf.*, *e.g.*, Recami 1979; Recami & Zanchin 1992) of the *sign s* of the quark strong-charges g_j in a hadron. These strong charges can have *three* signs, instead of two as in the case of the ordinary electric charge *e*. They can be represented, for instance, by $s_1 = \left(i - \sqrt{3}/2\right)$; $s_2 = \left(i + \sqrt{3}/2\right)$; $s_3 = -i$, which correspond to the arrows separated by 120° angles. The corresponding anti-quarks will be endowed with strong charges carrying the complex conjugate signs $\bar{s}_1$, $\bar{s}_2$, $\bar{s}_3$. The three quarks are represented by the "yellow" (Y), "red" (R) and "blue" (B) circles; the three anti-quarks by the "violet" (V), "green" (G) and "orange" (O) circles. The latter are complementary to the former corresponding colors. Since in real particles the inter-quark forces are saturated, hadrons are white. The white colour can be obtained either with three-quark structures, by the combinations YRB or VGO (as it happens in baryons and antibaryons, respectively), or with two-quark structures, by the combinations YV or RG or BO [which are actually quark-antiquark combinations], as it happens in mesons and their anti-particles.‡‡

At this point, we can make some simple remarks. First of all, notice that, if we put conventionally $m \equiv g$, then the strong universal constant N becomes

$$N \approx \rho\, G \approx \hbar c / m_p^2 \,. \qquad (4)$$

On the other hand, if we choose units such that $[N] = [G]$ and moreover $N = G = 1$, we obtain $g = m\sqrt{\rho}$ and, more precisely (with $n = 2$ or $n = 3$),

$$g_o = g/n \approx \sqrt{\hbar c/G} \equiv \text{Planck mass},$$

which tells us that—in suitable units—the so-called "Planck mass" is nothing but the *magnitude* of the rest strong-mass [= strong charge] of a typical hadron, or rather of quarks (see note ‡‡).

From this point of view, we should *not* expect the "micro black-holes" (with masses of the order of the Planck mass), predicted by various authors, to exist; in fact, we *already* know of the existence of quarks, whose *strong charges* are of the order of the Planck mass (in suitable units). Moreover, the fact—well known in standard

‡‡ Let us recall that the hadron constituents (2 for mesons and 3 for baryons) were named *quarks* by M.Gell-Mann. This Anglo-Saxon word, which usually means mush or also curd, is usually ennobled by literary quotations (for example, Gell-Mann was inspired—as is well known—by a verse from J. Joyce's *Finnegans wake*, 1939). Here we wish to note that Goethe had used the word properly in his *Faust*, verse 292, where Mephistopheles referring to mankind exclaims: "In Jeden Quark begräbt er seine Nase"!
By considering quarks to be the real carriers of the strong charge (*cf.* Figure 1), we can call "colour" the *sign* s_j of the strong charge (Recami 1979); namely, we can regard hadrons as endowed with a zero total strong charge, each quark possessing the strong charge $g_j = s_j |g_j|$ with $\Sigma s_j = 0$. Therefore, when passing from ordinary gravity to "strong gravity", we shall replace m by $g = ng_o$, quantity g_o being the average *magnitude* of the constituent quarks *rest*-strong-charge, and n their number (Recami 1979).

theories—that gravitational interactions become as strong as the "strong" ones for masses of the order of the Planck mass in our opinion simply means that the *strong gravity* field generated by quarks inside hadrons (strong micro-universes) is nothing but the strong nuclear field.

"Strong Gravity"

A consequence of the above is that inside a hadron (*i.e.*, when we want to describe strong interactions among hadron constituents) it must be possible to adopt the same Einstein equations which are used to describe gravitational interactions inside our cosmos; provided the gravitational constant G (or the masses) and the cosmological constant Λ are suitably scaled, together with space distances and time durations.

Let us now recall that Einstein's equations for gravity essentially state the equality of two tensor quantities: the first describing the geometry (curvature) of space-time, and the second—which we will call the "matter tensor", GT_{mn}—describing the distribution of matter:

$$\mathrm{R}_{mn} - \frac{1}{2}\,½\, g_{mn} R_r^r - \Lambda g_{mn} = -kGT_{mn}; \quad \left[k \equiv \frac{8p}{c^4}\right]. \quad (5)$$

As is well known, $G \approx 6.7 \times 10^{-11}\ \mathrm{m^3/(kg \times s^2)}$, while $\Lambda \approx 10^{-52}\ \mathrm{m^{-2}}$.

Inside a hadron, therefore, equations of the same form will hold, except that instead of G we will have (as we already know) the quantity $N \approx \hbar c/m_p^2$, and instead of Λ the "strong cosmological constant" (or "hadronic constant") l will appear:

$$N = r_1 G\,; \quad l = r^2\Lambda; \quad r_1 \approx r, \qquad (6)$$

so that $\Lambda \approx 10^{30}\ \mathrm{m^{-2}} = (1\ \mathrm{fm})^{-2}$, or $l^{-1} \approx 0.1$ barn.

For the sake of brevity, we will call $S_{mn} = NT_{mn}$ the "strong matter tensor".

The preceding can be directly applied, with a satisfactory degree of approximation, to the case—for example—of the pion: *i.e.*, to the case of the cosmos/pion similarity. Almost as if our cosmos were a super-pion, with a super-quark (or "metagalaxy", adopting Ivanenko's terminology) of matter and one of anti-matter. Let us recall however that, as we already noted above, the parameter r *can* vary according to the particular cosmos and hadron considered. Analogously Λ, and therefore l, can vary too: with the further consequence that *a priori* their sign can also change, depending on the object (cosmos or hadron) under consideration.

As far as r_1 is concerned, an even more important remark has to be made. Notice that the gravitational coupling constant $Gm^2/\hbar c$ (experimentally measured in the case of the interaction of two "tiny components" of our particular cosmos) should be compared with the analogous constant for the interaction of two tiny components (partons? partinos?) of the corresponding hadron, or rather of a particular constituent quark. That constant is unknown to us. We know however, for the simplest hadrons, the quark-quark-gluon coupling constant: $Ng^2/\hbar c \approx 0.2$. As a consequence, the best value for r_1 we can predict —up to now— for these hadrons is $r_1 \approx 10^{38} \div 10^{39}$ [and, in fact, 10^{38} is the value which has provided results closest to the experimental data]: a value which, however, will vary—we repeat—with the particular cosmos and the particular hadron chosen.

The already mentioned "large numbers" empirical relations, which link the micro- with the macro-cosmos, have been obtained by us as a *by-product* of our scaled-down equations for the interior of hadrons, and of the ordinary Einstein equations. Notice, once more, that our "numerology" connects the gravitational interaction with the strong interaction, and *not* with the electromagnetic interaction (as Dirac suggested). It is noteworthy that the strong interaction, like gravity—but differently from electromagnetism—is highly non-linear and can be associated with *non*-Abelian gauge theories. One of the purposes of our theoretical approach consists, incidentally, in proposing an *ante litteram* geometrical interpretation of those theories.

Before going on, let us specify that the present *geometrization* of the strong field is justified by the circumstance that the "Equivalence principle" (which recognizes the identity, inside our cosmos, of inertial and gravitational mass) can be extended to the hadronic universe in the following way. The usual Equivalence principle can be understood, according to Mach, thinking of the inertia m_I of a given body as due to its interaction with all the other masses of the universe: an interaction which in *our* cosmos is essentially gravitational; so that m_I coincides with the gravitational mass: $m_I = m_G$. Inside a "hadronic cosmos", however, the predominant interaction among its constituents is the strong one; so that the inertia m_I of a constituent will coincide with its strong charge g (and not with m_G). We shall see that our generalization of the Equivalence principle will be useful for geometrizing the strong field not only inside a hadron, but also in its neighborhood.

Both for the cosmos and for hadrons, we shall adopt Friedmann-type models; taking advantage of the fact that they are compatible with the Mach Principle, and are embeddable in 5 dimensions.

The Interior of a Hadron

Let us see some consequences of our Einstein-like equations, re-written for the strong field, and therefore valid inside a hadron:

$$R_{mn} - \frac{1}{2} g_{mn} R^r{}_r - \Lambda g_{mn} = -l\, S_{mn}\,; \quad [S_{mn} = NT_{mn}]. \quad (7)$$

In the case of a spherical constituent, *i.e.* of a spherically symmetric distribution g' of "strong mass", and in the usual Schwarzschild-deSitter r,t coordinates, the

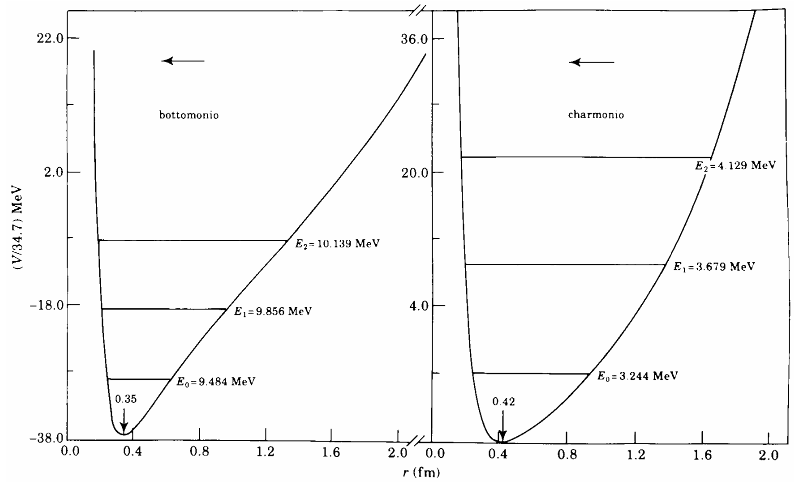

Figure 2 – The curves of two typical inter-quark potentials V_{eff} yielded by the present theoretical approach: *cf.* Eq. 8. We also show the theoretical energy levels calculated for the $1 - {}^3s_1$, $2 - {}^3s_1$ and $3 - {}^3s_1$ states of "Bottomonium" and "Charmonium", respectively [by adopting for the bottom and charm quarks the masses $m(b) = 5.25$ and $m(c) = 1.68$ GeV/c^2]. The comparison with experiment (Quigg 1985) is satisfactory: see the text.

known geodesic motion equations for a small test-particle (let us call it a *parton*, with strong mass g'') tell us that it will feel a "force" easy to calculate (Recami 1982 and refs. therein; Recami, Martínez & Zanchin 1986; Italiano *et al.* 1984; Ammiraju *et al.* 1991; Zanchin 1987), which for low speeds [*static limit*: $v<<c$] reduces to the (radial) force:

$$F = -\frac{1}{2}c^2 g''\left(1 - \frac{2Ng'}{c^2 r} + \frac{1}{3}l r^2\right)\left(\frac{2Ng'}{c^2 r^2} + \frac{2}{3}lr\right) \quad (8)$$

Notice that, with proper care, also in the present case one can introduce a language in terms of "force" and "potential"; for example in Eq. 8 we defined $F \equiv g''d^2r/dt^2$. In Figure 2 the form of two typical potentials yielded by the present theory is depicted [*cf.* Eq. 8′].

At "intermediate distances"—*i.e.*, at the Newtonian limit—this force simply reduces to $F \approx -\frac{1}{2}c^2 g''(2Ng'/c^2r^2 + 2lr/3)$, *i.e.* to the sum of a Newtonian term and an elastic term *à la Hooke*. Let us notice that, in this limit, the last expression is valid even when the test particle g'' does not posses a small strong mass, but is—for example—a second quark. Otherwise, our expressions for F are valid only *approximately* when g'' is also a quark; nevertheless, they can explain some important features of the hadron constituent behaviour, for both small and large values of r.

At very large distances, when r is of the same order of (or greater than) the considered hadron radius [$r \gtrsim 10^{-13}$ cm $\approx$ 1 fm], whenever we confine ourselves to the simplest hadrons (and thus choose $l \approx 10^{30}$ m^{-2}; $N \approx 10^{38\div39}$ G), we end with an *attractive* radial force which is proportional to r:

$$F \approx -g''c^2 l r/3. \quad (9)$$

In other words, one naturally obtains a confining force (and a confining potential $V \div r^2$) able *a priori* to explain the so-called *confinement* of the hadron constituents (in particular, quarks). Because of this force, the motion of g'' can be regarded in a first approximation as a harmonic motion; so that our theory can include the various and interesting results already found by different authors for hadronic properties—for instance, hadron mass spectra—just by *postulating* such a motion.

Up to now we have supposed l to be positive. But it is noteworthy that confinement is also obtained for negative values of l. In fact, with less drastic approximations, for $r \gtrsim 1$ fm one obtains:

$$F \approx -\frac{1}{3}g''c^2 l\left(r + \frac{lr^3}{3} - \frac{Ng'}{c^2}\right), \quad (9')$$

where, for r large enough, the l^2 term dominates. However, when considering "not simple" hadrons (so that l, and moreover N, may change values), other terms, such as the Newtonian term, $-Ng'^2/r^2$, or even the *constant* term $+Nl\,g'^2/3$ which corresponds to a linear potential, can become important. Finally, this last equation predicts that, for inter-quark distances of the order of 1 fm, two quarks must attract each other with a force of some *tons*: quite a large force, especially when we recall that it has to act between two extremely tiny particles (the constituents of mesons and baryons), whose magnitude increases with distance.

We now consider distances that are not too great, always at the static limit. It is then important to add to the radial potential the usual "kinetic energy term" (or centripetal potential), $(J/g'')^2/2r^2$, in order to account for the orbital angular momentum of g'' with respect to g'. The effective potential (Italiano *et al.* 1984; Ammiraju *et al.* 1991; Zanchin 1987) between the two constituents g', g'' thus assumes the following form

$$V_{\text{eff}} = \frac{1}{2}g''c^2\left[2\left(\frac{Ng'}{c^2}\right)^2\frac{1}{r^2} - \frac{2Ng'}{c^2}\frac{1}{r} - \frac{2lNg'}{3c^2}r + \frac{l}{3}r^2 + \frac{1}{2}\left(\frac{l}{3}\right)^2 r^4\right] + \frac{(J/g'')^2}{2r^2}, \quad (8')$$

which, in the region where GR reduces essentially to the Newtonian theory, simplifies into:

$$V_{\text{eff}} \approx -\frac{Ng'g''}{r} + \frac{(J/g'')^2}{2r^2}.$$

In this case the test particle g'' can stabilize (executing a circular motion, for example: in the next section we will give more details) at a distance r_e from the source-constituent at which V is minimum; *i.e.*, at the distance $r_e = J^2/Ng'g''^2$. At this distance the "effective force" vanishes. Thus we find, at short distances, the phenomenon known as *asymptotic freedom*: For not large distances (when the force terms proportional to r and to r^3 become negligible), the hadron constituents behave as if they were (almost) free. If we now extrapolated, somewhat arbitrarily, the expression for r_e to the case of two quarks [for example, $|g'| = |g''| = g_o \approx (1/3) m_p$], we would obtain the preliminary estimate $r_e \approx (1/100)$ fm. *Vice versa*, by supposing—for instance in the case of baryons, with $g = m \approx m_p$ and $N \approx 10^{40}\ G$—that the equilibrium radius r_e is of the order of a hundredth of a fermi, one would get the Regge-like relation $J/\hbar \approx m^2$ (where m is measured in GeV/c^2).

We can perform these calculations again, however, using the *complete* expression for V_{eff}. First of all, we observe that it is possible to evaluate the radius at which the potential reaches its minimum also in the case $J = 0$. By extrapolation to the case of the simplest quarks [for which $Ng^2/\hbar c \approx 0.2$], we always find at least one solution, $r_e \approx 0.25$ fm, for l positive and of the order of 10^{30} m^{-2}. Moving to the case $J = \hbar$ (which corresponds classically to a speed $v \approx c$ for the moving quark), with the same hypothesis we obtain the value

$$r_e \approx 0.9 \text{ fm.}$$

Actually, for positive l it exists the above solution *only*. For negative values of l, however, the situation is more complex; we summarize it for the case of the N and $|l|$ values adopted here. We find—again—at least one solution, which for $J = 0$ assumes the simple analytic form $r_e^3 = 3Ng'/c^2|l|$.

More precisely, for $l = -10^{30} m^{-2}$ we find the values 0.7 and 1.7 fm, corresponding to $J = 0$ and $J = 1$. These values, however, become 0.3 and 0.6 fm, respectively, for $\Lambda = -10^{29}$ m^{-2}. In the $J = 0$ case, at last, two *further* solutions are found, the smaller one [for $l = -10^{30}$ m^{-2}] being once more $r_e \approx 0.25$ fm.

Recalling that *mesons* are made up of two quarks (q, $\bar{q}$), our approach suggests for mesons in their ground state—when $J = 0$, at least—the model of two quarks oscillating around an equilibrium position. It is interesting to note that for small oscillations (harmonic motions in space) the dynamical group would then be SU(3). It is interesting, too, that the value $m_o = hn/c^2$, corresponding to the frequency $n = 10^{23}$ Hz, yields the pion mass: $m_o \approx m_p$.

Analogous results must, obviously, hold for our cosmos (or, rather, for cosmoses which are "dual" to the hadrons considered).

The Strong Coupling Constant

Here we simply wish to add that, in the case of a spherically symmetric, static metric (and in the coordinates in which it is diagonal), the Lorentz factor is proportional to $\sqrt{g_{oo}}$, so that the *strong coupling constant* $a_S = S$ in our theory (Recami & Zanchin 1994) assumes the form:§§

$$a_S(r) \cong \frac{N}{\hbar c} \frac{g_o'^2}{1 - \dfrac{2Ng_o'}{c^2 r} + \dfrac{l r^2}{3}}, \qquad (10)$$

since the strong mass g'' depends on the speed:

$$g'' = \frac{g_o''}{\sqrt{g_{oo}}} = \frac{g_o''}{\sqrt{1 - \dfrac{2Ng_o'}{c^2 r} + \dfrac{l r^2}{3}}}, \qquad (11)$$

just as the ordinary relativistic mass does. The behaviour of our "constant" $a_S(r)$ is analogous to that one of the perturbative coupling constant of the "standard theory" (QCD): *i.e.*, $a_S(r)$ decreases as the distance r decreases, and increases as it increases, once more justifying both confinement and "asymptotic freedom". Let us recall that [see note §§], when $g''_o = g'_o$, the definition of a_S is $a_S = S = Ng'^2 \hbar c$.

Since the Schwarzschild-like coordinates $(t;r,q,j)$ do not correspond, as is well known, to any real observer, it is interesting from the *physical* point of view to pass to local coordinates $(T;R,q,j)$ associated with observers who are *at rest* "with respect to the metric" at each point (r,q,j) of space: $dT = \sqrt{g_{tt}}\,dt$; $dR = \sqrt{-g_{rr}}\,dr$, where $g_{tt} = g_{oo}$ and $g_{rr} = g_{11}$. These "local" observers measure a speed $U = dR/dT$ (and strong masses) such that $\sqrt{g_{tt}} = \sqrt{1 - U^2}$, so that Eq. 11 assumes the transparent form

$$g'' = \frac{g_o''}{\sqrt{1 - U^2}}. \qquad (11')$$

Once (thanks to the geodesic equation) the speed U is calculated as a function of r, it is easy to find, for example, that for negative l the minimum value of U^2 again corresponds to $r = [3Ng'_o/|l|]^{1/3}$. For positive l we get a similar expression, *i.e.*, $r_o = [6Ng'_o/l]^{1/3}$, which furnishes a limiting (*confining*) value of r that cannot be reached by any of the constituents.

Lastly, we consider the case of a geodesic circular motion, as described by "physical" observers, *i.e.*, by our local observers (even if we find it convenient to express everything as a function of the old Schwarzshild-deSitter coordinates). If a is the angular momentum per unit of strong rest-mass, in the case of a test-quark in motion around the source-quark, we find the interesting relation $g'' = g'_o\sqrt{(1 + a^2/r^2)}$, which allows us to write the strong coupling constant in the particularly simple form (Recami & Zanchin 1994)

$$a_S \cong \frac{N}{\hbar c} g_o' \left(1 + \frac{a^2}{r^2}\right). \qquad (10')$$

§§ Actually, if we considered a (light) test-particle g'' in the field of a "heavy" constituent g' (a quark for instance), we would rather obtain only a square root at the denominator; namely $a_S \approx [Ng'_o g''_o/\hbar c][\sqrt{(1 - 2Ng''_o/c^2 r + lr^2/3)}]^{-1}$. When we then consider two heavy constituents (two quarks) endowed with the same *rest* strong-mass $g''_o = g'_o$, we ought to tackle the two body problem in GR; however, in an approximate way, and looking at an *average* situation, we can propose a formula like Eq. 13, where r is the distance from the common "centre of mass".

We can now observe, for instance, that—if $l < 0$—the specific angular momentum a vanishes along the customary geodesic $r = r_{qq} = [3Ng'_o / |l|]^{1/3}$; in this case the test-quark can remain *at rest*, at a distance r_{qq} from the source-quark. With the "typical" values $r = 10^{41}$; $r_1 = 10^{38}$, and $g'_o = m_p/3 \approx 313$ MeV/c^2, we obtain $r_{qq} \approx 0.8$ fm.

Strong Interactions among Hadrons

From the "external" point of view, when describing the interactions among hadrons (as they appear to us in *our* space), we are in need of *new* field equations able to account for both the gravitational and strong fields which surround a hadron. We need actually a *bi-scale* theory (Papapetrou 1980), in order to study, for example, motion in the vicinity of a hadron of a test-particle possessing both gravitational and strong mass.

The preceding suggests—as a first step—representing the strong field around a source-hadron by means of a tensor field, s_{mn}, like (in GR) the gravitational field tensor e_{mn}. Within our theory (see *e.g.* Caldirola, Pavsic & Recami 1978; Recami 1982; Recami, Martínez & Zanchin 1986; Recami & Zanchin 1992; Recami 1983a, 1983b; *cf.* also Ammiraju, Rodrigues & Recami 1983), the Einstein gravitational equations are *modified* by introducing, in the neighborhood of a hadron, a strong deformation s_{mn} of the metric, acting only on objects having a strong charge (*i.e.*, an intrinsic "scale factor" $f \cong 10^{-41}$) and not on objects possessing only a gravitational charge (*i.e.*, an intrinsic scale factor $f \cong 1$). Outside a hadron, and for a "test-particle" endowed with both charges, the *new* field equations are:

$$R_{mn} + l s_{mn} = -\frac{8p}{c^4}\left[S_{mn} - \frac{1}{2} g_{mn} S^r_r\right]. \qquad (12)$$

They reduce to the usual Einstein equations far from the source-hadron, because they imply that the strong field exists only in the very neighborhood of the hadron: namely that (in suitable coordinates) $s_{mn} \to h_{mn}$ for $r >> 1$ fm.

Linear approximation: – For distances from the source-hadron $r \gtrsim 1$ fm, when our new field equations can be linearized, the total metric g_{mn} can be written as the *sum* of the two metrics s_{mn} and e_{mn}; or, more precisely (in suitable coordinates):

$$2g_{mn} = e_{mn} + s_{mn} \cong h_{mn} + s_{mn}\,.$$

The quantity s_{mn} can then be written as $s_{mn} = h_{mn} + 2h_{mn}$, with $|h_{mn}| << 1$; so that $g_{mn} \approx h_{mn} + h_{mn}$ (where, we repeat, $h_{mn} \to 0$ for $r >> 1$ fm). For the sake of simplicity, we also confine ourselves to the case of positive l [on the contrary, if $l < 0$, we should (Italiano *et al.* 1984; Ammiraju *et al.* 1991) put $s_{mn} = h_{mn} - 2h_{mn}$].

One of the most interesting results is that, at the static limit (when only $s_{oo} \neq 0$ and the strong field becomes a scalar field), we get that $V = h_{oo} = ½ (s_{oo} - 1) = g_{oo} - 1$ is exactly the *Yukawa potential*:

$$V = -g\frac{\exp\left[-\sqrt{2|l|}r\right]}{r} \cong -\frac{g}{r}\exp\left[\frac{-m_p rc}{\hbar}\right], \qquad (13)$$

with the correct coefficient—within a factor 2—also in the exponential (Recami 1982, 1983a; Caldirola, Pavsic & Recami 1978).

Intense field approximation: – Let us consider the source-quark as an *axially* symmetric distribution of strong charge g: a study of the metrics in its neighborhood will lead us to consider a Kerr-Newman-deSitter (KNdS)-like problem and to look for solutions of the type "*strong* KNdS black holes". We find that—from the "external" point of view—hadrons can be associated with the above mentioned "*strong black-holes*" (SBH), which turn out to have radii $r_S \approx 1$ fm.

For $r \to r_S$, *i.e.* when the field is very intense, we can perform the approximation just "opposite" to the linear one, by assuming $g_{mn} \approx s_{mn}$. We then obtain equations which are essentially identical to the "internal" ones [which is good for the matching of the hadron interior and exterior!]; hence what we are going to say can also be valid for quarks, not only for hadrons. Before going further, let us observe that l can *a priori* assume a certain sign outside a hadron, and the opposite sign inside it. In the following we shall confine ourselves to the case $l < 0$ for simplicity.

In general for negative l we find (Zanchin *et al.* 1994) three "strong horizons", *i.e.*, three values of r_S, that we shall call r_1, r_2, r_3. If we are interested in hadrons which are *stable* with respect to the strong interactions, we have to look for those solutions for which the SBH Temperature [= strong field strength at its surface; see *e.g.* Bekenstein, 1974; Hawking 1975] almost vanishes. It is worth noticing that the condition of a vanishing field at the SBH surface implies the coincidence of two, or more, strong horizons (Zanchin *et al.* 1994, and refs. therein); and that such coincidences imply in their turn some "Regge-like" relations among m, l, N, q and J, if m, q, J are—now—mass, charge and intrinsic *angular momentum* of the considered hadron, respectively. More precisely, if we choose *a priori* the values of q, J, l and N, then our theory yields the *mass* and *radius* of the corresponding stable hadron. Our theoretical approach is, therefore, a rare example of a formalism which can yield—at least *a priori*—the masses of the stable particles (and of the quarks themselves).

Mass Spectra

We arrived at the point of checking whether and how our approach can yield the values of the hadron masses and radii: in particular for hadrons that are stable with respect to strong interactions; we can guess *a priori* that such values will possess the correct order of magnitude. Several calculations have been performed by us, in particular for the meson mass spectra (Ammiraju *et al.* 1991, and refs. therein; Recami & Zanchin, 1992; Recami, Zanchin & Vasconselos 1995), although these results have yet to be presented in an organized form.

Here we will quickly outline just some of the results. First, let us consider the case of the simultaneous coincidence of all three horizons ($r_1 = r_2 = r_3 \equiv r_h$). We get a system of equations that—for example—rules out the possibility of intrinsic angular momentum (spin) J and electric charge q being simultaneously zero [practically ruling out particles with $J = 0$]; it also implies the interesting relation $l^{-1} \cong 2r_h^2$; and finally it admits (real and positive) solutions only for *low* values of J, the upper limit of the spin depending on the parameters chosen.

The values we obtained for the (small) radii and for the masses suggest that the "triple coincidences" represent the case of *quarks*. The basic formulae for the explicit calculations are the following (Zanchin *et al.* 1994). First of all, we put $N = r_1 G$, so that $g \equiv m$. We then define, as usual, $Q^2 \equiv Nq^2/Kc^4$; $a \equiv J/mc$; $M \equiv Nm/c^2$, and moreover $d \equiv 1 + la^2/3$. Then, the radii of the stable particles (quarks, in this case) are given by the *simple* equation $r = 3M/2d$; but the masses are given by the solution of a system of two Regge-like relations: $9M^2 = -2\delta^3/l$; $9M^2 = 8\delta(a^2 + Q^2)$.

The cases of "double coincidence", *i.e.* of the coincidence of two (out of three) horizons only, seem to be able to describe stable baryons and mesons. The fundamental formulae become, however, more complex (Zanchin *et al.* 1994). We define $h \equiv a^2 + Q^2$; $s \equiv d^2 + 4lh$; and $Z \equiv 3\delta^2 - 4l\delta h + 18lM^2$. The stable hadron radii are then given by the relation $r \equiv 3Ms/Z$; while the masses are given by the non simple equation $9M^2 s(ds - Z) + 2hZ^2 = 0$, which relates M to a, Q and l. Of course, some simplifications are met in particular cases. For example, when $l = 0$, we get the Regge-like relation:

$$M^2 = a^2 + Q^2, \qquad (14)$$

which—when q is negligible—becomes $M^2 = cJ/G$, that is [with $c = G = 1$]:

$$m^2 = J\,. \qquad (14')$$

On the other hand, when $J = 0$, and q is still negligible, we obtain [always with $c = G = 1$]:

$$9m^2 = -l^{-1}. \qquad (15)$$

Also in the cases of "triple coincidence" simple expressions are found, when $|la^2| << 1$. Under such a condition, we find the simple system of two equations:

$$9M^2 \approx 8(a^2 + Q^2); \quad 9m^2 \approx -2l^{-1}, \qquad (16)$$

where the second relation is written with $c = G = 1$.

All the "geometric" evaluations in this section are referred—as we have seen—only to *stable* hadrons (*i.e.*, to hadrons corresponding to SBHs with "temperature" $T \approx 0$), because we do not know of general rules associating a temperature T with the many experimentally discovered *resonances* [which will correspond (Caldirola, Pavsic & Recami 1978; Recami 1983a, 1982; Recami & Zanchin 1986) to temperatures of the order of 10^{12} K, if they have to "evaporate" in times of the order of 10^{-23} s)]. Calculations suited to comparing our theoretical approach with experimental mass spectra (for mesons, for example) have, therefore, been performed up to now by taking recourse to the trick of inserting our interquark potential V_{eff}, found above, into a Schroedinger equation. Also many calculations—kindly performed by our colleagues Prof. J.A. Roversi and Dr. L.A. Brasca-Annes of the "Gleb Wataghin" Physics Institute of the State University at Campinas (S.P., Brazil)—have not yet been reordered! Here we can specify, nevertheless, that potential (8′) has been inserted into the Schroedinger equation in spherical (polar) coordinates, which has been solved by a finite difference method (Ammiraju *et al.* 1991).

In the case of "Charmonium" and of "Bottomonium", for example, the results obtained [by adopting (Quigg 1985) for the quark masses the values m(charm) = 1.69 GeV/c^2; m(bottom) = 5.25 GeV/c^2] are the following (Figure 2). For the states $1 - {}^3s_1$, $2 - {}^3s_1$ and $3 - {}^3s_1$ of Charmonium, we obtained the energy levels 3.24, 3.68 and 4.13 GeV, respectively. Instead, for the corresponding quantum states of Bottomonium, we obtained the energy levels 9.48, 9.86 and 10.14 GeV, respectively. The radii for the two fundamental states turned out to be r(c) = 0.42 fm, and r(b) = 0.35 fm, with r(c) > r(b) [as expected from "asymptotic freedom"]. Moreover, the values of the parameters obtained by our computer fit are actually those expected: $r = 10^{41}$ and $r_1 = 10^{38}$ (just the "standard" ones) for Charmonium; and $r = 0.5 \times 10^{41}$ and $r_1 = 0.5 \times 10^{38}$ for Bottomonium.

The correspondence between theoretical and experimental results (Quigg 1985) is satisfactory, especially if we bear in mind the approximations adopted (in particular, treating the second quark g'' as a test-particle).

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Corrigenda

Volume 3, No. 3-4:

Page 126, col. 2, line 27 from bottom, after "We have", insert:

(Fig. 1)

Page 126, col. 2, line 16 from bottom, the second equation should read:

$|v'| = |vc/(c\pm v)|$

```
O' P Q
------------------------------------->
 O Q
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Fig. 1 (Walton; from p. 33)

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( O' )
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 ( O )
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Fig. 2 (Walton; from p. 33)

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( ( O' )
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 ( ( O )
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Fig. 3 (Walton; from p. 33)