Simple Experiments to Test the Dependence of Gravitational Action on Chemical Composition

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The celebrated experiments performed by R. von Eötvös and coworkers have been considered for decades as bearing conclusive evidence for the rigorous proportionality between inertial and gravitational masses of all substances (weak equivalence principle), and thus for the independence of gravitational mass on the chemical composition of bodies. Renewed interest in the subject was awakened by the 1986 paper by Fischbach et al., which cast doubt on the conclusiveness of Eötvös' experiments. Since the appearance of this paper, various experiments concerning both Newton's law and the weak equivalence principle have been carried out. Here we propose another class of composition-dependent experiments. Preliminary results obtained in one such experiment seem to suggest a dependence of the gravitational mass on the chemical composition of substances. Samples of lead and aluminum, weighing the same at see level, appear to exhibit an effective mass differential Dm(r)/m$_o$*, at 3260 m, of the order of magnitude of* 10^{-5}*, corresponding to an relative effective mass differential per meter of the order of* 10^{-9}*.*

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1. Introduction

The celebrated experiments performed by R. von Eötvös and coworkers (1922) have been considered for decades as providing conclusive evidence for the rigorous proportionality between inertial and gravitational masses of all substances (the weak equivalence principle), and thus for the independence of gravitational mass of the chemical composition of bodies.

New interest for the subject was awakened by the paper of by Fischbach *et al.* (1986), which cast some doubts on the conclusiveness of Eötvös' experiments. Indeed, Fischbach and coworkers pointed out that Eötvös' data left room for a "gravity-like" short range *fifth force*, determined by hypercharge and thus depending on the chemical composition of substances. It was thus suggested that one could have, at one time, a violation of both the inverse-square law and the weak equivalence principle. The fifth force found its first motivation in elementary particle physics, where, as it was suggested, it could account for small effects in kaon physics. General reasons for expecting a fifth force of this nature, falling exponentially with the distance, have been subsequently analyzed by various authors (see, for instance, Barbieri 1986).

At the same time, Fischbach's paper made people realize that a lot of experimental work on the laws of gravity, such as geophysical determinations of the gravitational constant G, and thus, indirectly, checks of the inverse-square law, had been going on in various contexts, and that the situation concerning both Newton's law and the weak equivalence principle was in general far from being settled.

Since the appearance of Fischbach's paper, various experiments on these matters have been devised (most of them have been actually carried out). I refer the reader to general reports such as those of Bizzeti (1989), Adelberger *et al.* (1991), and Fischbach and Talmadge (1992) for comprehensive reviews of the subject.

The effects being sought should manifest themselves as an apparent deviation from the usual $1/r$ Newtonian potential to the general form

$$V(r) = -\frac{Gm_k m_l}{r}\left(1 + ae^{-r/l}\right). \qquad (1$$

In the Earth's gravitational field, a composition-dependent force would manifest itself in the constant a depending on the sample body. A gravity-like short range fifth force can therefore be detected "through a modification of the inverse-square law ($G(r) \neq$ constant) and/or through a composition-dependence of the net acceleration; both effects are generally present in a given experiment. In practice, however, experiments are usually designed to isolate one or the other of these signals for a new force...". (Fischbach and Talmadge 1992, p. 209) Experiments may thus conveniently be classified as composition-independent and composition-dependent.

As a general comment, it may be said that "the anomalies reported in the original Eötvös experiment remain to be understood", but that "no compelling evidence has yet emerged that would indicate the presence of a fifth force" (Fischbach and Talmadge 1992, p. 214), although there have been weak signals that could be interpreted as evidence for it from composition-dependent experiments (Boynton *et al.* 1987), (Thieberger 1987); claims of significant departures from Newton's law have come from gravity tower experiments (Eckhardt *et al.* 1988); deviations manifest themselves also in preliminary results from hydroelectric lake experiments (Mäuller *et al.* 1990; Focardi 1994).

I am here proposing yet another class of composition-dependent experiments, and one which, surprisingly, does not seem to require the level of accuracy required by previous experiments. In this paper I report on preliminary results obtained in one such experiment, which seem to suggest a dependence of the gravitational mass on the chemical composition of the substances.

I would like to mention that I had already found the first indications for the effect some years ago. When I first presented my results at the Department of Physics of the University of Bologna, I received criticisms and suggestions regarding ways of dealing with side effects that one should take into account by the physicists who attended my informal seminar; other suggestions came subsequently from physicists at the Universities of Trieste, Pisa and of the Istituto Colonnetti of Torino. In the following years, I have tried to deal with all the criticisms and follow all the suggestions. Since the results are still there, I have decided to overcome my own doubts and to communicate them, with the hope that someone else will either confirm or disprove them in similar experiments.

The structure of the paper is as follows. In Section 2, I describe the principle of the experiment; in Section 3, the experimental setup, the possible sources of experimental errors and the means I have envisaged to overcome them. Finally, in Section 4, I present the experimental data with some preliminary comments.

2. The Principle of the Experiment

In an ideal gravity tower experiment (perfectly spherical and homogeneous Earth, weightless tower of an indefinite length), overall behaviour as described by Eq. (1) would be directly revealed by a gravimeter or, in principle, by accurate enough weighings of samples of different chemical compositions at various heights. Actual gravity tower experiments exploit towers that rise about 600 meters above local ground level. In order to explore a wider range of differences of level, one must necessarily have recourse to mountains employed as towers. In this case, however, a test body of definite composition is subject to so many side effects as to make it very doubtful that it will ever be possible to reveal differences from the inverse square law. However, if a dependence on the chemical composition is also present, it might manifest itself as a differential variation with the altitude of the acceleration towards the Earth of test bodies, or, in other terms, of their weight, depending on the chemical composition.

From (1) one finds that the order of magnitude of the relative gravitational effective mass difference between two test bodies at the altitude r, a and b, can be expressed as

$$\frac{\Delta m(r)}{m_o} = \boldsymbol{a}_a\left(1 - \frac{m_b \boldsymbol{a}_b}{m_a \boldsymbol{a}_a}\right)\left[\left(1 + \frac{r}{l}\right)e^{-r/l} - 1\right] \qquad (2$$

where $m_o = m_a \cong m_b$. The necessary condition for the effect to be there is of course $m_b \boldsymbol{a}_b \neq m_a \boldsymbol{a}_a$; I am here interested in its r-dependence. However, I want to make a preliminary comparison of its foreseeable order of magnitude, determined by $\boldsymbol{a}_a$, with that of the effects analysed in the experiments recalled above. The experiment I want to describe here has been realized by means of a research balance. Due to the extreme accuracy reached in all the experiments mentioned above, in which instruments such as delicate torsion balances and very accurate gravimeters are used, it would seem completely hopeless to have recourse, in order to test these effects, to as simple an object as a balance. Let me, however, point out the following. In the typical case of the Eötvös experiment one easily derives (Appendix B), for test bodies a and b, the following expression for the Eötvös effect (m^i = inertial mass, $m^G(r)$ = effective gravitational mass at altitude r):

$$\begin{aligned}\left(\frac{m^i}{m^G(r)}\right)_a - \left(\frac{m^i}{m^G(r)}\right)_b &= \frac{G_\infty M}{ar} \\ &= \frac{m^i}{m^G}(\boldsymbol{a}_a - \boldsymbol{a}_b)\left(1 + \frac{r}{l}\right)e^{-r/l}\end{aligned} \quad . (3$$

The order of magnitude of the effect is therefore determined by the *difference* between the values of the constants $\boldsymbol{a}$, assumed to depend on the chemical composition, and not, as in the effect above, by the order of magnitude of the constants themselves. The limit of 10^{-9} on the difference of the inertial to gravitational mass ratios does not, therefore, necessarily imply a similar limit on the effect discussed here, and thus results obtained in an experiment of the type described here showing an effect need not be incompatible with (even much more precise) experiments of a different nature.

I am thus in fact proposing, as I have anticipated, yet another class of composition-dependent experiments, and one which, surprisingly, does not seem to require the level of accuracy required by previous experiments. I am not aware of experiments of this class having been reported. However, indirect information regarding the effect foreseen could perhaps be contained in the data collected by the National Bureau of Standards (NBS) (Pontius 1975; Schoonover *et al.* 1980) in order to test the assumption that "the measurement of the difference in mass between two objects would be the same in all laboratories" (Pontius 1975, p. 379). This difference, arising from the buoyant force of the atmosphere, is given by the product of the computed air density and the difference in the displacement volumes. One expects it to be constant for any pair of samples regardless of location. Equivalently, one expects two samples of equal displacement volumes, hence subject to the same buoyant force, weighing the same at one location, to weigh the same at any other location. A lack of verification of this circumstance could only be attributed to a differential dependence of the effective weight of the samples from the location. The adequacy of the total correction in various laboratories located at altitudes ranging from near sea level to approximately 2000 m was tested by the Mass and Volume Section of the NBS (Pontius 1975). A first analysis of the data indicated a depencence on location; over the pressure range from 0.5 to 2 atm, the claimed magnitude of the effect was of the order of 1 mg in 1 kg 10^{-6}). The source of this anomalous result was subsequently sought in some aspects of the buoyancy correction. Experiments were repeated in a laboratory of the Sandia Corporation at an altitude of about 1600 m (Schoonover *et al.* 1980). The main conclusion was that the the initial alarm "seemed overstated" (Schoonover *et al.* 1980, p. 1348). At any rate, the data seem to allow room for further investigations.

However, it cannot be excluded that an effect is in fact there. In this case, an upper limit on $\Delta m/m_o$ would be 10^{-6}. Pontius (1975) makes reference to research balances with standard deviations on the order of 0.005 mg. I finally used a research balance with a sensitivity of 10^{-4} g.

In this paper I report on preliminary results obtained with this very simple method, which consistently seem to suggest, as I have anticipated, a dependence of the gravitational mass on the chemical composition of the substances.

3. Experimental Setup

In order to test the effect envisaged here, one should prepare bodies of different chemical composition which weigh the same at at given altitude in the Earth's gravitational field, and check whether the weight decrease with the altitude actually depends on the substance. In this experiment, the samples were weighed in places differing in level of about 3260 m, namely the city of Ravenna, at near see level, and a hut just below the top of Mount Marmolada on the Dolomites.

The first effect that one has to take care of in this type of experiment is the Archimedean lift. In contrast with the NBS experiments, I wanted to completely eliminate its effect from the start. To this end, one needs to use samples of different materials not only weighing the same, but of the same volume, so that they are subject to the same buoyant force. For this purpose, side by side with an aluminum sphere, (50.00 ± 0.01) mm in diameter, of the approximate weight of 180 g (details below), I built, at the lathe, from two hollowed out half spheres, a hollow lead sphere of the same external diameter and of the same approximate weight after welding. Electrical welding of the half shells was carried out by soft soldering technique and rectified at the lathe.

My second concern was possible leakage from the hollow sphere in presence of an external depression. Indeed, the presence of a small leak from the lead sphere, which would allow air to escape at the higher altitude, could simulate, totally or partially, the observed results. Electrical welding is highly effective in order to avoid such effects. Nevertheless, in order to check its airtightness, I had the lead sphere plunged in water at the centre of a decompression chamber, which was covered with a strong plate of glass and gasketed (Figure 1). A depression of 65/76 on the vacuometer was created in the chamber, but no air bubbles were seen to form at the weld and cross the water. Since, as it will turn out, the depression produced was about triple of that introduced by the difference of level in the experiment, I was confident that no leakage of air would introduce a differential Archimedean lift in the conditions in which the experiments were run.

Subsequently, following a suggestion of P.G. Bizzeti, the hollow lead artifact was set in a container where an air depression was created, again up to 65/76 of the vacuometer's index. This decompression was kept constant for 2 h 40′. The artifact was weighted immediately after being extracted from the chamber; no difference in weight was observed within the instrument's sensitivity.

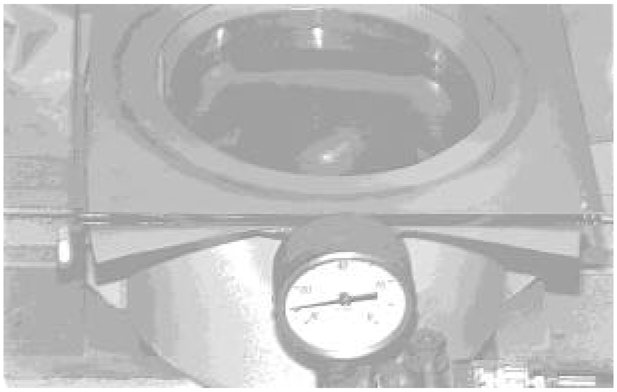

Figure 1 - The figure shows the lead sphere (on a support) plunged in a container filled with water nearly to the brim, inserted in the decompression chamber. The chamber is covered with a plate of bullet-proof glass and gasketed. The picture was taken while the decompression (vacuometer legible with a lens) was slightly larger than that mentioned in the text.

The diameter of the artifact was also measured, immediatly after its extraction from the chamber, using a micrometer accurate to a hundreth of a millimeter. No change of volume was measured. I could therefore be confident that the artifact would not change its volume due to the depression at high altitudes. Note in fact that the order of magnitude of the pressure difference between the two locations is 0.3 atmospheres, *i.e.* much lower than the one produced in the decompression chamber.

Finally, care must be taken to avoid the potential effect of variations in air density, due to changes in temperature, pressure and humidity, on the balance readings. However, these effects, which can be computed to be—especially that of humidity—extremely small, are neutralized by the use of artifacts of the same volume, surface and weight.

In the experiment, I used the above test bodies and a research balance (Mettler model AE240) with the following properties: sensitivity: 0.0001 g; range: 200 g.

In a previous experiment, I used a balance (Mettler model PM6100) with sensitivity 0.01 g, range 6100 g. This allowed me to use test bodies weighing nearly 6000 g.[★] In Ravenna, the two samples were weighed alternatively (series of ten weighings for each sample). I then let fresh air in, for about ten minutes, in order to allow for a change in the Archimedean lift. The series were repeated 1.30 hour later, when it could safely be assumed that the air had come to a standstill. The day after, I reached the hut by cableway at 10.30; I waited until 14 h so the balance could reach optimum working conditions and the samples would adapt themselves to the surrounding temperature. Then, shortly before beginning weighing, I checked the diameters of the two samples, with a caliper accurate to a hundredth of a millimetre at several points marked in ink where they had been mesasured in Ravenna. Within the instrument's sensitivity, no difference in diameter was found (this control was in fact made superfluous by the test run in the depression chamber described above). Within these limits, it could thus be concluded that not only the Archimedean lift was the same for the two samples, but also that it acted in the same ratio on samples of the same weight. Series of weighings were then carried out with the same criteria used in Ravenna.[†] The lead (aluminum) test body exhibited an average difference of weight between the two levels of 2.60 ± 0.01 g (2.57 ± 0.01 g) and 2.54 ± 0.01 g (2.51 ± 0.01 g) respectively for the first and the second series; note that, although the differences change from a series to the next one, the "difference of differences" (DoD) remains stable. I also observed that, for instance, in the first series, the differences in weight between the two levels were never lower, for lead, than 2.60 g, whereas those for aluminum never exceeded 2.57 g. However, the value of the DoD, of 0.04[‡] compared with the balance sensitivity, did not allow a clearcut conclusion. The feeling that an effect was there, and, at the same time, that it had not been clearly exhibited, invited to repeat the experiment with a balance of higher sensitivity. The experiment was thus repeated with the new balance, using experimental procedures similar to those just described. The results are reported in the next section, together with some very preliminary comments.

4. Experimental Results

Typical series of ten weighings are reported in Table 1, to give an immediate feeling of what is going on (data in g).

Table 1 - Typical series of ten weighings. Data are reported in g (± 0.0002 g). First two columns, sea level data; last two, data at 3260 m above sea level.

LEAD	ALUMINUM	LEAD	ALUMINUM
180.8836	180.8918	180.8120	180.8216
180.8835	180.8918	180.8122	180.8217
180.8834	180.8916	180.8122	180.8217
180.8832	180.8914	180.8116	180.8210
180.8832	180.8914	180.8120	180.8217
180.8833	180.8915	180.8120	180.8216
180.8833	180.8914	180.8119	180.8214
180.8832	180.8913	180.8120	180.8216
180.8831	180.8914	180.8122	180.8218
180.8830	180.8912	180.8121	170.8217
Averages:			
180.8833	180.8915	180.8120	180.8216

Note that the samples of aluminum and lead turned out to weigh slightly differently at (approximate) see level (weight difference: 0.0082 ± 0.0002 g). The weight difference at the higher site, however, was sensibly larger, namely 0.0096 ± 0.0002. One may thus put things, intuitively, in this terms: samples of lead and aluminum, weighing the same at see level, appear to exhibit a relative differential effective mass difference [Eq. (2)] $\Delta m(r)/m_o$, at 3260 m, of the order of magnitude of $0.0014/180 \cong 8 \times 10^{-6}$. A simple calculation shows that the result on $\Delta m(r)/m_o$ is consistent with that obtained in the experiment briefly discussed in the preceeding section (proportionality to the weight of the samples). This consistency between results obtained with test bodies widely differing in the volume/surface ratio rules out the possibility that the effect be due to a surface effect (adsorption of air molecules).[§]

[★] Actually, 5840 g. Also in this case, I used an aluminum sphere and a hollow sphere of lead of the same weight and volume with the same external diameter of 157.3 mm.

[†] In both cases, the aluminum and lead test bodies were weighed alternately a total of twenty times each

[‡] In the next footnote I will use in a proportion the numerical value of 0.037 which is in fact derived from the data

[§] The surface/volume ratio varies from 0.38 to 1.20 going from the first to the second experiment. If the effect would depend on some characteristics of the surfaces distinguishing between the different substances, this proportionality would not arise. To rule

At first sight, the results concerning the difference in weight *for each sample separately*, appear to be anomalous with respect to what turns out on the basis of the inverse square law. For an artifact of 180 g, the difference in weight one obtains from the law is in fact

$$180-\left[180\left(\frac{6.3700\cdot 10^{6}}{6.37326\cdot 10^{6}}\right)^{2}\right]\mathrm{g}=0.185\,\mathrm{g}\,.$$

The results above give instead 0.071 g for lead, and 0.070 g for aluminum. However, the measurements at the highest location were carried out some 200 km from the sea. Now, the geodetic line rises gently following the slope of the ground up to about 2000 m above sea level, that is the average altitude around the mountain top. From that altitude, the slope changes abruptly to nearly vertical. In the situation sketched here, one should rather turn to a modified formula, in which the effective difference of level is between 2000 and 3260 m; one then obtains

$$180-\left[180\left(\frac{6.3700\cdot 10^{3}+2\cdot 10^{3}}{6.3700\cdot 10^{3}+3.26\cdot 10^{3}}\right)^{2}\right]\mathrm{g}=0.072\,\mathrm{g}\,.$$

Of course, this calculation cannot be taken too seriously: it does only give us an indication of the type of effect involved. I may add, however, that in various previous circumstances I had noted the same kind of effect. Finally, I want to stress that the main result of the experiment is completely independent of the effect just discussed, since the balance is used in it only as a basis of comparison.

In order to assess both the repeatability of the experiment and the stability of the balance and of the samples, the measurements were repeated at sea level after the measurement at 3260 m. A typical series of ten weighings is reported in Table 2.

Table 2

LEAD	ALUMINUM
180.8837	180.8919
180.8836	180.8918
180.8837	180.8920
190.8838	180.8920
180.8837	180.8920
180.8836	180.8918
180.8840	180.8922
180.8839	180.8922
180.8838	180.8920
180.8838	180.8919
Averages:	
180.8838	180.8920

The average values of this second sea level series differ from those of the first series, due to changes in the environmental conditions (especially temperature). Note, however, that the difference between the averages for lead and aluminum is the same as for the first sea level series, namely 0.0082 ± 0.0002 g. Note that allowing for a hundreth of a millimetre difference in the diameter *d* would, according to

$$\Delta V=\frac{p}{2}d^{2}\Delta d\ ,$$

corresponds to a difference in volume of about 40 mm^3; the corresponding Archimedean lift would be of the order of magnitude of 5×10^{-5} g, which is more than one order of magnitude lower than the measured weight difference.

The relative differential effective mass difference *per meter* is of the order of magnitude of 10^{-9}; it is perhaps worth pointing out that the effect is thus, at the lab scale, of the order of magnitude of the effects sought in other experiments.

The effect needs certainly confirmation, but seems to be there. I might say that it seems to be obstinately there. The results reported here follow in fact a series of similar results obtained in previous experiments, which were run utilizing balances of lower precision— as already mentioned—and smaller differences of level (Mount Fumaiolo and Mount Corno alle Scale, on the Apennines), but nonetheless already showed consistently, if not with the same level of accuracy, the same kind of effect. Some of these were run using artifacts of the same weight and different volume, as well as of the same wolume and different weight; they all showed consistently an effect; however, criticisms addressed to my results concerning the evaluation of the buoyancy corrections convinced me to resort to the solution presented in this paper. I would also like to point out that some of the previous experiments were run using artifacts of copper, zinc and bronze. Results for these materials were intermediate between those found for lead and aluminum.

I may add that, following suggestions by E. Polacco, I wanted to check possible contaminations of magnetic and electric effects. As far as the former are concerned, I had built a frame holding a strong magnet in position above the balance at a distance of 5 mm from the surface of the sample to be weighted. Weighings of the lead and aluminum artifacts, with or without the magnet in place, did not give observable differences. As regards possible electric effects, I put balance and artifacts within a Faraday box, realized in terms of a semispheric net of zincked iron wires covering an aluminum plate. Again, no effect was found.

It was also suggested to run the experiment at various external pressures, by putting the balance under a vacuum bell. Since this turned out to be technically difficult, I had recourse to an equivalent setup, obtained by creating various depressions in a chamber weighted together with the artifacts. These measurements were carried out between the first and the second experiment described in this paper, using artifacts of about 2100 g.

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out surface effects, I had also previously made the following check: I weighed alternately, at sea level and at a high altitude, aluminum cylinders of the same volume and weight, but differing in area in the ratio 227/711. No surface dependence was detected.

No noticeable difference between solid and hollow artifacts was found at 300/760 and 580/760 of the normal atmospheric pressure.

As a final remark, I may observe that the differential effective mass difference would seem to exceed the upper bound that could roughly be derived from the NBS experiments.

Acknowledgments

I have already mentioned helpful criticisms, at an early stage of this work, from members of the Department of Physics of the University of Bologna, namely Profs. S. Bergia, D. Cattani, S. Focardi, A. Forino, M. Galli. Further criticisms and helpful suggestions came subsequently by Prof. Marson (University of Trieste), Prof. E. Polacco (University of Pisa), Prof. P. G. Bizzeti (University of Florence) and Dr. Plassa, of the Istituto Colonnetti of Torino. I am indebted to Prof. A. Bray, of the Politecnico di Torino, for a conversation, and to Dr. R. S. Davis, of the Bureau International Poids et Mesures (Sèvres, France), for a useful correspondence. I wish to acknowledge the participation of Prof. S. Bergia in a previous run of the experiment, which utilized the difference of level between the city of Bologna and a hut on Mount Corno alle Scale, and his help in drafting this paper. All of these contributions are gratefully acknowledged; of course, these physicists do not share any responsibility for the way their suggestions have been applied and for the conclusions of this paper, which are mine alone.

Finally, I wish to thank the referees of the first version of this paper, who, with their stimulating remarks, have helped me in improving it.

Appendix A

It is customary to write the force determined by the potential (1) as

$$\vec{F}(r) = -G(r)\frac{m_k m_l}{r^3}\vec{r}\,, \qquad \text{(A1}$$

$$G(r) = G_\infty\left[1+\boldsymbol{a}_l\left(1+\frac{r}{l}\right)e^{-r/l}\right], \qquad \text{(A2}$$

where G_∞ actually denotes the Newtonian gravitational constant. Then, at fixed m_k, say m_k = the Earth's gravitational mass M, everything goes as if the l-th test body had an r-dependent effective gravitational mass

$$m_l(r) = m_l\left[1+\boldsymbol{a}_l\left(1+\frac{r}{l}\right)e^{-r/l}\right]. \qquad \text{(A3}$$

The effective mass difference between test bodies a and b is then

$$m_l(r) = (m_a - m_b) + (m_a\boldsymbol{a}_a - m_b\boldsymbol{a}_b)\left(1+\frac{r}{l}\right)e^{-r/l}\,.$$

Equality of effective masses must be *defined* at some level. If one fixes $\Delta m(0) = 0$, then

$$\Delta m(r) = (m_a\boldsymbol{a}_a - m_b\boldsymbol{a}_b)\left[\left(1+\frac{r}{l}\right)e^{-r/l} - 1\right].$$

whence Eq. (2) of the text follows if one sets $m_o = m_a \cong m_b$.

Appendix B

In the gravitational field of the Earth (mass M), modified gravitational "constant" $G(r)$ (Eq. A2), the acceleration a of a test body of inertial mass m^i and gravitational mass m^G is determined by

$$\frac{G(r)M}{r}m^G = m^i a \qquad \text{(B1}$$

For constant m^i and m^G, the test body's acceleration depends on $\boldsymbol{a}$, which may depend on the body's chemical composition, through $G(r)$. This corresponds to a violation of the weak equivalence principle, due to the replacement of the gravitational mass m^G in terms of the effective gravitational mass (Eq. A3). Indeed, the ratio between m^i and the effective gravitational mass is now

$$\frac{m^i}{m^G\left[1+\boldsymbol{a}_k\left(1+\frac{r}{l}\right)e^{-r/l}\right]} = \frac{G_\infty M}{ra} \cong \frac{m^i}{m^G}\left[1-\boldsymbol{a}_k\left(1+\frac{r}{l}\right)e^{-r/l}\right].$$

The difference between the ratios for two test bodies k and l is the given by:

$$\frac{m^i}{m^G}(\boldsymbol{a}_k - \boldsymbol{a}_l)\left(1+\frac{r}{l}\right)e^{-r/l}$$

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***Notice*: Dr. Jean-Claude Pecker has been awarded the Lodén Prize for constructive criticism of the Big Bang cosmology. The Prize, awarded by the Uppsala Observatory in Sweden, is named for the Swedish astronomer Lars Olof Lodén, who retired last year. Dr. Pecker is the first recipient of the award. We extend our congratulations to Dr. Pecker on his signal achievement.**