A Fractal Universe with Discrete Spatial Scales: In Memory of Toivo Jaakkola

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The work of this paper is based on work which has been described in a preliminary form elsewhere (Roscoe 1995), and it applies the formalism developed there to the problem of deriving the cosmology for a universe which is in a state of gravitational equilibrium. It predicts that, in such a universe, material is distributed in a fractal fashion with fractal dimension ***two*** *whilst redshifts necessarily occur in integer multiples of a basic unit and, given a certain model for light propagation, the measured magnitudes of peculiar velocities will increase in direct proportion to cosmological redshift.*

The first of these predictions is strongly supported by the results of the most modern pencil-beam and wide-angle surveys, whilst the second conforms with the results of very recent rigorous analyses of accurately measured redshifts of nearby spiral galaxies and the third is in qualitative agreement with the very limited data available. The observational support for these predictions is described in detail in the text.

Toivo Jaakola was convinced that all the evidence supported the idea of an infinite self-sustaining equilibrium universe. Many of us agreed with some of his arguments, and I was no exception. The following article falls into this pattern: it describes a self-sustaining equilibrium universe; but infinite it is not, and homogeneous it is not ...

1. Introduction

The following work describes the application of the gravitation theory described earlier (Roscoe 1995) to the problem of deriving a cosmology. This latter presentation is a preliminary and incomplete development of work now completed, and in preparation for publication elsewhere. Preprints are available on request. The underlying gravitation theory, which is predicated upon the idea of a discrete and finite model universe, is distinguished in the fact that, according to it, concepts of spatial and temporal measurement are undefined in the absence of mass—in this sense, it conforms to the strictest possible interpretation of Mach's Principle.

There is evidence, discussed in §2, to suggest the real Universe is in a state of approximate thermodynamic equilibrium; this possible state is used to justify the cosmological principle that the model universe is in a state of exact gravitational equilibrium. The mass-distribution corresponding to this state is calculated in §3 and §4, and is found to be fractal with a fractal dimension of *two*. This mass-distribution prediction is very strongly supported by the results of several modern surveys, and this evidence is discussed in §5.

The discrete nature of material in the model universe is considered in §6, and is found to imply a discretization of distance scales which leads, in §7, to the conclusion that redshifts must increase in integer multiples of a basic unit; the evidence supporting this is discussed in §8. The discretization of distance scales occurs in such a way that spatial and temporal measurement scales in remote localities undergo systematic change, discussed in §9, which has implications for kinematics and the nature of light, discussed in §10 and §11 respectively. The predicted kinematics has implications for the apparent behaviour of the *peculiar velocities* of galaxies; these are discussed in §12 where it is shown how one consequence of the scale-change phenomenon is that the estimated magnitudes of peculiar velocities will appear to vary linearly with the cosmological redshift. The evidence supporting this conclusion is discussed in §13. The discussion of §3 also leads to the idea of a *material vacuum*, existing in the model universe, and the implications of this are briefly considered in §14.

The equations of motion, derived for a spherically symmetric distribution of material particles, with an isotropic velocity distribution, are given by

$$\ddot{\mathbf{r}} = -\frac{\partial V}{\partial r}\hat{\mathbf{r}}\ ,$$

where r is the position vector defined with respect to the global mass-centre,

$$V \equiv -\frac{1}{2}\langle \dot{\mathbf{r}}, \dot{\mathbf{r}} \rangle = -\frac{r_0 g A}{2} + \frac{B}{2A}\dot{\Phi}^2\ , \qquad (1)$$

g is the gravitational constant, r_o is a constant defined below, and A, B are defined by

$$A \equiv \frac{M}{\Phi},$$
$$B \equiv -\left(\frac{M}{2\dot{\Phi}^2} - \frac{M'M'}{2a_0 M}\right), \qquad (2)$$
$$M' \equiv \frac{\mathrm{d}M}{\mathrm{d}\Phi},\ \ \Phi = \frac{1}{2}\langle \mathbf{r}, \mathbf{r} \rangle.$$

The function M is the mass-distribution function, for which a broad admissible class is given by

$$M(r) = m_0 \left(\frac{r}{r_0}\right)^{g\,1}\ , \qquad (3)$$

where m_o has dimensions of *mass*, and r_o is the radius of the volume containing mass m_o. It is to be noted from this expression that $M/r^{g\,1}$ is a global constant, so that the particular choice of r_o has no significance for (3). Finally, the defining relationship between time scales and distance scales is given by

$$\mathrm{d}t^2 = \left(\frac{\Phi^2}{r_o g M^2}\right) g_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j . \qquad (4)$$

whilst the metric tensor is given by

$$g_{ab} = A\delta_{ab} + B x^a x^b . \qquad (5)$$

It follows from (2), (4) and (5) that if $M = 0$, so that there is no mass, then concepts of time and distance are undefined.

The foregoing equations of motion can be identified with those given in Roscoe (1995) by making the substitution $M = \alpha U$. It is to be noted that the potential form of the equations is not given in this early development, nor is the interpretation of $M \equiv \alpha U$ as a mass-distribution made there. Preprints of the complete development are available on request.

2. A Simple Cosmological Principle

There is some evidence, briefly discussed below, which suggests the observable universe might be in a state of approximate thermodynamic equilibrium with respect to the various energy sources within it. If this is the case, it would follow that gravitational energy must be included as one of these sources; correspondingly, the most simple realistic cosmological principle applicable to the model universe is the condition that it is in a state of *gravitational equilibrium*. However, before the consequences of this most simple possible of cosmological principles are worked through, we shall consider some of the evidence supporting the argument that the cosmic ray flux, the cosmic background radiation (both extragalactic sources) and our own galaxy's starlight field are in thermal equilibrium. We are indebted to Assis and Neves (1995) for much of the following discussion

One of the earliest (if not the earliest) predictions of a background temperature to space, and estimations thereof, is that of Guillaume (1896) who used Stefan's Law (1879) to calculate the equilibrium temperature, arising from stellar radiation, of an inert body placed in the interstellar space of contemporary understanding; this was equivalent to calculating the 'temperature of space', and the figure arrived at was 5.6 K. A similar black-body calculation was given by Eddington in 1926 (reprint 1988), and he arrived at the figure 3.18 K, calling it explicitly the 'temperature of interstellar space'.

It was known by 1928 (Millikan & Cameron) that cosmic rays have an extragalactic origin and, subsequently, Regener (1933 or 1995 for an English translation) calculated the equilibrium temperature of an inert body (having the necessary dimensions to absorb cosmic rays) which is placed in a 'sea' of cosmic radiation, and found this to be 2.8 K. Regener went on to argue that, because of the extragalactic origin of cosmic rays, and because of the (assumed) extreme weakness of starlight in inter-galactic space, then 2.8 K must be the 'temperature of intergalactic space'.

The earliest *Hot Big Bang* predictions for the existence of the CBR with a black-body spectrum were given by Alpher & Herman (1949) and Gamow (1952,1953), and these authors estimated the 'temperature of space' variously in the range 5 K to 50 K; After the observations of Penzias & Wilson (1965), we are now aware that the CBR does exist as an additional extragalactic energy field, with a temperature of 2.7 K.

So, there are at least three independent sources of energy—galactic starlight, cosmic rays and the CBR—which have been used to estimate the 'temperature of space', giving answers which suggest that the three sources are in near thermodynamic equilibrium. In addition, Sciama (1971) has pointed out that the turbulent energy density of interstellar gases and the energy density of the interstellar magnetic field is similar to that of the aforementioned sources, and so the net picture is entirely consistent with the idea of a universe which is in an approximate thermodynamic equilibrium.

3. The Equilibrium Universe

If the model universe is in gravitational equilibrium, then the net gravitational force at every point within it is necessarily zero, so that $\ddot{\mathbf{r}} = 0$ everywhere. Consequently, the potential is constant everywhere so that, by (1),

$$-\frac{r_o g A}{2} + \frac{B}{2A}\dot{\Phi}^2 = V_o , \qquad (6)$$

where V_o is the value of the constant potential. Using the definitions of A, B, Φ given at (2), this equation can be written as

$$-\frac{r_o g M}{r^2} - \frac{\dot{r}^2}{2}\left\{1 - \frac{1}{4\alpha_o}\left(\frac{r}{M}\frac{\mathrm{d}M}{\mathrm{d}r}\right)^2\right\} = V_o .$$

An easy means of solving this equation is arrived at as follows: The equation gives the form of $M(r)$ which is consistent with the constraint $\ddot{\mathbf{r}} = 0$ for all motions in the model universe. Of all possible trajectories of this type, there will be a subclass which pass directly through the centre-of-mass, and will therefore have zero angular momentum about this point. These *particular* trajectories satisfy $\dot{r} =$ constant where, because the speed of the particle concerned is arbitrary, then *constant* is arbitrary; consequently, these trajectories can be considered specified by $\dot{r}^2 = 2\lambda_1$, for arbitrary positive values of λ_1. The above equation for $M(r)$ can be now written

$$\left(-\frac{r_o g M}{r^2} - V_o\right) - \lambda_1\left\{1 - \frac{1}{4\alpha_o}\left(\frac{r}{M}\frac{\mathrm{d}M}{\mathrm{d}r}\right)^2\right\} = 0 .$$

Since λ_1 is simply a measure of an arbitrary constant speed, then this equation must be decomposable into

$$\left(-\frac{r_o g M}{r^2} - V_o\right) = 0 \quad \text{and} \quad \left\{1 - \frac{1}{4\alpha_o}\left(\frac{r}{M}\frac{\mathrm{d}M}{\mathrm{d}r}\right)^2\right\} = 0 . \qquad (7)$$

According to the first of these equations,

$$M(r) = -\frac{V_o r_o}{g}\left(\frac{r}{r_o}\right)^2 ,$$

which satisfies the second equation if $\alpha_o = 1$. This solution is a special case of the more general admissible form (3) so that, finally, the mass-distribution function appropriate to an equilibrium model universe is

$$M(r) = m_o\left(\frac{r}{r_o}\right)^2 \qquad (8)$$

where, by comparing the two forms of $M(r)$, the value of the constant potential is found to be given by

$$V_o = -\frac{g\,m_o}{r_o} .$$

Since m_o, in (8), has dimensions of *mass*, it must be interpreted as the amount of mass contained in a sphere of arbitrarily chosen radius r_o. It is to be noted that the definitive constant value—lacking all arbitrariness—given to the constant potential in the present equilibrium case can only be interpreted to represent some kind of *absolute* ground state energy, or vacuum energy, associated with the system.

Finally, if (7) is compared with (6), it can be seen how the second of (7) is equivalent to $B = 0$ so that, with (2), (5) and (8), the metric tensor for the equilibrium universe is given as

$$g_{ab} = \left(\frac{M}{\Phi}\right) d_{ab} = \left(\frac{2\, m_o}{r_o^2}\right) d_{ab} \,. \qquad (9)$$

4. The Model Fractal Universe

The equilibrium model universe is characterized by $\ddot{\mathbf{r}} = 0$, which means that *all* points in the space are dynamically equivalent. Consequently, there is no dynamical experiment in the space which can distinguish between any pair of points, and hence there is no way of determining the position of a global mass-centre. Since a unique origin for the mass distribution (8) cannot now be defined, then it must be considered true about arbitrarily chosen origins in the space, and this amounts to the statement that mass is distributed in a self-similar, or fractal, fashion with a fractal dimension of *two*.

A direct corollary of this argument is the fact that, if $M(r)$ has any form, other than (8), then potential gradients must exist, so that $\ddot{\mathbf{r}} \neq 0$ necessarily. As a consequence, it becomes possible to determine a unique global-mass centre and so the corresponding $M(r)$ cannot be describing a fractal distribution of mass, since such distributions are necessarily isotropic about *all* points in the space. So, in conclusion, the only possible *fractal* distribution of mass in the model universe is the one which has fractal dimension two.

5. A Fractal Universe, The Evidence

A basic assumption of the *Standard Model* is that, on some scale, the universe is homogeneous; however, in early responses to suspicions that the accruing data was more consistent with Charlier's conceptions of an hierarchical universe (Charlier 1908, 1922, 1924) than with the requirements of the *Standard Model*, de Vaucouleurs (1970) showed that, within wide limits, the available data satisfied a mass distribution law $M(r) \approx r^{1.3}$, whilst Peebles (1980) found $M(r) \approx r^{1.23}$. The situation, from the point of view of the *Standard Model*, has continued to deteriorate with the growth of the data-base to the point that, (Baryshev *et al.* 1995)

> *...the scale of the largest inhomogeneities (discovered to date) is comparable with the extent of the surveys, so that the largest known structures are limited by the boundaries of the survey in which they are detected.*

For example, several recent redshift surveys, such as those performed by Huchra *et al.* (1983), Giovanelli and Haynes (1986), De Lapparent *et al.* (1988), Broadhurst *et al.* (1990), Da Costa *et al.* (1994) and Vettolani *et al.* (1994) *etc.* have discovered massive structures such as sheets, filaments, superclusters and voids, and show that large structures are common features of the observable universe; the most significant conclusion to be drawn from all of these surveys is that the scale of the largest inhomogeneities observed is comparable with the spatial extent of the surveys themselves. So, to date, evidence that the assumption of homogeneity in the universe is realistic does not exist. By contrast, evidence for the fractal nature of the matter distribution is becoming increasingly strong; for example, Coleman *et al.* (1988) analysed the *CfA1* redshift survey of Huchra *et al.* (1983), and found $M(r) \propto r^{1.4}$ for this sample; subsequently, the *CfA2* survey of Da Costa *et al.* (1994), which is an extension of the *CfA1* survey out to about twice the depth, has been analysed by Pietronero and Sylos Labini (1995) to reveal $M(r) \propto r^{1.9}$. The pencil beam survey data accumulated in ESO Slice Project (Vettolani *et al.* 1994), which reaches out to 800 Mpc, has been similarly analysed (Pietronero and Sylos Labini (1994)) to conclude that, within this data, the distribution of galaxies conforms to the fractal law $M(r) \propto r^2$ up to the sample limits and, according to Baryshev *et al.* (1995), this same result of fractal distribution of dimension ≈ 2 has been found in the analysis of other deep redshift surveys such as those of Guzzo *et al.* (1992) and Moore *et al.* (1994).

To summarize, for more than twenty years evidence has been accumulating that material in the universe appears to be distributed in an hierarchical, or fractal way—in direct opposition to the requirements of the *Standard Model*—and the results of the most modern deep and wide angle surveys are consistent in suggesting the distribution law $M(r) \propto r^2$, valid about arbitrarily chosen centres. This empirical law, which describes a self-similar mass distribution of fractal dimension two, *is in direct conformity with the mass distribution law derived, for a universe in gravitational equilibrium, in this paper.*

6. Discrete Mass Implies Discretized Distance Scales

The model universe was defined, in the first instance, to consist of a *finite* amount of *discrete* material, and it was the finite quality which allowed the definition of the global mass-centre, and hence enabled the theory to be developed as it has been; in the following, the discrete quality of mass in the model universe is considered, and shown to imply the discretization of distance scales. At first sight, this seems to be a surprising conclusion but, when it is remembered that, according to the theory, concepts of space and time cannot be formulated in the absence of mass, then it appears reasonable to expect that a discrete matter distribution must imply discrete space.

We begin by considering the mass distribution function which, according to (3), is given by

$$M(r) = m_o \left(\frac{r}{r_o}\right)^g .$$

When g is real then $M(r)$ varies continuously through real values with r, and so the discrete quality of the model universe cannot be made manifest in this case. However, the analysis which gave rise to the foregoing expression for $M(r)$ does not exclude the possibility of g assuming complex values so that, with g written as explicitly complex, the most general expression of $M(r)$ is

$$M(r) = m_o \left(\frac{r}{r_o}\right)^{g_o + i g_1}$$

for $i \equiv \sqrt{-1}$ and real g_o and g_1. The function $M(r)$ now only takes real positive values at the set of discrete points

$$r_k = r_o \exp\left(\frac{2\,kp}{g_1}\right), \;\; k = 0, \pm 1, \pm 2, \ldots \qquad (10)$$

and so, from point to point, $M(r)$ varies discretely over real values, as required for the model universe. It follows that, for perfect rigour, the whole analysis to this point should be recast from a continuum form into a discrete form, where r is discretized according to (10). However, for the sake of brevity and convenience, the discrete analysis will only be applied from (7) onwards.

Defining the derivative in (7) in terms of differences, according to

$$\frac{dM}{dr} \equiv \frac{M_k - M_{k-1}}{r_k - r_{k-1}},$$

and using (10), the first of (7) is found to be satisfied by

$$M_k \equiv M(r_k) = -\frac{V_o r_o}{g}\left(\frac{r_k}{r_o}\right)^2, \quad r_k = r_o \exp\left(\frac{2kp}{g_1}\right), \quad k = 0, \pm 1, \pm 2, \ldots$$

whilst the second of (7) is found to be satisfied only when

$$4a_o = \left[1 + \exp\left(-\frac{2p}{g_1}\right)\right]^2.$$

Notice that, according to (10), there is no such thing as an origin $r = 0$; it then becomes natural to interpret r_o as a form of 'reference surface' from which displacements are calculated. In this case, (10) gives, for the value of non-negative displacements,

$$\Delta_k = r_k - r_o = r_o\left[\exp\left(\frac{2kp}{g_1}\right)\right], \quad k = 0, 1, 2, \ldots$$

If g_1 is large compared to $2kp$, this gives

$$\Delta_k \approx \frac{2kpr_o}{g_1}, \quad k = 0, 1, 2, \ldots \qquad (11)$$

A crucially important point about the foregoing analysis is that is valid about *arbitrary* points in the space, because of the fractal nature of the equilibrium mass-distribution. So, although (11), taken as a statement about the nature of 'space' about any origin, appears paradoxical, this is only so when 'space' is imagined as something which has properties independently of its material content; but, when it is remembered that, here, 'space' is merely a *metaphor* for the relationships between material and that, in this case, the fractality of the matter distribution ensures it looks the same from all origins, then the idea of (11) being true about arbitrary points presents no difficulty of comprehension.

7. Quantized Redshifts

The considerations of the previous section, together with Hubble's Law, make the existence of the quantized redshift phenomenon axiomatic: specifically, Hubble's Law and (11) together give

$$cz = H(r_k - r_o) \approx H\frac{2kpr_o}{g_1}, \quad k = 0, 1, 2, \ldots$$

Apart from the quantal aspect, one interesting thing about this redshift-distance relationship is that it inherently requires redshifts to have a non-trivial 'zero-surface' from which the Hubble Law is valid. Such a surface is, in fact, well known to be a feature of the real redshift phenomenon, and Sandage (1986) puts this surface at about $r_o = 1.5$ Mpc.

8. Quantized Redshifts: The Evidence

The conclusions of §7, that redshifts necessarily occur as integer multiples of a basic unit, has been the substance of claims made for the past twenty years by Tifft & Cocke (1976, 1980, 1984, 1990); these claims have generated considerable dissension, but not much reasonable discussion. However, in independent studies Guthrie & Napier (1990, 1991, 1996) have tested the *specific* hypothesis of Tifft & Cocke, *that quantization at 72 km/s and 36 km/s exists in the redshifts of low redshift spirals*, in a statistically rigorous manner using independent data sets, characterized by their high accuracy and totalling several hundred objects. They found that, after the redshifts were corrected for the solar motion about the galactic centre, then Tifft & Cocke's basic hypothesis is confirmed at the level of virtual certainty for the samples analysed.

9. An Hierarchy of Measurement Scales

Equation (10), which defines the sequence of possible radial shells definable from the origin, gives

$$r_k - r_{k-1} = r_o \exp\left(\frac{2kp}{g_1}\right)\left[1 - \exp\left(\frac{-2p}{g_1}\right)\right],$$

which can be directly interpreted as the minimum possible distance interval definable at r_k. Since this interval increases with k, it follows that, from the perspective of an observer at the origin, there is an hierarchy of increasing local spatial scales at increasing distance and, by the comments at the end of §6, this hierarchy will be apparent from arbitrary origins.

To understand the behaviour of the temporal scales, it is necessary to refer to the defining relationship between time scales and distance scales given, for the general case, at (4) as

$$dt^2 = \left(\frac{\Phi^2}{r_o g M^2}\right) g_{ij}\, dx^i dx^j.$$

Using the prescriptions of the mass-function and the metric tensor in the equilibrium universe given at (8) and (9) respectively, together with the equivalence $dx^i dx^j d_{ij} \equiv |d\mathbf{r}|^2$, this can be written

$$dt^2 = \left(\frac{r_o}{2gm_o}\right)|d\mathbf{r}|^2,$$

where, as given in §3, m_o denotes the amount of mass contained in a sphere of arbitrarily chosen radius r_o. This can be put in a form more useful for present purposes by noting, from (8), that $M(r)/r^2$ is a *global* constant of the system, and so $2gm_o/r_o^2$ is also a global constant. Denoting this latter constant by a^2, the above equation can be written as

$$dt = \frac{1}{a\sqrt{r_o}}|d\mathbf{r}|. \qquad (12)$$

This expression is then the defining relationship between time scales and distance scales in the equilibrium universe. However, it is to be noted that, for given $d\mathbf{r}$, the elapsed time, dt, depends upon the arbitrary choice of the radius-parameter r_o; this can only mean that the choice of r_o amounts to choosing the *clock* with which the passage of time is to be measured. If physical substance is to be assigned to the chosen 'clock', then a reasonable working hypothesis would be that it consists of the ensemble of material, mass m_o, contained within the radius r_o sphere. Note that, according to this interpretation of (12), the more massive the clock, then the more slowly it records the passage of time.

To summarize, along with the hierarchical distribution of matter in the fractal universe, there are corresponding hierarchies of spatial and temporal measurement scales.

10. Kinematics

Whilst (12) defines the relationship between time and distance scales in the equilibrium universe, it also necessarily defines the equation of motion for particles in this universe, given by

$$|\dot{\mathbf{r}}| = a\sqrt{r_o} \equiv \sqrt{-2\,V_o}\,, \qquad (13)$$

where a is the universal constant defined in §9 and V_o is the ground state energy of the system, identified in §3; it is clear from (13) that the choice of r_o amounts to choosing the clock used to define the velocity $\ddot{\mathbf{r}}$. There are three fundamental peculiarities arising from this equation, considered in turn below.

Firstly, (13) says that, for a given clock, *all* particles in the model universe have velocities of the same magnitude and this velocity corresponds to the ground state energy of the equilibrium system, identified in §3; the absence of any other constraint implies that the directions of these velocities must be uniformly random. Thus, according to the equilibrium model, the distribution of material particles in the model universe has kinematic properties which exactly mirror those existing in an isotropic distribution of photons. With the exception of any statements about the distribution of mass in these material particles, what emerges is a material analogue of the cosmic background radiation. This seems very odd when set against conventional experience, but it must not forgotten how this experience relates exclusively to a world of electromagnetic and non-equilibrium gravitational forces, neither of which is part of the equilibrium model, and both of which occur on a scale far below that presumed for this model.

Secondly, by definition, (13) assumes the existence of some absolute rest-frame which, at the beginning of this development, was assumed given by the global mass-centre. However, in §4, it was indicated that, in an equilibrium universe, it becomes impossible to locate the global mass-centre and therefore impossible to define an absolute state-of-rest in terms of it. Since (13) is an equation which arises from the assumption of universal equilibrium, it follows that, implicit to the whole development, there must be another means of determining the absolute rest-frame and the answer lies in the considerations of the previous paragraph: Specifically, the predicted kinematic structure will lead to a Doppler redshift field that will appear (statistically) isotropic when viewed from the absolute rest-frame, but will be subject to a dipole displacement when viewed from any other frame. So, the absolute rest-frame is that in which no dipole effects exist in the observed redshift field.

Thirdly, in (13) a is a universal constant, and r_o is the radius of the sphere which contains mass m_o; however, the model universe is defined to be finite, and so it follows that there is a limiting value of r_o defining the smallest sphere which contains the total of the universal mass. Denoting this value as r_* then, by (13),

$$|\dot{\mathbf{r}}| \le a r_*^{1/2} \equiv \sqrt{\frac{2gm_*}{r_*}}\,. \qquad (14)$$

That is, although the choice of clock in the model universe is arbitrary, there is one *fundamental* clock, which consists of the whole ensemble of mass in the model universe, and according to which, velocities attain their maximum value.

11. The Nature of Light

To the extent that the presented theory possesses the concepts of a universal time together with a separate three-dimensional physical space, then it is a 'classical theory'. However, consideration of (10), according to which a radial displacement from a given origin can only have certain admissible values, shows immediately that a photon can no longer be considered as something with a continuous trajectory, but must be considered as a sequence of *resonances* at discrete locations. The theory tells us nothing about the rate at which these resonances propagate, and so further progress can only be had by introducing an *ad hoc* propagation model. Since it is necessary for any such model to be consistent with the kinematic structure of the theory, then it must have the general form of (13), but with a replaced by another value appropriate to light-propagation. So, consider the form $c_o = b\sqrt{r_o}$ where c_o is the speed of light measured by the r_o-clock and b is a universal constant. Since c_o can never be zero, then this equation implies that r_o must have a minimum non-zero value which represents the minimum dimension of a physical clock. Consequently, we can write

$$c_o = c_{\min}\sqrt{\frac{r_o}{r_{\min}}} \equiv b\sqrt{r_o} \qquad (15)$$

for a simple light-propagation model which is consistent with the kinematic structure defined by (13).

12. Peculiar Velocities

In the context of the *Standard Model*, the phrase 'peculiar velocities' refers to real motions that galaxies might possess, defined with respect to some fundamental rest-frame, and generated by local gravitational gradients. In the present context, the basic assumption is that the peculiar motions arise wholly out of the kinematic structure of the equilibrium universe and, in the following, the extent to which the observations support this assumption is considered.

In effect, any practical determination of the peculiar velocity of a distant object involves an estimation of the object's distance made on the basis of magnitude information; this is then used to estimate the corresponding *cosmological redshift* which should be associated with the object. This estimated cosmological redshift is compared with the actually measured redshift of the object, and the difference between the two redshifts is assumed to give that component of the measured redshift which arises in consequence of a radial Doppler effect; the radial component of the object's peculiar velocity is then inferred from that. More specifically, in the conventional way of doing things, the 'Doppler shift' of an object at distance r_o is estimated as a wavelength-shift defined relative to the observer's local measurement standards, and the peculiar velocity calculated from that.

However, by the considerations of §9, it is known that, from the point of view of any observer, measurement standards vary with radial location. So, suppose the 'Doppler shift' of the r_o-object is estimated as a wavelength-shift defined relative to the measurement standards at r_o, rather than to the observer's measurement standards, and suppose this estimate is labelled $z_o^{(D)}$; in this case, a Doppler shift is being estimated purely in terms of r_o-scales, and so any expression which relates $z_o^{(D)}$ to velocities, $z_o^{(D)} = f(v/c)$ say, must define v and c in terms of the r_o-clock. Therefore, using (13) and (15), the Doppler shift of an object estimated in terms of the scales at the object can be expressed as

$$z_o^{(D)} = f\left(\frac{|\dot{\mathbf{r}}|}{c_o}\right) = f\left(\frac{a}{b}\right),$$

which is a *global* constant, since a and b are global constants. If the estimated cosmological component of an object's measured redshift is taken as an indicator of the measurement scales at the object, then this latter result simply means that the Doppler component of the wavelength-shift *expressed in terms of our local scales* must increases in direct proportion to the cosmological component of the wavelength shift. Consequently, the corresponding estimates of peculiar velocity magnitudes will appear to increase linearly with distance. So, they will be small for small r_o and large for large r_o.

13. Peculiar Velocities: The Evidence

The available evidence falls into three broad categories: (a) objects with redshifts in the range (0–500) km/s, (b) objects with redshifts in the range (800–2000) km/s and (c) objects with redshifts in the range (2000–15000) km/s. All the evidence is indirect, and differs in type between the cases.

For the first category, Karachentsev & Makarov (1996) analyse the local velocity field using a sample of 103 galaxies with maximum redshifts of 500 km/s. There are two surprising results arising from their analysis, only one of which they note: the noted result is that the dispersion of the radial components of the peculiar velocities is 72 ± 2 km/s throughout their sample volume and this value is *the same for dwarf and giant galaxies*. This is contrary to the standard expectation which, by the equipartition of kinetic energies in a random 'gas', would have the small objects moving more rapidly that the large objects; according to the presented view, all objects have peculiar velocities of identical magnitudes, independently of their size, and so this view is consistent with the Karachentsev & Makarov result. The second, un-noted, point arises from the fact that the quoted values for the dispersion of peculiar velocities, given for increasing sample volume, have a *remarkably* stable value; this value varies by no more than about 3\% of the mean value, 72 ± 2 km/s, when the sample size gets above 12 objects and persists up to the full sample of 103 objects. The significance of this is profound, since it indicates very strongly that the peculiar velocities have *non-Gaussian* statistics—a conclusion which is also directly contrary to standard expectations, but which is consistent with the presented view, since the only randomness arises from the directions of peculiar velocities.

For the second category, Guthrie & Napier (1996) analysed a large sample of galaxies in the range (800–2000) km/s, primarily to test the Tifft hypothesis that redshift quantization is a real phenomenon. In the course of their analysis, they performed Monte-Carlo simulations in which the real redshift data was perturbed by normal random error; they found that when the mean of this error exceeded about 4 km/s, then the signal indicating the presence of redshift-periodicity disappeared. From this, they conclude that the peculiar velocity magnitudes of the objects in their data base have an upper bound of about 4 km/s—at face value, this latter figure conflicts in an obvious way with the Karachentsev & Makarov values for peculiar velocities. However, the alternative view, expressed in §12, is that peculiar velocities will be increasingly recognized to have very strange properties that will allow a consistent resolution of such apparent conflicts.

For the third category, the most extensive *specific* peculiar velocity survey completed to date is that by Lauer & Postman (1994, 1995) which had the specific aim of measuring the velocity of the local group with respect to an inertial frame defined by the 119 Abell clusters within 15,000 km/s. Since this inertial frame was to be defined with respect to a very large amount of matter distributed over the whole sky, it was expected to be approximately stationary in the CBR frame, with the effect that the calculated local-group velocity should approximate the COBE vector. However, the analysis of the radial component of the Abell peculiar velocities appears to indicate that these 119 clusters are participating in a bulk flow of approximately 689 km/s with respect to the CBR—a result which Lauer & Postman say surprised them; they conclude that, if the CBR can be considered as a valid frame of rest, then the calculated bulk flow must arise from the gravitational action of large material concentrations beyond 100 Mpc (*cf*. the discussions of Baryshev *et al.* (1995), §5). Furthermore, as Strauss, Cen, Ostriker, Lauer & Postman (1995) observe, this result is extremely difficult to understand on the basis of the *Standard Model*, or any of the popular variants.

By contrast, the results of these analyses can be readily understood from the perspective of the presented work: Specifically, from the considerations of §3, in which the impossibility of giving a dynamical meaning to the notion of a mass-centre in an equilibrium universe was indicated, then the Lauer & Postman concept of an Abell clusters inertial frame is dynamically meaningless—if the observed 'fractal two' nature of the real Universe is taken to indicate a condition of global equilibrium. It follows that the figure of 689 km/s quoted for the supposed bulk flow of the 119 Abell clusters is simply an estimation of a weighted mean of the radial components of the estimated peculiar velocities, measured in the CBR frame, and has no dynamical significance whatsoever. From this viewpoint, the bulk flow is not a bulk flow at all—therefore not requiring any mass-concentrations whatsoever to explain it—and the figure of 689 km/s calculated for the weighted mean velocity of the Abell clusters can be understood in terms of the individual clusters having large measurable radial velocities (*cf*. (13), and the associated clock-assumption) with a large dispersion arising from inhomogeneities in their distribution over the sky.

To summarize, the Karachentsev & Makarov analysis, involving objects with redshifts in the range (0–500) km/s, leads to the general inference that the statistics of peculiar velocities in this range are independent of object masses, contrary to standard expectations, and exhibit a strong uniformity independently of sample size which is contrary to the behaviour expected on the basis of the standard assumption that peculiar velocity magnitudes are random normal variables. The Guthrie & Napier analysis, involving objects with redshifts in the range (800–2000) km/s, leads to the inference that the magnitudes of peculiar velocities (assumed to have Gaussian statistics) of the objects concerned have an upper bound of ≈ 4 km/s, whilst the Lauer & Postman analysis, involving objects with redshifts in the range (2000–15000) km/s, leads to the inference that the peculiar velocities of the objects concerned are of the order 700 km/s. It is therefore possible to conclude that, overall, the observations provide *qualitative* support for the kinematic structure described by (13). An extremely interesting question is whether future observations will provide support for the *quantitative* kinematic structure described by (13); the resolution of this question will require considerably more data than is currently available. However, Lauer & Postman are planning a survey out to 24000 km/s, and the results of this might begin to provide an answer to the question.

14. The Role of the Vacuum?

Prior to §17 (Roscoe 1996), $M(r)$ was considered as simply an abstraction of the observed constraints on the particle-ensemble motions, and served merely as an aid to calculation; this is effectively how Newton viewed his law of 'Universal Gravitation' for which, famously, he 'posed no hypothesis'. However, in our latest work, (Roscoe 1996), $M(r)$ is given the interpretation of a global measure of the amount of mass associated with a specified volume, and it is subsequently shown how this interpretive model leads to an exact conformity with the distribution of condensed material which appears to be observed in deep space surveys, and to a conceptually simple model of the quantized redshift phenomenon which recent data analyses are giving increasing support to.

This interpretation of $M(r)$ leads to an interesting problem: specifically, it was shown in §15, Roscoe (1996) how the identification $M(r) = m_o\, r/r_o$ led directly to Newton's Theory of Universal Gravitation, known to be extremely accurate within our solar system; but this specification of $M(r)$ for local gravitational effects does not appear to be consistent with the given interpretation of this function in the conditions of the solar system—that is, the amount of mass in a sphere of radius r (centred on the solar system barycentre) does not appear to vary as $M(r) = m_o r/r_o$.

However, in §3, the existence of an absolute 'ground state'—or *material vacuum*—associated with gravitational phenomena in the model universe was identified so that, in the terms of the model, $M(r)$ must also included any mass this material vacuum might possess. In the world of our experience, the material vacuum was first predicted to exist by Nernst (1916) (as the *zero point radiation* having energy density at frequency n proportional to n^3), and first predicted to have detectable consequences by Casimir (1948); these consequences have been confirmed (in the Casimir effect) so that, today, the material vacuum is an accepted reality of the physical world. So, it is quite conceivable that this actual material vacuum has mass which plays a fundamental role in gravitational phenomena; for example, it has been speculated by many that ponderable material somehow condenses out of the vacuum—Nernst (1912, 1937) was perhaps the first—and, in this case, it might be expected that relationships exist between material condensates and the material vacuum from which they have condensed. Thus, it might be that material condensates create gradients in the physical properties of the material vacuum in their near vicinity, and one can then conceive the idea that, in an isolated system like that of our sun, these gradients are such that the distribution of *total mass* (vacuum mass plus condensate mass) behaves as $M(r) = m_o r/r_o$. According to this view, when condensed cosmological material is observed distributed according to $M(r) = m_o(r/r_o)^2$ and therefore in equilibrium according to the presented theory then, on the same scale, the mass of the material vacuum must also be distributed in the same way, if the equilibrium is to be maintained.

Whatever the realities of the situation, it seems reasonable to expect that, given the material vacuum exists and has mass, then it will exert gravitational influences according to the form of its distribution.

15. Conclusions

A cosmology is derived by imposing the most simple possible *Cosmological Principle* that the model universe is in a state of gravitational equilibrium. The resultant cosmology gives a *unique* specification of the mass distribution function and, according to this function, material in the model universe is distributed in a fractal fashion, having fractal dimension *two*. This prediction is in exact accordance with the most recent analyses of modern wide-angle and deep pencil-beam surveys, (Baryshev *et al.* 1995).

Fractality implies structure, and structure implies discreteness and this was one of the assumed properties of material in the model universe. Analysis then showed that this material discreteness necessarily implies a discretized distance scale which, together with Hubble's Law, makes the existence of a quantized redshifts in the model universe axiomatic; this is consistent with claims made by Tifft & Cocke over the past 20 years (1976, 1980, 1984, 1990), and with the results of a recent rigorous and independent analysis performed by Guthrie & Napier (1996).

The theory then predicts very strange behaviour of the peculiar velocity field at all distance scales; specifically, assuming a specific model of light propagation and the idea that cosmological redshift is an indicator of local measurement scales, it states that the estimated magnitudes of the peculiar velocities of objects should increase linearly with distance, r. Whilst there is insufficient evidence available at present to test this prediction quantitatively, there is evidence arising from the Guthrie & Napier analyses (1990, 1991, 1996) which implies the peculiar velocities are unexpectedly small at small distances, and evidence arising from recent peculiar velocity surveys (Lauer & Postman 1994) to suggest the peculiar velocities are unexpectedly large at large distances. Additionally, there is evidence from the Karachentsev & Makarov analysis that the statistics of peculiar velocity magnitudes, out to small distances, do not conform to any of the standard models of the peculiar velocity but are broadly consistent with the those expected of the presented model. These results, taken together, are consistent in a qualitative sense with the predicted behaviour.

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