Problems of Quantum Mechanics

A recent Special Issue on fundamental problems of quantum physics (Barut *et al.* 1995) confirmed widespread recognition of the persistent difficulties of quantum measurement theory and its canonical "Copenhagen interpretation." The measurement theory problems were faced with ingenuity, fortitude, and a shared hope for "hidden variables." What they were not faced with in respect to the latter was *consensus*.

That being the case, there remains room for consideration of still other approaches to hidden variables than those favored by the particular authorities chosen. I shall confine attention here to my own penchant on this subject, which will be summarized in the same spirit of "science criticism" as my previous essay (Phipps 1995), in which I touted the advantages of a Galilean invariant *covering theory* of Maxwell-Einstein electromagnetism first propounded by Hertz (1892).

As a persistent advocate of covering theories (Phipps 1987), I was struck by the scarcity in Barut *et al.* not only of these but of *specific* challenges to the perfection of the "accepted" mathematical formulation of ordinary non-relativistic quantum theory. It seems to be a widely-established premise that because Copenhagen has brought the subject to a dead end philosophically it must also be at a dead end mathematically and formally. Such is far from being the case. Indeed, if the topic is approached with a modicum of disenchantment, one notices a startling discrepancy (an "incompleteness," to be sure) in the area of *parametrization:*

There is a well-known outward resemblance, termed a "formal Correspondence," between the classical Hamilton-Jacobi equations and the Schroedinger equation. Remarkable as this Correspondence may be, its breakdown is even more remarkable, and the failure of physicists in general to take notice of that breakdown is the most remarkable of all these prodigies. By "breakdown" I mean the fact that *half the classical descriptive parameters are missing* from the quantum equations of motion. That is, in order to treat the interactions of point particles, Hamilton-Jacobi mechanics provides a *canonical formalism*, contrived to effect a transition from description in terms of a set of n "old canonical variables" to description in terms of a set of n "new canonical variables." Typically, the latter are chosen to be a set (Q_k, P_k) of *constants of the motion*, whereas the former are identified with the ordinary dynamical variables (q_k, p_k).

Both sets, $2n$ parameters in all (omitting time t), are needed for the description of specific physical events on the classical side—there being an essential relationship of *formal symmetry* between the two sets. Yet, if we believe the Schroedinger equation, only n descriptive parameters (formal analogues of the old canonical variables) have a role to play on the quantum side. The other n have simply gone fishing, taking their symmetry with them—the whole idea of a canonical formalism being thus summarily discarded. This is surely a curious occurrence, and one possibly deserving of more attention than it has received. With n missing *constants* thus identified, the search for hidden "variables" in quantum mechanics takes a new direction.

In passing from description of the large and crude to that of the small and delicate, how can there be a change in *number* of descriptive parameters? We are solemnly assured that a *formal Correspondence* exists, and Pauli (1933), for one, has made a great show of tracing it in ostensible rigor. The Correspondence is supposed to hold seamlessly over a continuously variable range of physical conditions. Yet a number, an integer, is ... well, *discontinuous*, is it not? How, then, did a jump in the head-count of parameters by even one integer occur? Yet, somehow, n of our parameters, half of our $2n$ Hamilton-Jacobi descriptors, have quietly disappeared in this process—have jumped straight into limbo. A critic feels obliged to inquire: How *rigorous* can the "formal Correspondence" be that drops any parameters at all?

And how about that formal parametric *symmetry* that lies at the heart of canonical transformation theory? At precisely what stage of a physical-descriptive transition from large to small did it decide to fold its tent? And at what stage of the retro-transition from small to large did the prodigal return? Why is the air not vibrant with the complaints of important theoretical physicists super-sensitized to symmetry through being saturated to their eyebrows with symmetry-based group theory? The core symmetry of classical physics has been ... raped? … murdered? ... held for ransom? ... yet we hear not a squeak out of any of these wheels. Bizarre! Bizarre! The sociology of physics appears every bit as fascinating as the discipline itself.

For such reasons there is no cause for a rational being to take aesthetic pleasure or satisfaction in the current formalism underlying quantum physics. To venerate it as unalterable for all time is intellectually indefensible. The reasons advanced against "accepted" quantum mechanics from Einstein on have been primarily physical or philosophical. Am I the first to object (Phipps 1987) on purely aesthetic grounds? Though there is much talk about the need to inject hidden parameters, one observes no rush of physicists to correct their subject's glaring parametric deficiency through the obvious method of *rigorizing the formal Correspondence.* Yet that is manifestly what needs to be done, at highest priority, surely before anything else is tried. If that fails, it will be time for hidden-variable hunters to broaden their search.

When this rigorizing is carried out (necessarily in a fairly unique way, since the Hamilton-Jacobi equations, except for trivial variants, have a well-defined *form*), one finds (Phipps 1960, 1979, 1987, 1988) that formal analogues of the new canonical variables have no choice but *to persist* within the quantum (operator or q-number) equations of motion. They do not play hooky in order to go fishing, nor allow themselves to be kidnapped, raped, or murdered, because the hickory stick of *strict form preservation* denies them opportunity for such extracurricular activities. But, as we shall see, these extra parameters persist within the quantum formalism in a latent or comparatively "harmless" form. That is, the new c-number parameters (Q_k, P_k) (which remain constants on both sides of the Correspondence transition) turn out to enter quantum theory through an indeterminate (constant) phase factor on the wave function—a phase factor that plays somewhat the same role as does the "gauge" in electromagnetism. The proof of this fact has been given repeatedly (Phipps 1960, 1979, 1987, 1988), but is so short and simple that it can easily be summarized here:

Let us write the Hamilton-Jacobi equations (Goldstein 1950) for *m* particles:

$$H = -\frac{\partial}{\partial t}S \qquad H = H\left(q_k, p_k, t\right) \tag{1a}$$

$$p_k = \frac{\partial}{\partial q_k}S \qquad S = S\left(q_k, Q_k, t\right) \tag{1b}$$

$$-P_k = \frac{\partial}{\partial Q_k}S \qquad k = 1, 2, \ldots, 3m\ . \tag{1c}$$

This shows the formal symmetry of (q_k, p_k) and $(Q_k, -P_k)$ on the classical side of the Correspondence transition. We propose to found that transition on *form preservation*, by reinterpreting all these *c*-number symbols, without the slightest alteration of their formal relationships, as operators (*q*-numbers). This means supplying a formal operand $\Psi_f = \Psi_f\,(q_k, Q_k, P_k, t)$, which can be the same on both sides of all of these simultaneous equations. Thus

$$H\,\Psi_f = -\frac{\partial}{\partial t}S\,\Psi_f \tag{2a}$$

$$p_k\Psi_f = \frac{\partial}{\partial q_k}S\,\Psi_f \tag{2b}$$

$$-P_k\Psi_f = \frac{\partial}{\partial Q_k}S\,\Psi_f\ , \tag{2c}$$

wherein the partial differential operators are understood to act upon everything to their right. Eq. (2) can now be postulated to hold for *all mechanics*, on both sides of, and throughout, the Correspondence transition. This is the case because we readily discover three distinct classes of exact mathematical solution of these simultaneous partial differential equations, each of which correlates with the physics on one side or the other of this transition. Thus:

Class I. Ψ_f = const. Since the operand in this case cancels trivially from the postulated equations of motion (2), we are left, as the most elementary class of *exact* solutions, with the Hamilton-Jacobi equations (1) describing classical mechanics.

Class II. S = const. = $\hbar/i$. This class of solutions of (2) describes ordinary quantum states of motion; *i.e.*, atomic physics. This is the case because (2) reduces to

$$H\,\Psi_f = -\frac{\hbar}{i}\frac{\partial}{\partial t}\Psi_f \tag{3a}$$

$$p_k\Psi_f = \frac{\hbar}{i}\frac{\partial}{\partial q_k}\Psi_f \tag{3b}$$

$$-P_k\Psi_f = \frac{\hbar}{i}\frac{\partial}{\partial Q_k}\Psi_f\ . \tag{3c}$$

If we make the substitution

$$\Psi_f = e^{-\frac{i}{\hbar}\sum_k Q_k P_k}\,\Phi\left(q_k, t\right), \tag{4}$$

then (3a,b), from which the constant phase factor has been canceled, will be recognized as the Schroedinger-Dirac equations,

$$H\Phi = -\frac{\hbar}{i}\frac{\partial}{\partial t}\Phi\ , \quad p_k\Phi = \frac{\hbar}{i}\frac{\partial}{\partial q_k}\Phi\ , \tag{5}$$

and Eq. (3c) will be observed to be identically obeyed by the formal solution (4). Hence the Class-II solutions of (2) recapture established quantum mechanics, apart from a constant phase-factor $\exp\left(-i\alpha\right)$, $\alpha = \hbar^{-1}\sum_k Q_k P_k$, attached to the quantum wave function, Eq. (4). This phase factor affects nothing directly observable, since it is simply a constant that can be absorbed into the wave-function normalization factor. In this, as we remarked, it resembles an electromagnetic "gauge."

Class III. $S \neq$ const., $\Psi_f \neq$ const. This most general class of solutions requires concomitant solution of all three parts of Eq. (2), wherein S and Ψ_f are treated as *simultaneous unknowns*. An example of such a solution has been given (Phipps 1960, 1987), based on a Dirac-type Hamiltonian. This solution exhibits bound states of electron-positrons on the scale of nuclear dimensions. Those stationary states lie within the Pauli *Zwischengebiet* (region of total particle energy between $\pm m_o c^2$), and are consequently characterized by real mass-energy but imaginary momentum. Transitions of particles occupying such states to or from *observable* states of real momentum are presumably what physicists refer to as nuclear "beta processes." The reason such light particles can exist stably in nuclear confinement is that the Heisenberg postulate is violated locally near a massive center of Coulombic force. This can be seen from the fact that the Heisenberg postulate is generalized to

$$p_k q_j - q_j p_k = \frac{\partial}{\partial q_k}S q_j - q_j\frac{\partial}{\partial q_k}S = S\,\delta_{jk}\ , \tag{6}$$

as follows directly from (2b). The Heisenberg postulate corresponds to the special case $S = \hbar/i$, *viz.*, that of the Class-II solutions. But in the Class-III solutions the value of the *p,q* commutator S can be a more general (non-constant) function of distance from a force center.

Such generality introduces a technical question concerning the Hermitean property of operators. Let $S = (\hbar/i)s$, where $s = (q_k, Q_k, t)$ is some real scalar function. Then we see that the classical-analogue operators for energy and momentum in (2) become non-Hermitean. For example, $p_k = \left(\hbar/i\right)\left(\partial/\partial q_k\right)s$ is seen to be the product of two Hermitean operators, which is known to be non-Hermitean.

Fortunately, the simple but vital "reification" transformations

$$\mathsf{H} = Hs^{-1}\ , \quad \mathsf{P}_k = p_k s^{-1}\ , \quad \Psi = s\,\Psi_f\ , \tag{7}$$

yield a reversion to the familiar Hermitean formalism, $\mathsf{H}\ \Psi = -\left(\hbar/i\right)\left(\partial/\partial t\right)\Psi$, $\mathsf{P}_k = \left(\hbar/i\right)\left(\partial/\partial q_k\right)$, *etc.*, where for known classical-analogue operators the transformed operators H and P_k are in all cases found to be Hermitean. Thus the only effect of this "generalization of quantum mechanics" (amounting to a new identification of the time-conjugate energy operator) is to alter the effective energy and momentum operators that are to be used in physical problems. For instance, in a non-relativistic one-body problem the effective Hamiltonian becomes

$$\mathsf{H} = Hs^{-1} = \left[\frac{1}{2m}\mathbf{p}\cdot\mathbf{p} + V\right]s^{-1} = -\frac{\hbar^2}{2m}\nabla s\cdot\nabla + Vs^{-1}\ ,$$

which is seen to be Hermitean, since s is real. Our generalization of quantum mechanics thus fails to interest mathematicians; but it should interest physicists because of the *altered Hamiltonian*. (It is true on both sides of the Correspondence transition that *the physics is in the Hamiltonian*.)

It is tempting to go on about the quite fascinating nuclear-bound-state solutions generated by a relativistic Dirac-type Hamiltonian; but these matters have been treated elsewhere (Phipps 1987), and I wish to keep this critique primarily on the plane of aesthetics. Returning to the Class-II solutions:

There is clearly no recovery of determinacy, because the phase factor on the wave function affords no predictive nor localizing information not present in the traditional quantum description. In the world described by accepted quantum theory, however, the capacity is lost not only for prediction but, more seriously, for *retrodiction*. The latter loss proves catastrophic, since it contradicts our belief in the definiteness of factual history. That belief is grounded in every iota of human experience. *It is not acceptable that physical theory directly flout experience—such theory being granted no higher charter than to describe or explain experience.* The presently suggested approach completely corrects this most basic flaw of current quantum measurement theory: That is, it restores *objectivity* to physical description through provision of parameters essential to retrodiction. In short, we now have the parameters necessary to allow *after-the-fact description* (*via c*-numbers) of physical point events.

Even though the values we may assign to such retrodictive parameters (the Q_k, P_k) are not observationally verifiable (given the concept of "verifiability" as associated narrowly with prediction, rather than retrodiction), we can assert an epistemological gain, in that now it is permissible to suppose that *something actually happened*. History becomes irrevocably "real," in that it differs from the future by the localized definiteness associated with *c*-number describability. The future still and always lacks such definiteness, so there arises a formal dichotomy between past and future, reflecting the dichotomy of actual experience.

In ordinary quantum physics there is no such distinction between past and future, hence *no objectivity* ascribable to physical experience. This, I claim, is a far worse malady than indeterminacy of trajectories, since quantum-level trajectories (linking observable events) are pure inferences—impositions of the human mind, conditioned by the gross-scale experiences through which it has evolved. By contrast, point events (localized "happenings"), representing quantum process completions, once they join the fabric of accomplished fact known as *the past*, are at least roughly describable (as to location in space and time) by *c*-number parameters—which our formalism now contains ... and which ordinary quantum mechanics conspicuously lacks.

Such description implies a discontinuous "jump" of the erstwhile *constant* parameters in the wave function phase factor, symbolized by the quantity a introduced following Eq. (5), above. This jump of the a-value destroys any "phase knowledge" connecting "before" and "after." In the early days of quantum theory this was referred to as a "quantum jump" or "severance of the von Neumann chain" of phase connections. We now have an explicit parametric mechanism to accomplish such severance. The scheme works perfectly well, even though we can assign neither observational meaning nor verifiability to specific numerical values of a_{before} and a_{after}. All we need is the fact of an unknown (and unknowable) jump or discontinuity, Δa, accompanying any quantum process completion, which "ends the quantum descriptive problem" by irreversibly disconnecting quantum system phases before and after ... and sets the stage for the next descriptive problem.

The jump can take place only *after* the physical event occurs that is being described. The "event" is the raw material of experience. It occurs uncontrollably and independently of human volition. In descriptive parlance the jump acts directly on, or "reduces," the system wave function itself (not the "wave packet"). After this action we face a new descriptive problem, often involving new "particle" participants and a new Hamiltonian. The phase jump affects only retrodiction and has no predictive scope. Whatever physical happening it reflects occurs "out there"—as a part of objective reality. It takes place independently of "mind," which has no enabling role. Objectivity is thus restored to physics, but without any accompanying rehabilitation of quantum-level determinism.

In this way the equations of motion, (2), through their extra parametrization, do the "problem resetting" job of a Projection Postulate, but without *ad hoc* postulation ... to the notable benefit of logical economy. (In classical mechanics equations of motion *suffice* for physical description without supplementary postulation. Where there is rigor of formal Correspondence, this logical sufficiency should persist throughout the full range of description. A Correspondence-based theory must thus obey a rule of austerity, being denied resort to postulation-of-convenience.) The world is therefore no longer one big phase-connected, interminable "quantum problem." Thus, improvement of logical economy is one of the principal selling-points for the present covering theory.

The irreversibility of the phase-connection severance just described seems, in information theoretical terms, suited through "phase-knowledge loss" to distinguish between a factually determined past and an undetermined or unknowable future. The severance takes place at the most basic quantal level and marks the "completion" of each individual physical process. It seems thus a prime candidate for the elusive element of *mechanical irreversibility* that may reasonably be suspected to underpin process irreversibility, time-flow irreversibility, and the second law of thermodynamics.

Is it pure accident that a rigorization of formal Correspondence brings logical economies to measurement theory and a formal unification to all mechanics? Or is there some principle, from whose implementation these benefits might reasonably be expected to follow? Physicists have learned from Einstein that *associated with any form preservation is a relativity principle*. The formal Correspondence on which Schroedinger's (wave) mechanics is based is certainly, by definition, a form preservation—although until now an imperfect one. All we have done here is to perfect that form preservation by rigorizing it. Einstein's lesson, then, is: *cherchez la relativity principle*. So, what relativity relates?

To belabor the obvious, we appear to be concerned with a *principle of relativity of physical size*. For, what are we doing physically in extending the Hamilton-Jacobi formalism beyond its range of proven validity? We are pushing that descriptive formalism into the realm of very small physical systems, first at the atomic scale, then down to the nuclear and below. If we had taken such a relativity principle as our starting point, we could have deduced the Correspondence principle from it and could have proceeded deductively with all the developments indicated above. In fact, this was more or less the order of procedure adopted in (Phipps 1987).

In theoretical physics as a social enterprise, such an approach has been stymied historically; notably because Dirac (1947) chose to picture quantum mechanics as the discipline that sets an *absolute size* scale to the world. ("So long as *big* and *small* are merely relative concepts, it is no help to explain the big in terms of the small. It is therefore necessary to modify classical ideas in such a way as to give an absolute meaning to size.") One cannot talk size relativity in the presence of convictions like that one, nor is one encouraged to look beyond the atomic realm toward a new *trans-quantum dynamics* of the nuclear realm. So, fruitful as it might prove to be for future physical description, and elegant as it may be on the plane of aesthetics, a principle of size relativity is clearly assured of weighty opposition from entrenched ways of thought.

Perhaps the dominant consideration to be emphasized here is that we have met an example of a true *covering theory* of ordinary quantum mechanics, Eq. (2), which includes that discipline and reduces to it in the formal limiting case $s \to \hbar / i$. At the same time the generalized mechanics embodied in (2) is an exact covering theory of classical Hamilton-Jacobi mechanics. To find a single set of equations that thus constitutes a covering theory of *both* classical and ordinary quantum mechanics, two major physical theories that together account for a range of applications from stars to atoms, is a unique occurrence in my experience.

This alone qualifies Eq. (2) as an aesthetic landmark; but the fact that it volunteers coverage also of a third range of physical experience through supporting the long-sought possibility of a *nuclear dynamics* (complete with radically simplified explanations of why beta particles emerge from and enter nuclei and the vacuum-plenum, why proton and positron charges and spins are identical, *etc.*), incorporated in rigorous formal detail within the grand mechanical tradition that bears the paw-mark of the lion, Newton, is additional inducement to explore its possibilities.

String theorists are said to feel the aesthetic attractions of their formal mathematical contentions to be so compelling as to transcend all need for empirical support. I confess to being similarly overpowered by the beauties of the position adumbrated above … though I hope not to the extent of putting theory before facts. If a theory meant to be descriptive of facts is more beautiful than those facts, it can hardly be descriptive of them.

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Seventieth Birthday of a Non-Effect: "Thomas Precession"

As an undergraduate in the early 60s, I was taught next to nothing about "Thomas Rotation" (TR), "Thomas Precession" (TP) or "Thomas Effect" (TE). Graduate courses, including general relativity with Nathan Rosen ("the EPR one", as Evert J. Post once called him: "Tell me, is your Rosen the EPR one?") didn't change the situation. Most textbooks and monographs on relativity are silent about TP (and the related Ehrenfest paradox)—a notable exception being Arzélies' *Relativistic Kinematics*. Advanced texts either mention, as an oddity, "Thomas's one half" (*e.g. Relativistic Quantum Mechanics I* by Landau and Lifschitz), or simply put it in the historical introduction (*e.g.* Weinberg's recent *The Quantum Theory of Fields*). The late Eugen Wigner discussed TP in 1939 "in terms of the O(3)-like little group which describes rotations in the Lorentz frame in which the particle is at rest." Exploiting the general ignorance of physicists, the mathematician Abraham Ungar has, in recent years, produced a whole "thomasian literature". The mathematical elegance of his presentation notwithstanding, I stress the plain fact that *there never was, is or ever will be a physical "Thomas Precession."*

It was Henri Poincaré who, in June 1905, first wrote the Lorentz transformations (LT) in a form which revealed a group structure in (1+1) dimensions. Einstein also pointed out that the LT form a group, "*wie dies sein muss*" (as it should be—!?); two successive transformations with velocities v_1, v_2 in the same direction are equivalent to a LT with a velocity v given by $v = (v_1 + v_2)/(1 + m\, v_1 v_2)$, where $m = 1/c^2$. He briefly mentioned the velocity composition law *in (3+1) dimensions*, but—strangely—failed to notice that in this case *the LT's do not form a group* anymore! Twenty years later, Einstein heard something about the Lorentz group that greatly surprised him (A. Pais): "It happened while he was in Leyden. In October 1925 Uhlenbeck and Goudsmit had discovered the occurrence of the alkali doublets... then Llewellyn Thomas supplied the missing factor, 2, now known as the Thomas factor. Uhlenbeck told me that he did not understand a word of Thomas's work when it first came out... Even the cognoscenti of the relativity theory (Einstein included!) were quite surprised... (It took Pauli a few weeks before he grasped Thomas's point.)" Citing Pais further: "at the heart of the Thomas precession lies the fact that the LT with velocity $\vec{v}_1$ followed by a second one $\vec{v}_2$ with velocity in a different direction does not lead to the same inertial frame as one simple LT with velocity $(\vec{v}_1 + \vec{v}_2)$." Indeed, it has been shown (Moeller, Arzélies) that for $\vec{v}_1 = \vec{v}$ and $\vec{v}_2 = \vec{v} + d\vec{v}$, the "Thomas angle" $\mathrm{d}\,\theta_T = -(\gamma - 1)(\vec{v} \times \mathrm{d}\,\vec{v})/v^2$ yields—after division by d*t*—the "Thomas precession" $\omega_T \equiv \mathrm{d}\,\theta_T / \mathrm{d}\,t$, too. The fact of deriving a *non-inertial* "effect" from/*via* manipulations of LTs relating only *inertial* frames of reference (IFR) relies on the "special" relativistic dogma (not included in the two postulates) that a non-inertial frame moving with velocity *v*(*t*) and *acceleration* $\vec{a}$ can at every moment be replaced by an IFR moving with *uniform velocity* $\vec{v}$! The plain fact is that the *Thomas precession violates the inertial nature of the transported system*! The TP is wrong from first principles! Furthermore, the prediction that the parallel transport of an IFR, (*i.e.* an extended object) around a circle will produce (N.B. *torque-free!*) a *net retrograde rotation* $\Delta\theta_T = -2\pi(\gamma - 1)$, where $\gamma \equiv (1 - \mu v^2)^{-1/2}$ was put to the test in 1974 by Thomas Phipps, Jr. The clear null result $\Delta\theta_T \equiv 0$ necessarily implies $\gamma \equiv 1$, $\mu \equiv 0$ and Galilean velocity composition: $\vec{v} = \vec{v}_1 + \vec{v}_2$. In other words, the "Thomas precession"—a genuine "special" relativistic prediction—is absent: there is no Thomas precession in *this* world! One can, if one wishes, celebrate the publication of Thomas's article in 1926, but there is no reason to celebrate the birth of a physical effect!

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