Nonlinear Nature of Gravitational Waves

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The gravitational wave should be a type of nonlinear wave and has characteristics corresponding this type of wave. In harmonic coordinates, we obtain a series of equations for the nonlinear wave and some simple nonlinear solutions. The soliton solution shows that Weber's detection may not be repeatable. Lastly, we think that the velocities of gravitational and electromagnetic waves should be different.

1 Introduction

Although the gravitational wave had been predicted by Einstein (1918) and the orbital decay of the Hulse-Taylor (1992) pulsar (PSR 1913+16) has provided remarkably good indirect confirmation of the existence of gravitational waves, but detection is still a difficult and interesting question (Schutz 1994). It is remarkable that the corresponding gravitational field discussed by Einstein (1918, 1955; Einstein and Rosen, 1937), Landau and Lifshitz (1951)and Fock (1959) is only weak. Let $g_{mn} = d_{mn} + g_{mn}$, and "it is assumed that the g_{mn} are small *i.e.* that the gravitational field is weak." "If the g_{mn} are everywhere sufficiently small compared to unity one obtains a first-approximation solution of the equations by neglecting the higher powers of the g_{mn} (and their derivatives) compared with the lower ones"(Einstein and Rosen 1937). Then the gravitational equations for empty space can be written in the form

$$\sum_a \frac{\P^2 g_{mn}}{\P x_a^2} = 0 , \qquad (1)$$

so one obtains plane gravitational waves. It is completely analogous to the electromagnetic wave, and physicists therefore conclude that gravitation is propagated with the speed of light (Fock 1959). Theoretically, the inference of the gravitational wave is used chiefly this way. Experimentally, the method for detecting gravitational waves is also analogous to the electromagnetic wave. This method is powerful for the proof of existence of gravitational waves, even the field is very weak; but it appears to overlook key differences between the gravitational and electromagnetic fields, and, perhaps, produces a shadow for the detection of the gravitational radiation.

First, we must distinguish the weak gravitational field from the weak gravitational wave. In celestial space the usual gravitational wave which propagates for a very long distance is weak, especially when it is similar to the electromagnetic wave, but the gravitational field, whose source radiates a gravitational wave, may not be weak. Of course, the wave of a weak gravitational field is necessarily very weak, and experimental detection is still very difficult. Conversely, only for strong gravitational fields may the waves be strong, while detection is less difficult; but under this condition the gravitational wave and its equation possess different characteristics from the electromagnetic wave. It may explain the waves detected by Weber (1969, 1970). Although he claims to have discovered gravitational radiation, other groups have not found anything.

2 Discussion of some known solutions

It is well known that the Einstein gravitational field equations of general relativity are nonlinear

$$R_{mn} - \frac{1}{2} g_{mn} R = 4kT_{mn} \qquad (2)$$

Their solutions are very difficult. Thus far, no universal analytic solution has been found, and only some particular solutions have been obtained, when the equations are subject to certain conditions or approximations are made. But so long as the equations are not the simplest linear equations at first approximation, their solution cannot be a simple linear superposition. Stumpf *et al*. (1993) assumed that spin-2 gravitons are composed of four spin-½ (sub)fermions whose spinor field equation is still nonlinear. In the past seventy years, various solutions have been discussed widely by physicists and mathematicians (Kramer 1980). The majority of these solutions are derived from the particular metric, and the corresponding g_{mn} are substituted into the field equations. Some workers have taken various simplified approximations. Others have combined the solutions from the Yang-Mills equations in gauge theory.

Astronomers consider that the possible sources of gravitational radiation may be continuous or pulsed or stochastic radiation (Hawking and Israel 1979). In vacuum the Einstein gravitational field equations are

$$R_{mn} - \frac{1}{2} g_{mn} R = G_{mn} = 0 \qquad (3)$$

They are necessarily nonlinear for g_{mn}. The gravitational field determines the property of the gravitational wave; therefore, the gravitational wave should consist of many nonlinear waves. Recently, Efroimsky (1994) has even considered the nonlinearity of the weak gravitational wave. However, the simplest solutions of the linear wave, which is completely analogous to the plane solution of electromagnetic wave, disagree with the gravitational field equations.

Einstein and Rosen (1937) investigated plane and cylindrical gravitational waves, and Rosen came to the conclusion that there were no exact plane-wave metrics filling all spacetime. Bondi (1957) derived the relations

$$2j' = ub'^2, b'' + 2u^{-1}b' - ub'^3 = 0\,, \qquad (4)$$

from Rosen's metric

$$\mathrm{d}s^2 = e^{2j}\left(\mathrm{d}t^2 - \mathrm{d}x^2\right) - u^2\left(e^{2b}\mathrm{d}h^2 + e^{-2b}\mathrm{d}z^2\right), \quad (5)$$

where j and b are functions of $u = t - x$. From Equations (4), we may obtain

$$b' = \frac{1}{u\left(1+cu^2\right)^{1/2}},\ b = \ln\frac{c_o u}{1+\left(1+cu^2\right)^{1/2}}, \qquad (6)$$

$$j' = \frac{1}{2u\left(1+cu^2\right)},\ j = \frac{1}{4}\ln\frac{c_o u^2}{1+cu^2}, \qquad (7)$$

$$-g_{oo} = g_{11} = e^{2j} = \frac{c_o^{1/2} u}{\left(1+cu^2\right)^{1/2}} \qquad (8)$$

$g_{11} \to 0$ for $u \to 0$; $g_{11} \to \left(c_o/c\right)^{1/2}$ for a $u \to \infty$. This corresponds already to a soliton. This sort of wave is a pulse, which exhibits a type of macroscopic quantized phenomenon. It is natural result that Weber's detection may not be repeatable.

Wainwright (1979) considered the gravitational wave pulse solution in a spatially homogeneous spacetime. Belinsky and Zakharov (1978) applied the inverse-scattering technique to the Einstein equations. Then Belinsky and Fargion (1980) constructed a gravitational two-soliton exact solution, but the velocity of propagation defined by them is greater than the velocity of light. Ibanez and Verdaguer (1983) obtained a four-soliton solution and the collision between two of them, then (1985) discussed a multisoliton solution where the velocity is never greater than *c*. Letelier (1984) obtained the one-, two- and *n*-soliton solutions of the cylindrically symmetric metric. Ferrari *et al.* (1987) constructed a general class of nondiagonal solutions of the vacuum Einstein equations describing gravitational colliding waves and shock waves. Cespedes and Verdaguer (1987) studied the collision of a gravitational wave pulse and a soliton wave. Tomimatsu and Ishihara (1987) discussed nonlinear wave propagation including the occurrence of shocklike discontinuity in the Gauss-Bonnet extended Einstein theory. Feinstein and Ibanez (1989) obtained curvation-singularity-free solutions for colliding plane gravitational waves with broken *u*-*v* symmetry. Abrahams and Shapiro (1990) obtained nonlinear and time-dependent solutions of potential flows in general relativity. Torre (1990) studied perturbations of gravitational instantons.

3 A series of nonlinear equations

By using the curvature tensor transformed by Fock (1959)

$$R_{mn} = -\frac{1}{2} g_{ms} g_{nr} g^{ab} \frac{\P^2 g^{rs}}{\P x_a \P x_b} - \Gamma_{mn} + \Gamma_m^{ab}\Gamma_{n,ab}\,, \qquad (9)$$

$$R = -\frac{1}{2} g^{ab} g_{mn} \frac{\P^2 g^{mn}}{\P x_a \P x_b} - \Gamma + \Gamma_m^{ab}\Gamma_{ab}^{n} \qquad (10)$$

and $\Gamma^a = 0$, $\Gamma_{mn} = 0$ in the harmonic coordinates, so in the vacuum the Einstein gravitational equations (3) become

$$\frac{1}{2} g^{ab}\left(g_{ms} g_{nr} \frac{\P^2 g^{rs}}{\P x_a \P x_b} - \frac{1}{2} g_{mn} g_{mn} \frac{\P^2 g^{mn}}{\P x_a \P x_b}\right) = \frac{1}{4} g^{ai} g^{bj}\left(\frac{\P g_{mi}}{\P x_j} + \frac{\P g_{mj}}{\P x_i} - \frac{\P g_{ij}}{\P x_m}\right) \times\left[\left(\frac{\P g_{na}}{\P x_b} + \frac{\P g_{nb}}{\P x_a} - \frac{\P g_{ab}}{\P x_n}\right) - \frac{1}{2} g_{mn} g^{mk}\left(\frac{\P g_{ka}}{\P x_b} + \frac{\P g_{kb}}{\P x_a} - \frac{\P g_{ab}}{\P x_k}\right)\right] \qquad (11)$$

From (11), some simplified equations are discussed.

If $m = j$, and $b = n = k$ (for instance, for a certain metric), Equations (11) will be

$$g^{ak}\left(g_{js} g_{kr} \frac{\P^2 g^{rs}}{\P x_a \P x_k} - \frac{1}{2} g_{jk} g_{jk} \frac{\P^2 g^{jk}}{\P x_a \P x_k}\right) = \frac{1}{2} g^{ai} g^{jk} \frac{\P g_{jj}}{\P x_i}\left(\frac{\P g_{kk}}{\P x_a} - \frac{1}{2} g^{jk} g_{jk} \frac{g_{kk}}{\P x_a}\right). \qquad (12)$$

Further let $a = j = r$ and $s = k = i$, then

$$g^{jk} g_{jk} g_{jk} \frac{\P^2 g^{jk}}{\P x_j \P x_k} = -g_{jj} g_{kk} \frac{\P g^{jk}}{\P x_j} \frac{\P g^{jk}}{\P x_k}, \qquad (13)$$

and if $j = k$,

$$g^{jj} \frac{\mathrm{d}^2 g^{jj}}{\mathrm{d}x_j^2} = -\left(\frac{\mathrm{d}g^{jj}}{\mathrm{d}x_j}\right)^2. \qquad (14)$$

The simplest equations are still nonlinear.

Assume $g^{mn} = g^{mn}_{(0)} + h^{mn}$ where for $h^{mn} >> g^{mn}_{(0)}$ strong field or for $h^{mn} << g^{mn}_{(0)}$ weak field, these equations are still nonlinear. For $h^{mn} >> g^{mn}_{(0)}$ the form of Equation (14) is the same

$$h^{jj}\frac{\mathrm{d}^2 h^{jj}}{\mathrm{d}x_j^2} = -\left(\frac{\mathrm{d}h^{jj}}{\mathrm{d}x_j}\right)^2 . \qquad (15)$$

$$h^{jj} = \left(2Cx_j + C_o\right)^{1/2}, \qquad (16)$$

$$h_{jj} = \left(2Cx_j + C_o\right)^{-1/2}. \qquad (17)$$

Let two integral constants $C > 0$ and $C_o > 0$; then $h_{jj} = C_o^{-1/2}$ for $x_j \to 0$; $h_{jj} \to 0$ for $x_j \to \infty$. And $h_{11} \approx 1/\sqrt{C_o}\left(1 + Cr/C_o\right)$ corresponds to the Schwarzschild metric $g_{11} \approx 1 + \left(2km/c^2 r\right)$. We may derive similar results with other forms of the curvature tensor.

4 Conclusion

So long as the gravitational equations are still nonlinear, their solutions, whether soliton or instanton, or kink, or chaos, *etc.* will show some properties different from the usual electromagnetic field and wave. The nonlinearity of the gravitational wave may exhibit some characteristics of nonlinear waves. For example, if the gravitational wave radiates as a solitary wave, its strength will be far higher than the present gravitational wave; the recurrence phenomenon occurs under periodic boundary conditions *etc*.; the gravitational wave possesses the self-induced transparency and the self-focusing effect, which appears to correspond to the gravitational lens effect, where a distant optical or radio-source can be focused to form two similar images. We believe that these features may be observed.

Christodoulou (1991) proved that gravitational waves from astronomical sources have a nonlinear effect on laser interferometer detectors on Earth . The effect is of the same order of magnitude as linear effects and its signature is a permanent displacement of test masses after the passage of a wave train. It is shown here that the nonlinear effect of gravitational waves cannot be neglected.

Although the velocities of gravitational and electromagnetic waves are different for bimetric or vector-tensor or stratified theories, they are the same in general relativity or scalar-tensor theories (Hawking and Israel 1979). We think that the two propagation velocities should be different *i.e*. the velocity of the gravitational wave will deviate from the velocity of light, at least since light deflects while the gravitational wave propagates along a straight line in a strong gravitational field, like an electromagnetic wave in an electromagnetic field, in particular when photon-photon interactions are neglected.

Our conclusion is that the gravitational equations and wave should be nonlinear, and should have some characteristics of nonlinear waves, while the velocities of gravitational and electromagnetic waves should differ.

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