

Acceleration Dependent Forces: Reply to Smulsky

In 1991 Richard Waldron published a paper against force laws that depend on the acceleration of the test body (Waldron 1991). We answered his criticisms showing two mistakes in his paper (Assis 1992). Contrary to Waldron's claims, we showed that these forces are compatible with Newton's second law of motion. In particular we analysed Weber's force, which depends linearly on the acceleration of the test body (Assis 1994).

In number 20 of *Apeiron* (October 1994), there is a letter by Dr. Smulsky claiming that we were mistaken in our refutation of Waldron's argument (Smulsky 1994). He also notes that there is a mistake in Waldron's derivation. We agree with Smulsky's statement as regards Waldron's derivation, but not as regards our own refutation. We invite the interested reader to analyse both papers, mine and Waldron's, in order to reach his own conclusions. In his letter Smulsky claims to correct Waldron's derivation, and also maintains that forces that depend on the acceleration of the test body are not compatible with Newton's second law. In this letter we show a mistake in Smulsky's derivation. He bases his proof in two equations, namely

$$F = f_0 + f_1 a, \quad (1)$$

$$F = ma, \quad (2)$$

where $f_i = f_i(r, v)$, $i = 0, 1$. (Although he also analyses the case of forces that depend quadratically on the acceleration of the test body, we will restrict our analysis to forces which depend linearly on the acceleration, as is the case in Weber's law). In Eq. (1) the coefficients f_0 and f_1 may depend on the position and velocity of the test body, but not on its acceleration. Smulsky concludes that if the force is multiplied by n , $F_1 = nF$, then the acceleration will also be multiplied by n , $a_1 = na$. This is fine and correct:

$$a_1 = \frac{F_1}{m} = \frac{nF}{m} = na \quad (3)$$

Then he says:

"The expression (1) is general and it is valid for force F_1 :

$$F_1 = f_0 + f_1 a \quad (4)$$

"If we substitute F_1 and a_1 in Eq. (4), we obtain $nF = f_0 + f_1 na$. Then

$$F = \frac{f_0}{n} + f_1 a. \quad (5)$$

"Since the left parts of Eqs. (1) and (5) are equal, the right parts must also be equal. But they are not equal. Therefore the initial sug-

gestion about the force depending on acceleration is wrong."

Where is his mistake? Obviously in Eq. (4). He should have written:

$$F_1 = g_0 + g_1 a_1. \quad (6)$$

Since the force is not the same anymore, the coefficients should be changed accordingly. By Eq. (1) and the fact that $F_1 = nF$, he might as well arrive at:

$$F_1 = nF = n(f_0 + f_1 a) = nf_0 + nf_1 a \quad (7)$$

Equating (6) and (7) and observing that $a_1 = na$ yields: $g_0 = nf_0$ and $g_1 = f_1$. And this solves all the problems which he raised. This shows once more that force laws that depend on the acceleration of the test body are compatible with Newton's second law of motion.

To see more readily the magnitude of Smulsky's error, we can analyse his derivation in a situation where the force does not depend on the acceleration (in this case he accepts that the force is compatible with Newtonian mechanics). In this case $f_1 = 0$. Then his equations (1) to (5) would read, respectively:

$$\begin{aligned} F &= f_0 \\ a &= \frac{F}{m} \\ a_1 &= \frac{F_1}{m} = \frac{nF}{m} = na \\ F_1 &= \frac{m}{m} F = F \end{aligned}$$

and $F = f_0 / n$. This cannot be the case, since he began with $F = f_0$ and arrived at $F = f_0 / n$. Obviously the incorrect equation which he utilized is the statement that $F_1 = f_0$. As $F_1 = nF$, he should have written $F_1 = nf_0$, not $F_1 = f_0$.

References

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Light Clock Gedanken Experiment

The Special Theory of Relativity's concept of time dilation is presented in physics textbooks using a device known as a light clock, which can be depicted as a vertical glass tube that has a light source at the base and a mirror at the top.

From the point of view of an observer past whom the device is moving, a pulse of light emitted by the source will travel from the base of the light clock to the mirror and back to its source in an angular path. Since this distance is, from the observer's point of view, greater twice the height of the device, and assuming constant light velocity, it is claimed that, from the stationary observer's point of view, an astronaut traveling along with the light clock has incurred time dilation and will age at a slower rate than the observer.

It is axiomatic that if such a device is fixed to a room that is moving past the observer any person contained within the room would, according to this concept, also incur time dilation, aging at a slower rate than the observer, and their clocks would similarly incur time dilation compared with the observer's clock.

In 1971, Hafele and Keating (1972) conducted an experiment by taking atomic clocks around the world in the same direction as the Earth's axial spin. It was found, as they anticipated, that the clocks incurred time dilation compared with the "stationary" laboratory clock back in Washington. They then took the clocks in the opposite direction, and it was found, also as anticipated, that the clocks incurred time contraction—ticking at a faster rate than the laboratory clock.

It is axiomatic that if an atomic clock is fixed to a room which is moving in the opposite direction to the Earth's axial spin (retrograde orbit) any person contained within the room would, in accordance with the Hafele Keating experiment, be aging at a faster rate than the "stationary" observer. This person's clock would tick at a faster rate than the observer's clock.

Let us imagine that we have an atomic clock and a light clock fixed to the outside of a room that is moving past a stationary observer in the opposite direction to the earth's axial spin. According to the light clock concept, passengers in that room will, compared with the stationary observer, incur time dilation, aging at a slower rate than the observer.