Inertial Induction and the Potential Energy Problem

Amitabha Ghosh
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur
India 208016

A model of dynamic gravitational interaction including velocity and acceleration dependent inertial induction terms has resulted in many interesting consequences. Exact equivalence of the gravitational and inertial masses and the redshifting of light from distant objects (as observed) without bringing in the concept of cosmological expansion are two of many results already published. In this paper it has been shown that the potential energy of a particle of mass m in an infinite, homogeneous Euclidean universe is not only finite but is exactly equal to $-mc^2$. *This implies that the total energy is zero, a fact which may have many interesting implications.*

Introduction

It has been shown (Ghosh 1984, 1986a, 1993) that a phenomenological model of inertial induction based on the Extended Mach's Principle yields a number of very interesting results. According to this model, the interactive dynamic gravitational force between two bodies depends not only on their relative separation, but also on their relative velocity and acceleration. If **F** be the force on body A due to B, then

$$\mathbf{F} = -\frac{Gm_A m_B}{r^2}\hat{\mathbf{r}} - \frac{Gm_A m_B}{r^2c^2}v^2\hat{\mathbf{r}}f(\theta) - \frac{Gm_A m_B}{rc^2}a\hat{\mathbf{r}}f(\phi) \tag{1}$$

where $\mathbf{r} = (r\hat{\mathbf{r}})$, $\mathbf{v}\,(= v\hat{\mathbf{v}})$ and $\mathbf{a}\,(= a\hat{\mathbf{a}})$ are the position, velocity and acceleration of body A with respect to B ($\hat{\mathbf{r}}$, $\hat{\mathbf{v}}$ and $\hat{\mathbf{a}}$ are the respective unit vectors); $f(\theta)$ and $f(\phi)$ (with $\cos\theta = \hat{\mathbf{r}}\cdot\hat{\mathbf{v}}$ and $\cos\phi = \hat{\mathbf{r}}\cdot\hat{\mathbf{a}}$) represent the inclination effects, c is the speed of light, m_A and m_B are the gravitational masses (actually these are relativistic gravitational masses; but in most cases the velocity is much smaller than c and the relativistic effect is not important) and G is the intensity of the gravitational interaction (it is assumed to be proportional to the energy of the agents transporting gravitational interaction, which might be called gravitons) whose local value, *i.e.*, when the separation of two interacting bodies is small, is G_o ($= 6.67\times10^{-11}\ \mathrm{m^3kg^{-1}s^{-2}}$).

A number of interesting results can be derived as shown in the subsequent sections.

Force on an isolated particle

Let us consider a particle of gravitational mass m in the universe which we assume to be (*i*) non-expanding and quasistatic, (*ii*) infinite with matter distributed uniformly and (*iii*) Euclidean. A mean rest frame exists for this non expanding quasistatic universe. If the velocity and acceleration of the particle with respect to this mean rest frame be **v** and **a** and ρ be the average density of matter in the universe, then the resultant force on the particle due to the velocity and acceleration dependent inertial induction with respect to the matter present in the universe is given by

$$\mathbf{F} = -\hat{\mathbf{v}}\frac{mv^2}{c}\int_0^\infty \frac{\chi G\rho}{c}\,\mathrm{d}r - \hat{\mathbf{a}}\frac{ma}{c^2}\int_0^\infty \rho G r\ \mathrm{d}r \tag{2}$$

where

$$\begin{aligned}\chi &= 4\pi\int_0^{\pi/2}\sin\theta\cos\theta\ f(\theta)\,\mathrm{d}\theta \\ &= 4\pi\int_0^{\pi/2}\sin\phi\cos\phi\ f(\phi)\,\mathrm{d}\phi\end{aligned} \tag{3}$$

To proceed further it is essential to know G as a function of r. First, let us write

$$\int_0^\infty \frac{\chi G\rho}{c}\,\mathrm{d}r = k \tag{4}$$

Then the expression of the force given by (2) takes the form

$$\mathbf{F} = -\hat{\mathbf{v}}k\frac{mv^2}{c} - \hat{\mathbf{a}}\frac{ma}{c^2}\int_0^\infty \chi G\rho r\ \mathrm{d}r \tag{5}$$

The above equation implies that if a particle of mass m moves with velocity **v** with respect to the mean rest frame of the universe, then it is subjected to a drag force equal to

$$\hat{\mathbf{v}}k\frac{mv^2}{c}$$

This drag will cause the particle to loose both momentum and energy (which goes to the matter in the rest of the universe). Every moving entity—even the graviton—is subjected to this drag. Consequently, if at any instant a graviton has an energy E (*i.e.*, a mass equal to E/c^2) it will be subjected to a drag

$$\frac{kE}{c}$$

assuming the gravitons to propagate with the speed of light. The energy E will then be reduced in proportion to the distance traveled r as

$$E = E_o \exp\left(-\frac{k}{c}r\right)$$

where E_o is the original energy. Hence

$$G = G_o \exp\left(-\frac{k}{c}r\right) \qquad (6)$$

Substituting G from (6) in (4) and (5) we get

$$\frac{cG_o r}{k} = k$$

or,

$$k = \sqrt{cG_o r} \qquad (7)$$

and

$$\mathbf{F} = -\hat{\mathbf{v}}\frac{mv^2}{c}k - \hat{\mathbf{a}}ma\frac{cG_o r}{k^2}$$

Substituting k in the above equation from (7) we get

$$\mathbf{F} = -\hat{\mathbf{v}}\frac{mv^2}{c}k - m\mathbf{a}$$

Thus the acceleration-dependent part of the inertial force (due to dynamic gravitational interaction with the rest of the universe) is exactly equal to $-m\mathbf{a}$. The velocity-dependent part represents a very small cosmic drag which acts opposite to the velocity with respect to the mean rest frame.

The value of k depends on c, which can be calculated when the form of the function $f(q)$ is known. Taking

$$f(q) = \cos q \qquad (9)$$

we get

$$c = \frac{4p}{3} \qquad (10)$$

finally yielding

$$k = \sqrt{\frac{4p}{3}G_o r}\,. \qquad (11)$$

When the estimated value of $r\ (= 7\times10^{-27}\ \mathrm{kg\ m^{-2}})$ is used, then we have

$$k = 1.41\times10^{-18}\ \mathrm{s^{-1}} \qquad (12)$$

Cosmic drag on a photon and cosmological redshift

If we consider a photon, the drag force it experiences is equal to

$$km_pC$$

where m_p is the relativistic mass of the photon at a given instant. If n be the frequency, then

$$m_p = \frac{hn}{c^2}$$

and the drag force can be written as

$$k\frac{hn}{c}$$

When it travels a distance dx the drop in energy is just

$$d\mathrm{E} = h\mathrm{d}n = k\frac{hn}{c}\cdot \mathrm{d}x$$

Solving, we get

$$\frac{n}{n_o} = \exp\left(-\frac{k}{c}x\right) \qquad (13)$$

where n_o is the frequency at the source and x is the distance it has traveled from the source. When $(k/c)x \ll 1$, the fractional redshift becomes

$$z = \frac{\Delta l}{l_o} \approx \frac{k}{c}s \qquad (14)$$

If this redshift is *interpreted* as a Doppler shift, the *corresponding* recessional speed is

$$v = cz \approx kx \qquad (15)$$

Thus k is nothing but the Hubble constant. The estimated value of k given by (12) is equivalent to $43\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$, which lies quite close to the current estimate of the Hubble constant, $50\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$.

This model, thus, predicts the observed cosmological redshift in a quasistatic universe without any expansion.

Potential energy

Equation (1) may give the impression that the concept of gravitational potential energy of two particles at a distance r from each other is not proper because of the velocity- and acceleration-dependent inertial drag terms. However, if we consider the situation to be quasistatic with the velocity and acceleration involved to be small, it is possible to define gravitational energy, as shown below.

The gravitational potential energy of a system of two particles of masses m_1 and m_2 with a separation of r can be defined as the negative of the work done in taking one mass away from the other to infinity with an infinitesimally small speed. Thus the gravitational potential energy of two particles of masses m_1 and m_2, separated by a distance r, is given by

$$U = -\int_r^{\infty} \frac{G_o \exp\left(-\frac{k}{c}x\right) m_1 m_2}{x^2}\,\mathrm{d}x = -\frac{G_o m_1 m_2}{r}\left[\exp\left(-\frac{k}{c}r\right) - \frac{k}{c} r Ei\left(\frac{k}{c}r\right)\right] \tag{16}$$

where $Ei(y) = \int_y^{\infty} \frac{\exp(-z)}{z}\,\mathrm{d}z$. Using the above formulation, the potential energy of a particle of mass *m* due to a spherical shell (of the uniform universe) of radius *r* and thickness d*r* with the particle at its centre is

$$\mathrm{d}U = -4\pi G_o \rho\, m r \left[\exp\left(-\frac{k}{c}r\right) - \frac{k}{c} r \cdot Ei\left(\frac{k}{c}r\right)\right]\mathrm{d}r \tag{17}$$

According to the conventional gravitational law, the potential of a particle of mass *m* due to an infinite homogeneous universe tends to infinity. However, with this model the potential will be finite, even for an infinite universe, as *G* is not a constant, but decreases exponentially with distance.

The potential energy of a particle in an infinite universe can be obtained by integrating the right-hand side of (17) from $r = 0$ to ∞. Thus

$$\begin{aligned} U &= -4\pi G_o \rho\, m \int_0^{\infty} \left[r \exp\left(-\frac{k}{c}r\right) - \frac{k}{c} r^2 \cdot Ei\left(\frac{k}{c}r\right)\right]\mathrm{d}r \\ &= -\frac{4\pi G_o m \rho\, c^2}{k^2}\left[1 - \frac{1}{3}\Gamma(3)\right] \\ &= -\frac{4\pi G_o m \rho\, c^2}{3k^2} \end{aligned} \tag{18}$$

Substituting $k = \sqrt{(4\pi/3) G_o \rho}$ from (11), the total potential energy becomes

$$U = -mc^2 \tag{19}$$

This result is very interesting, as it implies that the total energy is just

mc^2 (due to the mass) $-\, mc^2$ (potential energy) $= 0$,

and the total energy content of the universe is nil. Whatever positive kinetic energies we observe are due to the local fluctuations and the lumpy nature of the universe in the small scale.

Other consequences

Equation (1) leads to a number of detectable effects due to local interactions. It can explain (Ghosh 1986a,b, 1988, 1993): (*i*) the secular retardation of the earth's rotation without facing a close approach of the Moon, as in the case of the tidal friction model; (*ii*) the secular acceleration of the Phobos; (*iii*) the excess redshift of the spectrum of the solar limb; (*iv*) the unexplained redshift of electromagnetic waves while grazing past the Sun; (*v*) the transfer of angular momentum from a central spinning Sun to the planets; and (*vi*) the unexplained mismatch between the relativistic and astrophysical mass of white dwarfs. This model also provides (Ghosh *et al.* 1988) a servomechanism to distribute matter in spiral galaxies in a unique way which results in flat rotation curves.

Though in the previous works $f(\theta)$ was taken as $\cos\theta|\cos\theta|$ instead of $\cos\theta$, the changes in the results will be minimal and within acceptable ranges. Furthermore, it should be also mentioned that all the important basic results due to interaction of universal nature remain unchanged. The only noteworthy change is a slightly higher value of the theoretically obtained Hubble constant. The exact equivalence of gravitational and inertial mass, the existence of a velocity dependent cosmic drag and the exponential decrease of *G* are all true with the new form of the inclination effect. Furthermore, it should be noted that all important fundamental results due to interactions of a universal nature will remain unchanged. The magnitude of the Hubble constant will be a little higher in the present case.

Concluding remarks

The new result which has been presented in this article is the value of the potential energy of a mass particle in a infinite stationary universe. It is found to be $-mc^2$, implying that the total energy is zero. This finding may have important implications.

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References

Ghosh, A., 1984. *Pramana - Jr. of Physics* 23:671.

Ghosh, A., 1986a. *Pramana - Jr. of Physics* 26:1.

Ghosh, A., 1986b. *Pramana - Jr. of Physics* 27:725.

Ghosh, A., 1988. *Earth, Moon and Planets* 42:169.

Ghosh, A., 1993. Astrophysical and cosmological consequences of velocity dependent inertial induction, in: *Progress in New Cosmologies: Beyond the Big Bang* , ed. H.C. Arp *et al.*, New York, Plenum Press.

Ghosh, A., Rai, S. and Gupta, A., 1988. *Astrophysics and Space Science* 141:1.