Light Signals in Galilean Relativity

Adolphe Martin
2235 Brébeuf, Apt. 3
Longueuil, Québec J4J 3P9

In invariant Galilean space-time, the Einstein postulate of the constancy of light speed is replaced by the postulate of space and time invariance. The Doppler factor is used to determine the time of light reception by the observer moving relative to the source. The requirement that velocity addition be vectorial in three space dimensions or arithmetical on each space dimension is met. Finally, we compare four viewpoints: one in Einstein Relativity, three in Galilean Relativity, with Einstein speed v, with Proper speed v/L and with Galilean speed V, where L is the Lorentz factor $\left[1-(v/c_o)^2\right]^{1/2}$.

Introduction

In this paper, I will interpret Relativity in an invariant Galilean space-time. The presentation is a necessary background to solving the Einstein Simultaneity Train problem in Galilean space-time. As this particular problem is in one dimensional space, this presentation is also restricted to one space dimension.

The figure is an *x*-*t* diagrams where the *x*-axis represents distance and the *y*-axis represents time, these being sufficient to solve Relativity problems in one space dimension. Velocities are in light units, *i.e.* divided by the speed of light between fixed points, taken as $c_o = 1$.

Figure 1 is a representation of four different equivalent viewpoints or interpretations in resolving the problem of an observer moving relative to a light source. An Einstein speed $v = 0.8c_o$ is taken as an example because it emphasizes the differences between viewpoints and simplifies most numerical calculations. The arguments remain valid for all other velocities.

Einsteinian Viewpoint

The Einsteinian physicist considers two postulates:

a) The principle of relativity, which says that physical phenomena appear the same for two observers in relative motion.

b) The principle of constancy of the speed of light; that is, the speed of light for every observer is always $c_o = 1$, represented in the figure by lines at 45° to the time axis.

An observer moving with speed $v = 0.8c_o$ is assumed to pass at M at a distance 10, ahead of a source A of light emitted at time $t = 0$.

To calculate the time at which the light will reach the moving observer M, Einstein and all the others divide the distance 10 by the speed of light relative to the moving observer:

$$t = \frac{\text{AM}}{c_o - v} = \frac{10}{1-0.8} = 50$$

The distance covered during this time is $x = c_o t = 50$, giving the event point m_a (x,t), (50, 50). The light would be reflected back toward a fixed observer at A at the same speed c_o and would reach A at time $t_r = 50 + 50 = 100$ at event point (0, 100). This would correspond to a radar shot where the moving object is assumed to be localized at distance $x = c_o t = 50$ at time $t = (t_r - t_e)/2 = 50$ after the time of emission $t_e = 0$. The moving observer is said to pass at event point m_a (50, 50).

At time $t = 0$, the moving observer M was at a distance 10 from the source. This distance had been covered at speed 0.8 in 10/0.8 = 12.5 time units. Then M is observed to pass at world-line of A at time $t_p = -12.5$, called the time of passage, before the light emission from A.

The figure shows the Einstein viewpoint of an observer at rest in the light source reference frame, where all lengths and times are shown to the same scale.

Galilean Viewpoints

Now, let us assume that we are living in a Galilean space-time. The Galilean viewpoint considers two postulates.

a) The principle of relativity.

b) The principle of space and time invariance, that is, length and time intervals remain the same in all reference frames.

This enables us to represent two reference frames in relative motion on the same piece of paper without contracting or stretching the paper! Notice that all arguments are represented in the figure according to this assumption. Einstein variables and constants are in lower case while Galilean variables are in UPPER CASE.

The Galilean reference frame at rest with the source is exactly the same as Einstein rest frame with lengths and times at the same scales. Due to the principle of space and time invariance, a moving Galilean reference frame utilizes the same scales.

The Doppler factor is used to determine the time of reception of a light signal by the moving observer. When the Einstein observer, moving away from source A at speed v, receives the light, he observes that the wavelength is increased 3 times the wave length at speed $v = 0$. This is due to the experimentally proven Doppler effect which is given by the Einstein Doppler factor formula, inverted for wave lengths.

$$\text{with } b = \frac{v}{c_o} = 0.8 \qquad D = \frac{\sqrt{1-b^2}}{1-b\cos q} \qquad \text{or } D = \frac{1-b\cos q'}{\sqrt{1-b^2}}$$

$$\text{with } \cos(0) = 1 \qquad D_0 = \sqrt{\frac{1+b}{1-b}} = 3 \qquad \text{in one dimension}$$

$$\text{with } \cos(180) = -1 \quad D_{180} = \sqrt{\frac{1-b}{1+b}} = \frac{1}{3} \quad \text{in one dimension.}$$

The subscripts 0 and 180 refer to the angle in degrees between the relative velocity and the light velocity. The light frequency at the observer M, moving away from source A, is decreased 3 times. If the source A sent light toward the observer M from the time of passage t_p to the time $t = 0$ during a time interval of 12.5 time units, the frequency being reduced 3 times, the moving observer has to receive the

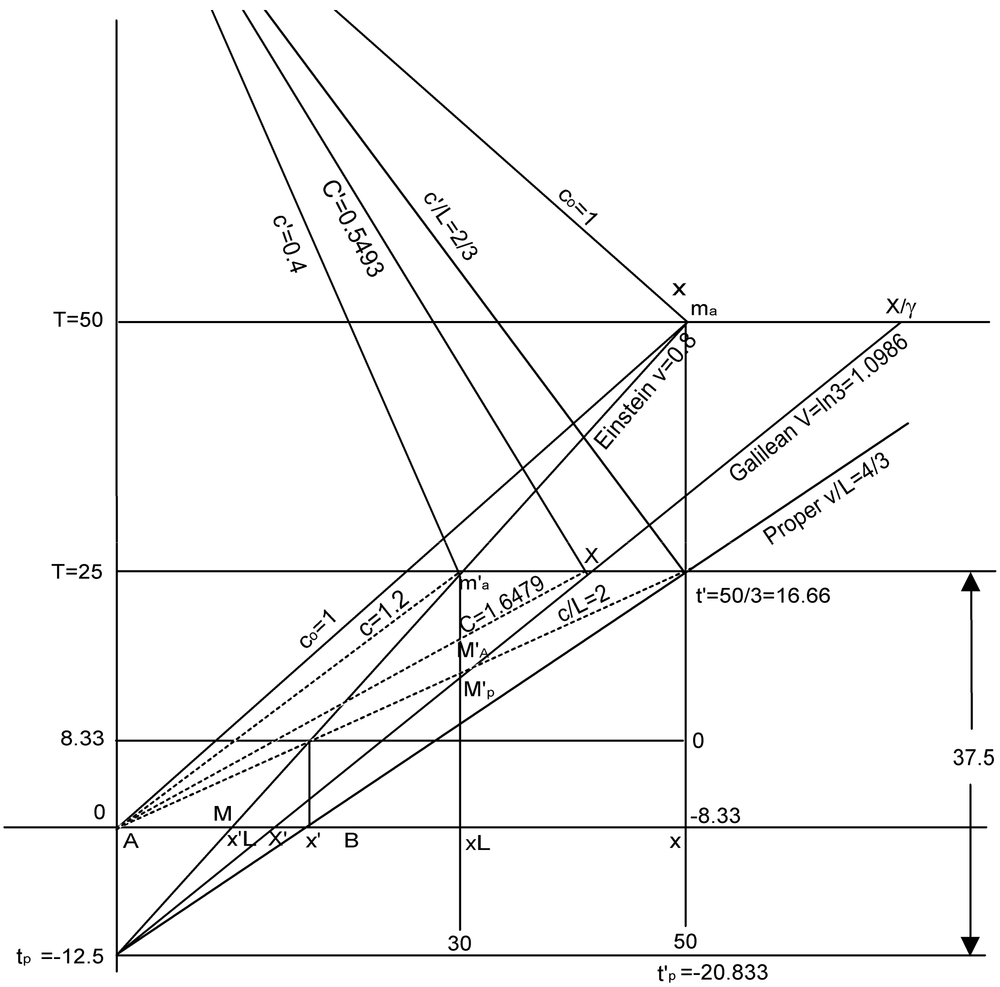

Figure 1. Relativity Relations between Einstein, Proper and Galilean speeds.

same number of waves during a 3 times longer interval, or $3 \times 12.5 = 37.5$ time units. This would bring the end of the reception at time $T = 37.5 - 12.5 = 25$.

According to the principle of relativity, an observer at A should also receive the light back from M at a 3 times reduced frequency. Because the emission from M lasts 37.5 time units, during the time interval t_p to T, the reception at A should last $3 \times 37.5 = 112.5$ measured from t_p or $t_r = 112.5 - 12.5 = 100$, exactly the return time of a radar shot as assumed by Einstein!

Since the Doppler effect is a fact of experience, all viewpoints have to respect it, including the Einsteinian. No matter what speed, equivalent to Einstein $v = 0.8c_o$, is assigned to the moving observer passing at A at time t_p, it has to be concluded that he receives the light emitted at A (0, 0) at time $T = 25$ in order to have the same Doppler factor. Similarly light reflected back to A from the moving observer is received at A at a single time $t_r = 100$, the observed return time. These are observed experimental facts to be respected by all viewpoints.

At velocity $b = v/c_o = .8$, the Lorentz factor is $L = \sqrt{1-(v/c_o)^2} = .6$.

We can calculate x' and t' by the Lorentz equations: taking $x = 50$ and $t = 50$, we obtain: $x' = (x - vt)/L = [50 - (.8 \times 50)]/L = 16.66$ and $ct' = (ct - bx)/L = [50 - (.8 \times 50)]/.6 = 16.66$. Calculating the values of x' and t' by the Lorentz equations, taking $x = 0$ and $t = t_p = -12.5$, we obtain: $x' = (x - vt)/L = 16.66$ and $t'_p = (t - vx/c_o^2)/L = -20.833$. Notice that the time interval from t_p' to t' is $16.66 - (-20.833) = 37.499$, or 37.5, exactly the same as the time interval from t_p to T. This proves that our Galilean reasoning is correct (or at least coincides with the Lorentz mathematics), and there is no time dilation.

The time interval $T - t_p = t' - t_p' = 0.6(t - t_p)$ is smaller than the time interval $(t - t_p)$ by exactly the Lorentz factor. Einsteinians then say that the moving frame time is dilated by the inverse of the Lorentz factor $(t' - t_p')/L = (t - t_p)$.

If the time origin was taken at t_p in both reference frames, to agree with the canonical Lorentz equations, then t_p and $t_p' = 0$, as $x' = 0$, and

$$T = t' = t\sqrt{1-\left(\frac{v}{c_o}\right)^2} = t \quad \text{the proper time}$$

again showing there is no time dilation, Einstein time t', the proper time and the Galilean invariant time T being numerically equal!

This is extremely important as it definitely proves, using only the Lorentz equations and the Einstein coordinates, without any further assumptions, that there is no time dilation! The time interval $t' - t_p'$ is strictly equal to the invariant time interval $T - t_p$ of the Galilean invariant space-time which was calculated based on the value of the Doppler factor as given by Einstein.

The Einstein Velocity *v*

We will consider first an observer moving with Einstein velocity v. At the moment of light reception $T = 25$, the Einstein observer moving with speed v is at a distance: $AM + vT = 10 + (0.8 \times 25) = 30$ or 0.6×50, which is exactly smaller by the Lorentz factor than $x = 50$ where the Einsteinian says the moving observer receives the light from A. In fact, if the Lorentz equations are written in Galilean form:

$$x'L = x - vt \qquad xL = x' + vt' \tag{1}$$

$$c_o t'L = c_o t - \frac{v}{c_o}x \qquad c_o tL = c_o t' + \frac{v}{c_o}x' \tag{2}$$

Equations (2) are essentially equations (1) where $x = c_o t$ and $x' = c_o t'$ on the x-axis.

We recognize, in Figure 1, distance $AM = x'L$ and distance of $m'a = xL$ as given by (2), with derived values $x' = AM/L = 10/0.6 = 16.66$ and $t' = x'/c_o = 16.66$. Thus, $x'L$ and xL are not contractions of x' and x, but space coordinates where the observer moving with speed v is at the time of light emission $t = 0$ and reception $T = 25$, respectively.

We see then why, for the Einsteinian who assumes $m'a$ at ma, the distances of the moving frame appear contracted by the Lorentz factor L, compared to those of the source rest frame. The Einsteinian says that $t' = 50/3 = 16.66$, as calculated by the Lorentz equations, is the time on the moving observer's clock at the time of reception of the light. It is then positioned on the right side of the figure at the reception time $T = 25$. The zero of the t' time scale is positioned at $T - t' = 25 - 16.66 = 8.33$ on the t or T time scale. It exactly corresponds to the time required by the Einsteinian moving observer, starting at $10 = x'L$, to reach the distance x': $(x' - x'L)/v = (16.66 - 0.6 \times 16.66)/0.8 = 8.33$.

Now x' is the coordinate of the moving observer in his own reference frame where he is considered to be at rest. At time $t' = 0$, the origin of the moving frame coincides with the origin of the rest frame at time $t = 8.33$.

We immediately see that the postulate of space and time invariance is respected. Consequently, we are living in a Galilean space-time.

Einsteinians use $AM/(c_o - v)$ to calculate t, a very Galilean formula, different from Einstein's own formula for the addition of velocities—a glaring inconsistency! This would mean that light from A takes 50 time units to traverse a distance 10 in the reference frame of the moving observer at the relative speed of $(1 - v/c_o)c_o = (1 - 0.8)c_o = 0.2$

c_o, not at speed c_o, contrary to Einstein's second postulate. These are exactly the same criticisms made in a recent article by Xu and Xu (1993).

To respect his postulate in the moving frame, the Einsteinian has to postpone the light emission from A at time $t = 0$ to $t = 8.33$, corresponding to his time $t' = 0$ and bring the reception back from $t = 50$ to $t' = 50/3 = 16.66$, corresponding to the Galilean time $T = 25$. Only then can he write with Lorentz $x' + vt' = xL$, again a Galilean formula! Notice in Figure 1 that this distance is $x' + vt' = (c_o + v)t' = 1.8 \times 16.66 = 30 = 0.6 \times 50 = xL$, the position of the observer moving with speed v at the time of reception of the light signal in Galilean space-time.

The Einstein Proper Velocity *v/L*.

In Figure 1, if a moving observer passing at (0, t_p) passed at (x, T), his speed would be greater by the inverse of the Lorentz factor: $v/L = 0.8/0.6c_o = 4/3$. This velocity is called proper velocity by modern Einsteinians, but they are convinced that it does not represent a physical velocity since it becomes greater than 1. When $v/L > c_o$, $v^2 > L^2 c_o^2 >$

$c_o^2 - v^2$, and for Einstein velocities

$$v > \frac{1}{\sqrt{2}} c_o > 0.7071 c_o$$

Nevertheless, assuming the moving observer has a proper velocity v/L greatly simplifies the relativity picture since, as the reader may have noticed, the moving observer would pass at x' at $t = 0$ and at x at $T = 25$ according to the very Galilean transformation $x' = x - (v/L)T$.

The light reflected at (x,T) or (50,25) has to be received at A at return time t_r or (0,100) to agree with the radar observation.

Costa de Beauregard (1968) wrote the Lorentz equations in hyperbolic form:

$$x' = x\cosh B - c_o t \sinh B \qquad x = x'\cosh B + c_o t' \sinh B$$

$$c_o t' = c_o t \cosh B - x \sinh B \qquad c_o t = c_o t' \cosh B + x' \sinh B$$

where

$$b = \frac{v}{c_o} = \tanh B, \quad \cosh B = \frac{1}{\sqrt{1-b^2}},$$

$$\sinh B = \frac{b}{\sqrt{1-b^2}}$$

The hyperbolic argument B is called celerity by Einsteinians.

These formulae give the same numerical results as the canonical Lorentz equations. In Einsteinian space-time, the velocity addition formula is really the equation for the addition of hyperbolic tangents

$$b = \frac{b_1 + b_2}{1 + b_1 b_2} \qquad \tanh(B_1 + B_2) = \frac{\tanh B_1 + \tanh B_2}{1 + \tanh B_1 \tanh B_2}$$

while the proper velocities add as hyperbolic sines:

$$\frac{v}{c_o L} = \sinh(B_1 + B_2) = \sinh B_1 \cosh B_2 + \cosh B_1 \sinh B_2$$

However, in Galilean space-time, because of the invariance of space and time, velocities should add vectorially in three dimension space, or velocity components should add arithmetically along each space dimension. Therefore, Einsteinian velocity and Proper velocity cannot be Galilean, since they do not add according to the Galilean way. The time of reception of a light signal by the moving observer is then:

$$T = t\left[1 - \cos\theta\left(\frac{1}{b} - \frac{L}{b}\right)\right] = t'\left[1 + \cos\theta'\left(\frac{1}{b} - \frac{L}{b}\right)\right]$$

$$T = t\left[1 - \cos\theta\left(\frac{\cosh B - 1}{\sinh B}\right)\right] = t'\left[1 + \cos\theta'\left(\frac{\cosh B - 1}{\sinh B}\right)\right] = T'$$

The Galilean Velocity *V*

In 1970, Jacques Trempe remarked that in the addition of two Einstein velocities, the Doppler factors D are multiplied while the B parameters of the hyperbolic functions add arithmetically, suggesting a simple logarithmic relation between D and B in a single space dimension:

$$B = \ln D_0 \qquad D_0 = e^B$$

$$-B = \ln D_{180} \qquad D_{180} = e^{-B}$$

The subscripts 0 and 180 refer to the angle between the relative velocity and the light velocity.

All this strongly suggests that B could be taken as the Galilean velocity parameter by making $B = V/c_o$. In this example, $B = V/c_o = \ln D = 1.0986$ for $D = 3$. The space coordinates, by the Galilean transformation, become:

$$X' = X - Bc_o T = X - VT \qquad X = X' + Bc_o T = X' + VT$$

Consequently, Jacques Trempe was able to write formulae for the light speeds C and C', which also apply to three-dimensional space. The outbound and inbound velocities of light are

$$C = \frac{c_o B}{\sinh B - \cos\theta(\cosh B - 1)}; \quad C' = \frac{c_o B}{\sinh B + \cos\theta'(\cosh B - 1)}$$

He then showed that the Lorentz equations are fully applicable to the same Galilean coordinates with $T = T'$:

$$X' = X\cosh B - CT\sinh B = X - VT$$
$$X = X'\cosh B + C'T'\sinh B = X' + VT'$$

$$C'T = CT\cosh B - X\sinh B$$
$$CT = C'T\cosh B + X'\sinh B$$

Notice that the two sets of equations are obtained by interchanging X with CT and X' with $C'T$.

The Lorentz transformation simply relates the same Galilean coordinates for the reception of the light signal by the moving observer with the Galilean light speeds C relative to the source and C' relative to the observer, while the Galilean transformation accomplishes the same thing with the Galilean relative speed V between the reference frames.

Summary of the Galilean Viewpoint

In Figure 1, light emitted at A, of velocity c, C or c/L, reaches the moving observer passing at $(0, t_p)$ with velocity v (Einstein), V (Galilean) or v/L (Proper) respectively at distances ma', X or x, at the same time $T = 25$. Light is reflected back toward A at the respective velocities c', C' or c'/L at the common event point (A, t_r) or (0, 100). The word velocity is used in the singular because the three sets (c, v, c'), (C, V, C') and $(c/L, v/L, c'/L)$ do not represent velocities pertaining to three different moving observers, but velocities pertaining to three different viewpoints of the same physical phenomenon, namely the reflection of light off a single moving object. A strict Galilean would use only C, V, C' but would soon find that solution of certain problems is complicated by the fact that he does not know his signal speed.

However using Einstein space-time, in which the light signal speed is normalized to the speed c_o between fixed points, the solution of problems is simplified. Knowing the relation between these four viewpoints enables us to solve problems in any of them and translate the results into any other viewpoint. This is due to the fact that, in Galilean invariant space-time, the three viewpoints are isomorphisms of one another when dealing with only two reference frames in uniform relative motion.

All lengths and distances of the three Galilean viewpoints T are proportional in the ratios: $v : V : v/L$ or $\tanh B : B : \sinh B$. In particular, the ratio of Galilean to Einstein distances is:

$$X/x = Y/y = Z/z = X'/x' = Y'/y' = Z'/z' =$$
$$CT/c_o t = C'T'/c_o t' = VT/(v/L)T = B/\sinh B$$

The first of the Galilean viewpoints uses the same coordinates as in Einstein space-time.

One must be careful not to confuse the first Galilean viewpoint using Einstein velocity in Galilean space-time, where light speeds c and c' are variable, with the Einstein viewpoint in Einstein-Minkowski space-time, where light speed c_o is constant. This is not an idle remark, as the great majority of critics of Einstein, claiming themselves as Galileans, use precisely the speeds $c_o + v$ and $c_o - v$ in their arguments. These speeds divided by c_o are hyperbolic tangents of $B = V/c_o$, and thus not Galilean speeds.

The variable speeds $c = c' + v$ and $c' = c - v$, like $C = C' + V$ and $C' = C - V$, and $c/L = c'/L + v/L$ and $c'/L = c/L - v/L$, are truly Galilean speeds in one dimensional space. Moreover, their ratio $c/c' = C/C' = (c/L)/(c'/L) = D$, is the Doppler factor even in three dimension space.

The chief difference between Einstein and Galilean Relativity stems entirely from Einstein assumption that the reception of a light flash by an observer at rest relative to the source A at (x,t), is the same event as the reception of the same light flash by the moving observer at (x',t') coordinates in his own reference frame given by the Lorentz equations. In Galilean space-time the two events, definitively different, are (x,t) and (X,T) in the source and fixed observer's reference frame, and in the moving observer's reference frame $(x - Vt, t)$ and $(X' = X - VT, T)$.

In Figure 1, if we were to draw lines from M, X', x' at $t = 0$ to (0, 25), they would reproduce at $\frac{1}{3}$ scale the same pattern as the lines from ma', X, x at $T = 25$ to (0,100) that we would see between T and t_r. They represent light emitted at $t = 0$ from the moving observer and received at A at $T = 25$.

According to Einstein, light emitted by the moving observer at M (distance AM = 10) at instant $t = 0$ should arrive with speed c_o at A at time $t = 10$ while light emitted at A at instant $t = 0$ should arrive at the moving observer at time 50! This is a flagrant breach of the relativity principle!

On the contrary, in Galilean invariant space-time, with light velocities dependent on the source-observer relative velocity, light emitted at A and moving M at instant $t = T = 0$ is received respectively at moving M and fixed A at the same time $T = 25$, in full compliance with the relativity principle. Similarly, light reflected from the moving M or the fixed A at time $T = 25$ is received at the return time $t_r = 100$ at the fixed A and moving M, respectively.

To comply with the relativity principle there should not be any time dilation. This is the only logical consequence if the light phenomenon depends solely on the relative velocity. Also the Galilean viewpoint is in accordance with the experimentally proven Doppler effect and the vectorial composition of velocities. The Galilean viewpoint eliminates the Einsteinian concepts of (Lorentz-Fitzgerald) space contraction and (Poincaré) time dilation.

Furthermore, it also removes the (Langevin) twin paradox, whereas, in this example, by placing event ma' at $m_a(x,t)$, a sizable chunk of time equal to $2(50 - 25) = 50$ is forgotten in the Einsteinian calculations of the moving twin's age, for a total time span of 2(50 + 12.5) = 125. According to the twin paradox the moving twin would be aged 125 – 50 = 75, after the return trip, while the stay-at-home twin would be 125, a ratio 0.6 = L according to the time dilation concept.

In Galilean space-time, on the other hand, the twins age at the same rate. Both would be 125 after the return trip. (This may be a disappointment for science fiction authors.) However, trips would be shorter in time since Galilean velocities greater than c_o are attainable! From $B = 1$, $D = e$ and $v = (e^2 - 1)/(e^2 + 1)c_o = 0.7616c_o$, the full range of Einstein velocities $0.7616c_o < v < c_o$ is represented by Galilean velocities $c_o < V <$ infinity. Moreover, there is only one Universe with one universal time, and time travel is absolutely physically impossible as it would entail the existence of an infinity of simultaneous universes.

However, the most important consequence for physics is the fact that the Lorentz transformation is fully applicable in Galilean space-time. This means that electromagnetism (invariant under the Lorentz transformation) and classical dynamics (invariant under the Galilean transformation) are both invariant in Galilean space-time. This constitutes a unification of electromagnetism with classical mechanics. In the search for a unified theory of the four forces, the unification of physics is a prerequisite.

A Ride on Einstein's Simultaneity Train

In view of the foregoing, we may look anew at the Einstein simultaneity train problem (Xu and Xu 1993).

For that purpose, in Figure 1, the Galilean and Proper observer do not pass at $(0,t_p)$ but at M(10, 0). Since the veloc ity is $V = c_oB$ and $v/L = c_o\sinh B$, respectively, the light velocity is still C and c/L, respectively. Thus, a line from M, parallel to $X'X$, intersects AX at MA' and a line from M, parallel to $(0,t_p)(x,T)$, intersects A(x,T) at MP'.

Point event MA' and MP' is the reception of light from A by the moving observer passing at M at time $t = T = 0$ with Galilean speed V and Proper speed v/L. The time of reception of the light event would then be inversely proportional to the velocity assigned to the moving observer. In this case, TA' = Tv/V; TP' = T$v/(v/L)$ = TL. We now find MA' and MP' are at exactly the same coordinate "xL" as ma'! In Galilean space-time, not only the angles are the same as in Einstein space-time, but in certain circumstances, the space coordinate on the x axis can be the same.

The full solution of Einstein Simultaneity Train problem will be pursued in a subsequent article.

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