Ampere Tension and Newton's Laws

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An experiment claiming to have resolved a long-standing controversy regarding the existence of so-called Ampere forces (longitudinal forces between electric current elements) is shown to have been in fact indecisive. A slight modification of the experiment, proposed here, should enable it to be crucial.

Introduction

A recent paper (Robson and Sethian 1992, hereafter termed R&S) reports new experimental results on the old problem of Ampere tension. The null results they achieved led them to conclude the nonexistence of Ampere longitudinal forces, interpreted as forces differing observably from those predicted by the Lorentz force law. Their bibliography confirms that the subject is controversial. As with any controversy, it is well to hear from both sides before reaching a judgment. Reasons will be offered here, with specific reference to the R&S experiment, why the issue is not yet empirically settled.

1. Shape-independence Theorem

All authorities agree that electric current flowing in a *closed external circuit* can exert no longitudinal force on a linear current-carrying test element. This is true for a wide variety of candidate force laws. If a longitudinal force component (such as exists according to Ampere's original inter-element force law) is to be observed, it is therefore essential to seek some measurable displacement of a *mobile portion* of a *single circuit*. To avoid force cancellation it is necessary that the fixed part of the circuit be in some sense asymmetrically configured with respect to the mobile portion. The experiment of R&S was of this kind, in that their fixed partial circuit was of *asymmetrical shape*. The following shows, however, that circuit shape asymmetry does not suffice for cruciality and that a different type of asymmetry is needed:

Theorem (shape-independence). For the electromagnetic force laws (*cf.* Whittaker 1960) of Lorentz (Grassmann, Biot-Savart), Ampere-Weber, Gauss-Riemann, and all others differing only by additive exact differential quantities, the longitudinal force component (if any) acting along the length of a straight current-carrying test element T, exerted by currents flowing in an external partial circuit C of given shape joining fixed endpoints E, E' (E, E', and C being nowhere coincident with T), is independent of the shape of C and depends only on the configuration of the gaps E–T and E'–T; *i.e.*, on the spatial locations of E, E' relative to T.

Proof. For simplicity we limit consideration to filamentary circuits. Given fixed relative positions of E, E', T, consider two alternative partial circuit shapes C', C" connecting E, E', as in Figure 1. Suppose that C', C" each carries a current I in the same sense, say, from E to E'. (T also carries a current. We view these as "mathematical currents" and need not specify how such partial circuits can carry physical currents.) Form a single closed circuit CC by connecting C', C" at the points E, E'. Let the sense of I in the C' portion of CC be reversed, so that a current loop is formed with current flowing unidirectionally (counterclockwise, as shown) in the complete circuit, but designated $-I$ in the C' portion from E' to E. We write as shorthand for the closed circuit CC = – C' + C".

By a well-known theorem (Maxwell 1954, Christodoulides 1988) any *closed circuit* carrying current entirely external to T exerts upon T, for all force laws of the class specified in the statement of the theorem, the same longitudinal force component as does the Lorentz law—namely, *zero*. So, closed circuit CC exerts zero force along the direction of T. If the circuit *portion* C', acting alone, exerts longitudinal force F on T, then a physical superposition of C' upon CC, call it CC + C', exerts longitudinal force $0 + F = F$ on T. But this superposition (wherein the superposed C' coincides geometrically with the – C' portion of CC), by the linearity of Maxwell's equations, is electromagnetically equivalent to the presence of C" alone—because the reversed current $-I$ in the C' portion of CC cancels the $+I$ within the superposed C'. In symbols, CC + C' = (– C' + C") + C' = C". So C' exerts *F*, CC + C' exerts *F*, and C" exerts *F*. Hence C' and C" exert the same longitudinal force on T . . . a force component therefore independent of shape of the external circuit portion in view of the arbitrariness of the shapes of both C' and C", *q.e.d.* The theorem is counterintuitive in that one normally thinks of longitudinal force applications (if physically possible) as requiring longitudinal current element alignments, or at least as influenced by alignment angles.

Corollary. If the gaps E–T and E'–T are symmetrical (of equal width) the force components balance and no net longitudinal force action on T can be observed [short of the breaking of material, as in exploding wire phenomena (Robson and Sethian 1992, Graneau 1985)].

The shape-independence theorem implies that asymmetries of the fixed external partial circuit shape, such as those employed by R&S, cannot in themselves produce observable imbalances of longitudinal forces acting on a straight test element; whereas, in light of the corollary just stated, gap-width asymmetries *can* do so for force laws such as Ampere's that assert the existence of longitudinal components of forces between current elements. Note that in order to apply the theorem it is essential that T be straight and that only force components parallel to its length be considered. It is not essential that the circuits be planar nor even that they be filamentary. The filaments of Figure 1 can in fact be deformed into current sheets of cylindrical symmetry, such as were employed in the R&S experiment. In the central vertical conductive cylinder, or *armature*, that forms the mobile element of their circuit all current elements are straight and parallel, so they form a cylindrical bundle of our elements "T" to each of which the theorem applies. The theorem is thus valid for the armature as a whole, and we can consider the R&S circular arc gaps (seen in cross section at a fixed azimuth) as corresponding to the gaps E–T and E'–T of Figure 1. By the corollary, if the gaps are of equal width at top and bottom, as they were in the R&S experiment, no net vertical force on the armature should be observable for any of the force laws named in the theorem. Regardless of shape of the external fixed portion of the circuit, *no experiment employing symmetrical gaps can be crucial* for the choice among force laws. (An experiment is crucial with respect to a theory if it has the capacity to falsify or refute it.)

According to the R&S method of analysis, their experiment could not have been crucial in any case. They integrate around the entire circuit in quantifying their predictions . . . but in reference to Ampere's law this is inappropriate insofar as it includes interactions among currents within the mobile armature. These self-actions (according to Newtonian mechanics) can have no effect on the state of motion of the armature's center of gravity (*i.e.*, no effect on any observable). They can stress the armature but cannot displace it. The full-loop integration, implying "bootstrap-lifting," is undesirable both because it conceptually defeats the purpose of single-circuit experimentation and because it is inconsistent with Ampere's assumption of the validity of Newton's third law. (That assumption was an explicit analytic input, not an empirical input, to Ampere's derivation of his law.) A fair test of any law demands of analysis that it not contradict the assumptions giving rise to that law. In order to predict observable motions of the armature consistently with Ampere's assumptions, it suffices to consider actions of *external* force-exerting current elements upon current elements within the mobile armature. What R&S derogate as "incomplete application of the (Ampere) law" is thus an essential requirement of logical consistency: one cannot jump back and forth between the presuppositions of two distinct logical disciplines such as relativity and Newtonian mechanics if one wants to know what either of them predicts about observables. And one cannot falsify any law empirically without adhering to its presuppositions analytically. Otherwise, it comes pre-falsified.

2. Proposed Modification of the Experiment

The shape-independence theorem proves very useful for simplifying quantitative force calculations. It enables us to substitute for an actual external circuit shape any "fictitious" partial circuit shape that (a) joins the terminal points E, E' (defining the upper arc gap E–T and the lower gap E'–T, respectively), (b) does not intersect the armature, and (c) simplifies analysis. In seeking to identify a possibly more "crucial" form of the R&S experiment, we shall here neglect the force contributions of currents flowing within the arc gaps themselves. This is a critical assumption that will be examined presently. Our theorem shows that the

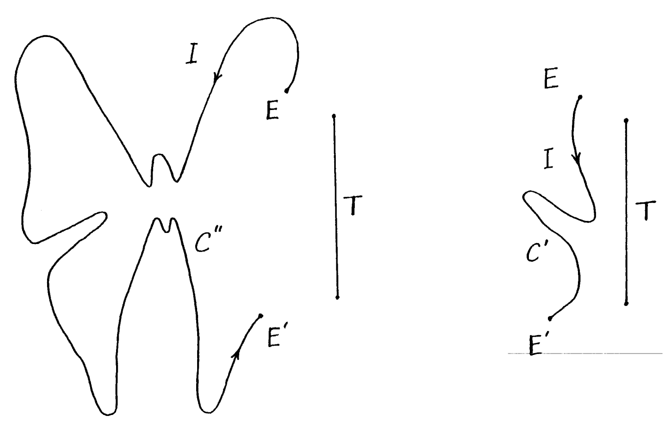

Figure 1. Arbitrary partial circuits employed in proof of shape independence theorem.

arc gaps must be asymmetrical. Suppose we choose to use exactly the R&S geometry except that we make their upper gap 1.5 mm wide and their lower gap 0.5 mm wide. (They chose both gaps to be about 0.5 mm.) Assume the length of the current-carrying portion of the armature to be 11.4 cm. In this case we may conveniently choose as our "fictitious" partial circuit a cylindrical current sheet of radius 1.26 cm coaxial with the armature (refer to Figures 5 and 6 of Robson and Sethian 1992), truncated at vertical height 11.4 cm, connecting to the upper-gap terminal point E and joined at its base to an annular horizontal conducting "base plate" having outer radius 1.26 cm and inner radius 1.16 cm, connecting to the lower-gap terminal point E'. By symmetry this coaxial cylinder of current, which encloses and is of the same height as the armature, exerts zero net force on the latter; so (always neglecting arc forces) the only observable force on the armature is that which is exerted upward by current flowing in the annular "base plate." This can be estimated by use of the F_2 integral given by R&S. The result thus predicted by Ampere's law, in conformity with Newton's third law, is a force constant $k' = -0.081$, which corresponds under the conditions of the R&S experiment to an upward impulse that would produce an initial armature motion of amplitude about 9 mm—which should be readily observable. Non-occurrence of this impulsive motion would be crucial for falsifying the Ampere law.

3. Effect of Arc-gap Currents

It remains to investigate the validity of the above neglect of forces exerted on the armature by currents within the arc gaps. This neglect is necessary to any prediction of observable motion of the armature because, if it were true that it makes no difference whether the gaps are filled with solid metal or with plasma, then we would be back to symmetrical (zero width) gaps—a case indicated by our corollary to yield zero net force on the armature. The following case for neglect of gap current force contributions is based on Newton's second law, about which there is no controversy:

Consider an elementary mechanical problem: Two bodies of masses M and m lie initially at rest at positions x_M, x_m on a horizontal x-axis. They repel each other with force $F(r)$, where $r = |x_M - x_m|$, due to electrostatics or some other cause. Newton's second law states that

$$\frac{d}{dt}(MV) = -\frac{d}{dt}(mv) = F$$

Case I. *Both masses free to move after release.* At $t' = 0$ the bodies are released and allowed to move apart along the x-axis under mutual repulsion without restraint. Before release the total momentum of the bodies is zero. After release it is still zero: $M\dot{x}_M + m\dot{x}_m = 0$ or $\dot{x}_m = -(M/m)\dot{x}_M$. Here $V = |\dot{x}_M|$ and we let

$$u = \frac{d}{dt}r = |\dot{x}_M - \dot{x}_m| = \left(1 + \frac{M}{m}\right)|\dot{x}_M| = \left(\frac{M+m}{m}\right)V$$

whence

$$V = \left(\frac{m}{M+m}\right)u$$

Thus

$$\frac{d}{dt}(MV) = M\frac{d}{dt}V = M\left(\frac{m}{M+m}\right)\frac{d}{dt}u = \mu\frac{d}{dt}u$$

where μ is reduced mass. We have

$$\frac{d}{dt}(MV) = \mu\frac{d}{dt}u = \mu\frac{dr}{dt}\frac{d}{dr}u = \mu u\frac{d}{dr}u = F(r)$$

which integrates to

$$\frac{1}{2}\mu u^2 = \int F(r)dr$$

It follows that

$$\frac{1}{2}\mu u^2 = \frac{1}{2}\left(\frac{Mm}{M+m}\right)\left(\frac{M+m}{m}\right)^2 V^2 = \frac{1}{2}MV^2\left(\frac{M+m}{m}\right)$$
$$= (KE)_M\left(\frac{M+m}{m}\right) = \int F(r)dr$$

Alternatively, it is formally permissible to write this in terms of an effective "reduced force,"

$$(KE)_M = \int F_{red}(r)dr$$

where

$$F_{red}(r) = \left(\frac{m}{M+m}\right)F(r)$$

Thus the freedom of recoil motion of the mass m (inversely related to its "sustainable reaction") causes the effective force it exerts on the mass M to be reduced from the "formula value" by a factor of the mass ratio $m/(M+m)$. It should be emphasized that this coefficient of force reduction is a *measurable quantity*. It is reflected in reduced kinetic energy and *reduced ultimately observable motion* imparted to the force sensor or test body M. (Velocity reduction $\propto \sqrt{m/(M+m)}$.)

Case II. *Only M free to move after release, m held fixed*. Since $x_m =$ const, we have

$$u = \frac{d}{dt}r = |\dot{x}_M - 0| = |\dot{x}_M| = V$$

As before,

$$\frac{d}{dt}(MV) = M\frac{d}{dt}u = M\frac{dr}{dt}\frac{d}{dr}u = Mu\frac{d}{dr}u = F(r)$$

which integrates to

$$\frac{1}{2}Mu^2 = \int F(r)\mathrm{d}r = \frac{1}{2}MV^2 = (KE)_M$$

So, in this case the effective force is the full formula force, $(KE)_M = \int F(r)\mathrm{d}r$, with no mass coming into the discussion. (Actually, mass is present in a "hidden" form, because our holding *m* fixed in space is the same as letting *m* become infinitely massive—but without the gravitational consequences.) So, if we want the full "formula force," *F(r)*, to be exerted on a test body *M* we must hold the force-exerting agent *m* fixed, assign it "infinite inertial mass", or rigidly attach it to some large actual mass such as the earth.

These two cases represent the extremes of complete motional freedom (Case I) and perfectly rigid constraint (Case II) of the force-exerting agency (the body of mass *m*). A simple mechanical model interpolating between these extremes is that in which the mass *m* slides upon a horizontal table of known coefficient of sliding friction. This may be left as an exercise for the reader. It illustrates the interpolation of "effective force" acting on *M* between the full "formula force" *F*(*r*) (Case II) and the maximally "reduced force" $F(r)m/(M+m)$ (Case I), and shows the possibility of quantifying the degree of firmness of "footing" of the force exerter with respect to any of the main masses of the universe. Although only Newton's second law has been employed in the foregoing argument, its major impact is upon our grasp of the third law: Force "equality" in the third law is to be understood in the sense that *observable action is limited by sustainable reaction*.

Application to estimating the "effective force" during capacitor discharge exerted by charge carriers (whether ions or free electrons) of mass *m* within the arc gaps of the R&S apparatus upon their mobile armature of mass *M* is immediate. Since friction (viscosity of the arc plasma) is sure to be small, and since the current-carrying charges in the arc are otherwise free (not anchored to the earth), it follows that we have what appears to be a close analogy to the above Case I—both the armature of mass *M* (= 56 gm) and the current carrying charges (force exerters) of mass *m* in the gaps being free to move under mutual action-reaction. *M* is so much greater than *m* $[(m/M+m) << 1]$ that one may be justified in treating the "reduced force" exerted on the armature by gap currents as reduced approximately to zero and thus in omitting the gaps from integration, as was done above. If solid metal conductor were the medium filling the arc gaps, instead of plasma, such metal would be effectively anchored to the earth by rigid connection through the rest of the external fixed portion of the electrical circuit. Thus we would replicate our above Case II, with exertion of the full formula force (and resulting force cancellation, as occurred for the equal gaps in the experiment actually done).

4. Discussion

The foregoing argument from analogy (which is not to be confused with proof) plainly supports the possibility of a crucial experiment based on modification of the R&S experiment to provide asymmetrical arc gaps. If a null result were observed this would seemingly falsify Ampere's law. Unfortunately, even if no motion of the armature occurred, cruciality might still prove elusive because of the following ambiguity. When the equations of motion of the two-body problem given above are integrated in the *time domain* for a force law of inverse r^2 character, such as Ampere's law, it is found that there is a characteristic time t_o that marks a transition (in the free-body Case I) from displacements of the test mass *M* that are indistinguishable from those of Case II (clamped force exerter *m*) to displacements that are readily distinguishable. Thus in the limit of large *M*/*m* we find roughly $t_o \approx \sqrt{2Mr_o/F_o}$, where r_o is initial separation of the bodies and F_o is the magnitude of repulsive force between them at that separation. As far as observables are concerned, no difference exists between Cases I and II for times $t << t_o$. For the R&S case this inequality is probably well satisfied, since t_o should be of the order of milliseconds and the gap transit time of ions of the order of microseconds. Hence, it would seem that force integration external to the armature should include the gaps and should by our corollary lead to (a) cancellation of Ampere forces (effectively zero widths of both gaps), (b) predicted immobility of the armature, and (c) no falsification of the Ampere law (*i.e.*, no cruciality), in congruence with the R&S outcome.

On the other hand, one could equally well argue that the force reduction factor *m*/(*M* + *m*) found in our analysis of Case I applies uniformly at all times greater than zero, but reveals its presence observably only after time t_o. This latter assumption would encourage a view of our proposed modified form of the R&S experiment as crucial by allowing empirical falsification of the Ampere law.

Of course, if the experiment were done and the Ampere law were *not* falsified (*i.e.*, if the armature moved in the case of asymmetrical gaps), this quite revolutionary result—entailing falsification of the Lorentz force law—would require rewriting the textbooks and agonizingly reappraising the physics of "universal covariance" (including certain physical concepts such as that of field momentum), as well as the status of Newton's third law. From a reaffirmation of Newton's third law concerning nonlocal action (Graneau and Graneau 1993) implications would extend throughout the cosmos, even to the validation of Mach's principle. Regrettably, the above-mentioned ambiguity as to the possibility of crucial experimentation arises out of an unobservable distinction (involving what one chooses to assume about action-reaction conditions prior to $t = t_o$) and thus bears a disturbing resemblance to the medieval issue of how many angels can dance on a pinhead. It is clear why controversy has persisted for over 170 years and why, even now, no single experiment is likely to settle it to everybody's satisfaction.

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