# Principle of Random Wave-Function Phase of the Final State in Free-Electron Emission in a Wiggler

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*The typical values of R = quantum energy/potential amplitude in electron energy of an incident radiation treated in quantum electrodynamics, an electric wiggler, and a magnetic wiggler are $10^8$, $10^{-4}$ and $10^{-10}$, respectively. Therefore, the interaction of a free electron with these fields cannot be described with the same principle. The wave-function phase of the final state through a transition route in a wiggler emission is unrelated to that through any other route, while that in photon scattering treated in quantum electrodynamics is route-independent. From this, we can explain the reason why the power of free-electron two-quantum Stark (FETQS) emission driven by macroscopic motion in an electric wiggler scales as $1/\hbar^2$, contradicting the correspondence principle, and is many powers of ten (e.g., $10^{19}$) times stronger than the value calculated with the quantum electrodynamic concept.*

## Introduction

The so-called Smith-Purcell (SP) radiations (Smith and Purcell 1953; and Doucas *et al.* 1992) were observed extraordinarily stronger than any free-electron emission that can be conceived with classical electrodynamics under any equivalent condition (Kim 1993 c). Purcell (Smith and Purcell 1953) thought of the SP radiation as Larmor radiation from the surface charge induced by the electron; the surface charge acts as the source of the radiated energy in Purcell's concept. We reject Purcell's concept, and identify the SP radiation with electromagnetic wake whose energy is derived from the electrostatic field energy around the electron, which is dragged by the moving electron (Kim, Chen and Yang 1990; Kim 1993 d). Calculation of the electromagnetic wake based on the Liénard-Wiechert potentials shows that the electromagnetic wake can account for at most one millionth of the measured power in the Smith-Purcell experiment (Kim 1993 d). Regardless of whether Purcell's concept or ours is valid, the SP radiation cannot come from the surface charge for three reasons.

(i) Doucas *et al.* (1992) measured radiation in the direction normal to the grating. If the SP radiation came from the surface charge which wiggles in the direction normal to the grating, the emission power would be zero at the direction normal to the grating. There was no indentation in the curve of the power per unit wavelength per unit solid angle *vs.* wavelength in that direction (*cf.* Figure 6 of Doucas *et al.* 1992).

(ii) Smith and Purcell (1953) measured the $l = 2$ harmonics (here $l = 1$ harmonic means the fundamental one). Since the surface charge moves in the plane normal to the grating containing the electron path, the motion of the surface charge is equivalent to electron motion in a plane-polarized magnetic wiggler. According to classical electrodynamics, the emission in a plane-polarized wiggler can exhibit only odd $l$ harmonics (Kincaid 1977). Therefore, Smith and Purcell should not have seen the first ($l = 2$) harmonics if the measured radiation came from the surface charge.

(iii) The principle of time-reversal invariance cannot be violated, whatever the process. In the SP experiments, the electron must be decelerated by emitting light at colatitude $\theta$ with the electron beam if the SP radiation is really to come from the electron. Inversely, if we inject laser light into the electron at angle $\theta$, the electron will be accelerated, assuming that the SP radiation comes from the electron. If the SP radiation were to come from the surface charge, the laser light would heat only the grating surface, and no electron acceleration could be observed. Mizuno *et al.* (1987) have experimented this inverse SP effect, and measured an extraordinarily large energy gain.

For the above reasons, we are convinced that the SP radiation comes from the electron, but not from the surface charge induced by the electron. Further, we are convinced that the SP radiation must be free-electron two-quantum Stark (FETQS) emission driven by practically-uniform axial motion in order to explain the extraordinarily large emission power (Kim 1993 c, d). The quantum-mechanically calculated FETQS emission agrees excellently with experiment in emission power, polarization, wavelength, and spectral width (Kim 1993 c). The power P of FETQS emission driven by macroscopic motion scales as $1/\hbar^2$, so that its magnitude is extraordinarily large. The scale law $P \propto 1/\hbar^2$ clearly excludes the correspondence principle. Yet the correspondence principle is considered one of the fundamental laws underlying modern physics. It is supposed that a free electron in practically-uniform motion on a straight line in free space cannot radiate, even when its energy is very high. In this paper, we investigate why such new facts, which contradict the laws which have been granted as valid, must be accepted.

## Photon in quantum electrodynamics

Quantum electrodynamics (QED) treats scattering of incident radiation by a free electron such as Compton scattering, but not scattering of an electromagnetic wave. In order to see quantum effects in laboratory, incident radiation should behave as a photon. Usually, photon in QED is X-ray and γ-ray. X-ray means radiation whose wavelength is between 100 **Å** (0.1 keV) and 0.01 **Å** (1 MeV) and γ-ray means radiation whose wavelength is near or shorter than 0.01 **Å** (1 MeV).

A radiation field behaves as a classical electromagnetic wave when

$$eA_o >> \hbar w \qquad (1)$$

In order for 1 **Å** X-ray (12 keV) to behave as a classical electromagnetic wave to an electron, its potential amplitude must be several orders larger than 12 keV. The intensity of the electromagnetic wave is related to its potential amplitude by the relationship:

$$I = \frac{pcA_o^2}{2l^2} \qquad (2)$$

To have a potential amplitude of 12 keV, the intensity should be about $10^{22}$ watt/cm$^2$. The usual laser intensity is $10^8$ watt/cm$^2$, and the largest laser intensity is now about $10^{18}$ watt/cm$^2$, which is achieved with excimer lasers.

We find from the above that, for a typical incident radiation treated in QED

$$\frac{\hbar w}{eA_o} >> 10^7 \qquad (3)$$

Thus, incident radiation treated in QED by no means behaves as a classical electromagnetic wave in the interaction with a free electron.

## First-Order Classical Field

An electric wiggler treated as a first-order classical field is, in every respect a classical field, but by no means a photon since its potential amplitude satisfies

$$\frac{ef_o}{\hbar\Omega} >> 1 \qquad (4)$$

For a magnetic wiggler, $\phi_o$ should be replaced by $A_{mo}$. The static wiggler with wavelength $l_\omega$, is equivalent to a temporally oscillating field with frequency $\Omega \approx pc/l_w$ . to a relativistic beam electron. In the Smith-Purcell (SP) experiment (Smith and Purcell 1953), $l_w = 10^{-4}$ cm, and the configuration is equivalent to an electric wiggler with quantum energy 1 eV, which is of the order of $ef_o$. For the practical electric wiggler $l_\omega$ is of order 1 cm, which is equivalent to frequency of $10^{-4}$ eV. The potential amplitude of a practical electric wiggler is order 100 V/cm, and thus $ef_o/\hbar\Omega = 10^6$ for a practical electric wiggler. The potential amplitude of the present magnetic wiggler (Elias *et al.* 1976) is of order 1 MeV, and thus $eA_{mo}/\hbar\Omega = 10^{10}$ for a magnetic wiggler.

A field is treated as a first-order perturbing field when the potential energy is much less than the kinetic energy in the direction of the force provided by the field, *i.e.*,

$$\textit{Potential Energy} << \textit{Kinetic Energy in Force Direction} \qquad (5)$$

The electric wiggler is a first-order field in a perturbation calculation since its potential amplitude in terms of the electron energy is much less than the axial kinetic energy, that is, $ef_o$ << axial K.E. ≈ $E$, where $E$ is the beam energy.

## Zeroth-Order Classical Field

If a field is a classical field, and its potential energy is much larger than the kinetic energy in the direction of the force provided by the field, then the field is called a zeroth-order classical field. We have found that a magnetic wiggler is a zeroth-order classical field (Kim 1992 a, 1993 a)

The force from a magnetic wiggler acts in the transverse direction. The transverse momentum is given by

$$p_\perp = \frac{eA_{mo}}{c} \qquad (6)$$

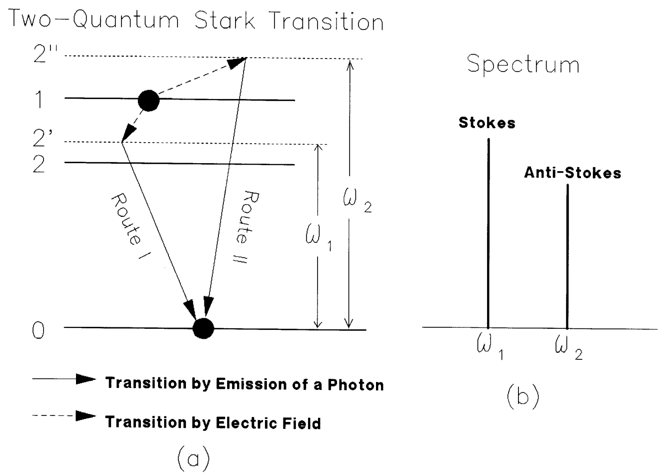


**Figure 1**. (a) Level diagram showing bound-electron two-quantum Stark transition.
(b) Spectral lines of bound-electron two-quantum Stark emission

The transverse kinetic energy is much less than the potential energy since transverse *K.E.* is

$$\left[(eA_{mo})^2 + c^2p_z^2 + m^2c^4\right]^{1/2} - \left(c^2p^2 + m^2c^4\right)^{1/2} \approx \frac{e^2A_{mo}^2}{2E} << eA_{mo} \quad (7)$$

Thus, a magnetic wiggler should be treated as a zeroth-order classical field (Kim 1993 b).

Even for a practical electric wiggler, we find that

$$\text{C.R.} = \frac{R \text{ in QED}}{R \text{ in Electric Wiggler Emission}} >> 10^{12}$$

Thus, it is most improbable that QED is still appropriate for free-electron emission in an electric wiggler.

## Wave-Function Phase

If $\Phi(\vec{r},t)$ is a wave function of a state, then $\exp(i\Lambda)\Phi(\vec{r},t)$ is also a wave function of the same state, where $\Lambda$ is the phase (Kim 1993 c). The phase is conserved until the next transition takes place. Thus, we can see a diffraction pattern in the usual double slit experiment (Kim 1993 c).

In atomic (or bound electron) two-quantum Stark emission, the applied oscillating electric field is a first-order perturbing classical field. A classical field is different from a quantum field in the sense that "phase" applies to the former, while the latter has no phase. In an interaction of an electron with a classical oscillating field, the wave function phase of the final state should vary with the phases of the classical field which the electron sees at the beginning and end of the interaction. However, there is an uncertainty as to the phases at which the electron starts and ends the interaction. Thus, the wave function phase after an interaction is unrelated to that after any other interaction. This can be seen in another way. As any other classical field, a first-order classical field consists of very large numbers of quanta. Even though an electron in a first-order classical field cannot absorb or emit more than one wiggler quanta simultaneously or consecutively, it performs the cycles of alternatively emitting and absorbing a wiggler quantum since the wiggler provides another wiggler quantum at any time after the electron absorbs a wiggler quantum. After one cycle, the electron state returns to the initial state only with the wave-function phase being changed. Since the electron does this act before and after an interaction with a photon, the wave-function phase of the final state through a transition route is unrelated to that through any other route. In reality, an electric wiggler does not act only as a first-order perturbing field; it acts as a low-order perturbing field as well. Thus, in reality, the electron can absorb more than one wiggler quantum simultaneously or consecutively so that it can exhibit higher harmonic, *e.g.*, second harmonic, lines.

Let us consider a three-level atom with two closely spaced upper levels (1 and 2) and one lower level (0), as shown in Figure 1 (a). The transition between level 2 and 3 through the emission or absorption of a photon is forbidden. The transition amplitude from level 1 to level 2 by two-quantum Stark emission can be, written as

$$\begin{aligned} T(1 \to 2) &= \left|A(1 \to 2' \Rightarrow 0) + \exp(i\Lambda)A(1 \to 2'' \Rightarrow 0)\right|^2 \\ &= \left|A(1 \to 2' \Rightarrow 0)\right|^2 + \left|a(1 \to 2'' \Rightarrow 0)\right|^2 + \\ &\quad + \exp(i\Lambda)A(1 \to 2' \Rightarrow 0)A^*(1 \to 2'' \Rightarrow 0) + c.c.. \end{aligned} \quad (8)$$

Now, $\left|A(1 \to 2' \Rightarrow 0)\right|^2$ and $\left|A(1 \to 2'' \Rightarrow 0)\right|^2$ represent the line intensity at frequency $w_1$, which is called the Stokes line and the one at frequency $w_2$, which is usually called the anti-Stokes line, respectively, as shown in Figure 1 (b).

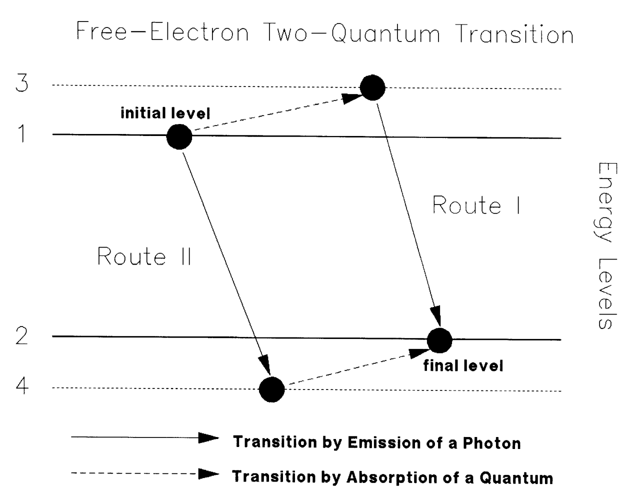


**Figure 2**. Level diagram of free-electron two-quantum transition when the kinetic energy is much greater than the photon energy. In this situation, the negative-energy electron going backward with time in the Dirac sea need not be considered.

Obviously, $\exp(i\Lambda)\langle A|1 \rightarrow 2' \Rightarrow 0\rangle\langle A^{*}|1 \rightarrow 2'' \Rightarrow 0\rangle + c.c.$, which is called the quantum interference term, has no physical meaning. It should disappear. Since $\Lambda$ is a random phase, it indeed disappears on averaging over $\Lambda$.

Let us call the principle that the phase of the final state should be random the *quantum wiggler electrodynamics* (QWD) *principle of random wave-function phase*.

The radiation acts as photon to the electron if its frequency $w$ is normally such that, $\hbar w >> >> >> eA_o$, where $A_o$ is its potential amplitude. When $\hbar w >> >> >> eA_o$ we observe that only one photon is incident on the electron, so that once the incident photon is absorbed, no further absorption can take place. Accordingly, harmonic Compton scattering, which is possible when the free electron absorbs more than one photon simultaneously or consecutively, does not occur. In a usual quantum electrodynamic calculation for a two-photon process associated with phase; the photon is considered as a particle, but not as a wave. Therefore, the electron identifies transition route *I* in which it interacts first with A and next with B, with another route *II* in which it interacts first with B and next with A; the electron at the final state can remember only that it has interacted with particles A and B, but does not remember with which of these it interacted first. Therefore, the wave-function phase of the final state is the same for both routes, and thus the transition probability amplitudes coherently contribute to scattering (not emission). This coherence is required to explain Compton scattering of a photon incident in an oblique direction to the electron velocity by a free electron whose kinetic energy is much greater than the photon energy. In this, the differential scattering cross section becomes of the form

$$\frac{\mathrm{d}s}{\mathrm{d}\Omega} = C\left|\frac{A_o + A_1\hbar + A_2\hbar^2 + \cdots}{\hbar} - \exp(i\Lambda)\frac{A_o + A_1\hbar + B_2\hbar^2 + \cdots}{\hbar}\right| \quad (9)$$

with $\Lambda = 0$ so that the two $\Lambda$'s cancel each other, giving

$$\frac{\mathrm{d}s}{\mathrm{d}\Omega} = x\hbar^2 + \cdots, \quad (10)$$

according to QED (Bjorken and Drell 1964; Sakurai 1967). (When the kinetic energy is less than the photon energy, we must think of the negative-energy electron going backward in time in the Dirac sea). On the other hand, if we apply the QWD principle of wave-function phase to the Compton scattering, $\Lambda$ is random, and $s = 2C|A_o|^2/\hbar^2$ on averaging over $\Lambda$, so that the differential scattering cross section becomes tremendously large, and does not agree with the correspondence principle. The QED concept, which agrees with experiment in this case, has been misunderstood as being appropriate even to the interaction of a free electron with a classical oscillating field. We will call the belief that the wave-function phase of the final state is route-independent the QED concept of *coherent wave-function phase*.

Even though QED agrees with experiments of Compton scattering, it is known that QED does not agree with a free-electron laser using a magnetic wiggler (MFEL). The magnetic wiggler is the same as an electromagnetic wave propagating in the negative *z*-direction. If we view this electromagnetic wave as a photon, the free-electron emission in a magnetic wiggler is just Compton scattering (actually the Compton scattering formula applies to the case where the kinetic energy is less than the photon energy, so that coupling with the negative-energy electron in the Dirac sea should be considered). Then, the total power and gain become small quantum effects which vanish in the classical limit. This is obviously wrong, since we can see quite obviously that any MFEL is lasing. It has been shown that, if the magnetic wiggler is treated as a zeroth-order classical field, the spontaneous emission power and laser gain are classical quantities which do not vanish in the classical limit (Kim 1992 a, 1993 a).

An electric wiggler is a first-order classical field, since $ef_o >> \hbar\Omega$ and $ef_o << E$. If the interaction follows the QWD principle, the laser gain $m$ and spontaneous emission power P of FETQS emission driven by macroscopic motion are of the form:

$$m = A_o/\hbar^2$$
$$\mathrm{P} = B_o/\hbar^2 \quad (11)$$

The above form contradicts the correspondence principle, since it diverges as $\hbar \rightarrow 0$. Accordingly, we have rejected the correspondence principle in favour of the non-correspondence principle (Kim 1993 c). These values are many orders of magnitude greater than would be expected if the interaction followed the QED principle of coherent wave-function phase, as explained in (1).

## Conclusions

If a wiggler is an incident quantum, we can see only a fundamental (or $l = 1$) harmonic line. However, both a magnetic wiggler and an electric wiggler (Smith and Purcell 1953) exhibit first, second, third, *etc.* harmonic lines. Therefore, it is obvious that a wiggler is not an incident quantum, but acts as a flux of wiggler quanta so that a free electron can absorb more than one wiggler quantum simultaneously or consecutively.

We have found that free-electron emission in an electric wiggler should not be treated with QED or classical electrodynamics. lt should be treated with quantum wig gler electrodynamics (QWD), in which the wave-function of the final state of a route involving an interaction with a wiggler is random.

We have also found that the correspondence principle is just a groundless faith.

The foregoing conclusions are summarized in Table 1.

**Table 1**. Comparison of incident radiation in QED, Electric Wiggler, and Magnetic Wiggler

| | **Incident Radiation in QED** | **Electric Wiggler** | **Magnetic Wiggler** |
|---|---|---|---|
| $\frac{\text{Quantum Energy } \hbar\Omega}{\text{Potential Amp. } eA_o \text{ or } ef_o}$ | $> 10^7$ | $1 \leftrightarrow 10^{-6}$ | $< 10^{-10}$ |
| $\frac{\text{K.E in Force Direction}}{\text{Potential Energy}}$ | Doesn't apply | << 1 | >> 1 |
| Perturbation Order | First | Low Order e.g., First, Second | Zeroth |
| Higher Harmonic | No | Yes | Depending on $\frac{eA_{mo}}{mc^2}$ |
| Wave-Function Phase $\Lambda$ of Final State | Route-Independent | Random | Random |
| Treatment | QED | QWD | QWD |
| Application | Physics Only | ultra-strong tunable laser X-ray laser | tunable, but too weak laser for application |

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