The Confrontation Between Relativity and the Principle of Reciprocal Action

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In this paper it is shown how Einstein's Theory of General Relativity violates the well established Principle of Reciprocal Action. Then some implications of this fact are discussed.

In a pure electromagnetic field two types of mass may be associated with a particle. These are the rest mass, m_0, and the inertial mass, m_i, determined by the instantaneous linear velocity u according to the relation

$$m_i = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}} m_0 \tag{1}$$

where c is the speed of light *in vacuo*.

In a pure gravitational field two additional types of mass may be identified for a particle. These are the passive and active masses. The passive mass, m_p, of a particle is its gravitational field responding or experiencing mass. The active mass, m_a, is the gravitational field producing or generating mass. Therefore, a theory of gravitation necessarily begins with choices for the passive and active masses—an equivalence principle.

Now, the equivalence of passive and inertial masses has been overwhelmingly verified experimentally since Galileo. But the choice of the active mass has remained controversial until today.

For his theory of gravitation called General Relativity, A. Einstein chose the Strong Equivalence Principle which may be formulated as follows:

> *In all reference frames the active mass of a particle is equal to its rest mass.*

The Strong Equivalence Principle was stated by the well known relativity authorities J. A. Wheeler and E. F. Taylor (1966) in the following words:

> *Rest mass of a system ... governs the gravitational attraction it exerts on a test particle.*

But, at the birth of General Relativity in 1915, the Strong Equivalence Principle was only a guess by A. Einstein with no basis in experimental fact. This fact, as well as its ominous consequence— it placed General Relativity on a precarious footing—was acknowledged and lamented by its very author in the following words (de Broglie 1979) concerning the right hand side (the side based on the Strong, Equivalence Principle) of his gravitational field equations

$$R^{ab} - \tfrac{1}{2} R g^{ab} = -\frac{8\pi G}{c^4} T^{ab} \tag{2}$$

"It resembles a building of which one wing (the left hand side) is constructed in fine marble, whereas the other (the right hand side) is low quality wood."

As far back as 1923 the great relativity authority A. S. Eddington reiterated the point in the following words (emphasis added):

> *The principle of equivalence [Strong Equivalence Principle] has played a great part as a guide in the building of the generalized relativity theory, but now that we have reached the new view of the nature of the world it has become less necessary ...[it] is* ***essentially a hypothesis to be tested experimentally as opportunity offers. Moreover, it is to be regarded as a suggestion rather than a dogma admitting of no exceptions.***

And, until today, the Strong Equivalence Principle on which Einstein's General Relativity critically depends has continued to remain a guess with little or no prospect for experimental verification, as expressed by W. Rindler (1977), the well known relativity authority, in the following words (emphasis added):

> ***It must be said in fairness that the empirical evidence for the [strong] equivalence principle is very poor.*** *The checks mentioned under (b) above concern two aspects of light propagation and it can be maintained that what they really test is only the Weak Equivalence Principle, once we grant the corpuscular nature of light (photons).*

Another relativity authority, C. M. Will (1979), stated the same point in the following words (emphasis added):

> *The strong experimental support for the fundamental criteria for the validity of gravitation theories, coupled with theoretical arguments such as Schiff's conjecture, serve to bolster our faith that whatever theory of gravitation is correct, it must be a metric [relativistic] theory ...* ***it must be kept in mind, however, that this assumption rests on potentially unsteady ground, and continued experimental and theoretical effort is crucial in maintaining a solid foundation for gravitation [general relativity] theory.***

Now it may be stressed that, as pointed out by W. Rindler (1977), all the so-called "tests" of General Relativity are consequences of the equivalence only of passive and inertial masses. The anomalous planetary precession, for example, is a test of the equivalence of the passive and inertial masses for the planets, and is in no way a test of the equivalence of the active and rest masses for the sun the source of the gravitational field in which they move. Similarly, the gravitational deflection and redshift of starlight in the solar system are tests of the equivalence of the passive and inertial "masses" of the photon and are in no way tests of the equivalence of the active and rest masses for the sun. In fact, any theories of mechanics based on the equivalence of the passive and inertial masses will duplicate the above effects, as is well established (Howusu 1991) for the theory of General Mechanics. Consequently, these effects are by no means exclusive to General Relativity.

Not only is Einstein's theory of General Relativity theoretically unproved and experimentally unverified, but the Strong Equivalence Principle has the consequence of contradicting the experimentally well established and philosophically very satisfactory Principle of Reciprocal Action. H. Bondi (1979) registered his suspicion and concern about this fact in the following words:

> *It is true that in the field equations we are interested in the active [field producing] mass, but in the spirit of reciprocity common to physics, exemplified in Newton's third law, it would be hard to imagine that kinetic energy did not contribute equally to active mass.*

Since this issue is of such fundamental importance in physics, it is illustrated in this paper, using as an example rotational motion, and its implications are examined.

Consider two homogeneous spherical bodies of nonzero rest masses M_{01} and M_{02} respectively, separated by a distance d. Let the sphere of rest mass M_{01} be given a rotational motion about an axis through its center with a uniform angular speed w_1. Then, if this sphere has a radius R_1, it follows that its inertial mass M_{i1} during the rotation is given by (see appendix for derivation)

$$M_{i1}(w_1) = K(w_1)M_{01} \tag{3}$$

where

$$K(w_1) = \sum_{n=0}^{\infty}\left(n^{-1/2}\right)(-1)^n\left(\frac{R_1 w_1}{c}\right)^{2n}\frac{3I_n}{2(2n+3)} \tag{4}$$

and

$$I_n = \begin{cases} 1 & ;n=0 \\ 2\cdot\frac{2}{3}\cdot\frac{3}{5}\cdots\frac{(2n-2)}{(2n-1)}\cdot\frac{2n}{(2n+1)} & ;n>0 \end{cases} \tag{5}$$

Now, suppose that the theory of General Relativity reduces smoothly and continuously to the theories of Newtonian Mechanics and Universal Gravitation for particles of nonzero rest masses at low speeds compared with the speed of light *in vacuo* as is required of all theories in physics. Then, by the Law of Universal Gravitation and the Strong Equivalence Principle ,the gravitational fields E_{g1} and E_{g2} established by the two spheres are given respectively by

$$E_{g1}(d) = -\frac{GM_{01}}{d^2} \tag{6}$$

and

$$E_{g2}(d) = -\frac{GM_{02}}{d^2} \tag{7}$$

Therefore, by the equivalence of the passive and inertial masses and the Law of Universal Gravitation, it follows that the gravitational forces F_1 and F_2 experienced by the two spheres respectively due to their mutual gravitational interaction are given by

$$F_1 = -\frac{GK(w_1)M_{01}M_{02}}{d^2} \tag{8}$$

and

$$F_2 = -\frac{GM_{01}M_{02}}{d^2} \tag{9}$$

Thus it follows from (8) and (9) that

$$\frac{F_1}{F_2} = K(w_1) \tag{10}$$

which to the order of c^{-2} gives explicitly

$$\frac{F_1}{F_2} = 1 + \frac{R_1^2 w_1^2}{5c^2} \tag{11}$$

Thus, the rotating sphere experiences a greater gravitational force from the stationary sphere than it exerts on it, completely contrary to the Principle of Reciprocal Action. This is a necessary consequence of the Strong Equivalence Principle on which Einstein's theory of General Relativity is based. And it implies that irrespective of the magnitudes of M_{01} and M_{02}:

(a) F_1/F_2 is greater than unity,

(b) F_1/F_2 increases with the angular speed w_1,

and

(c) F_1/F_2 increases with the radius R_1.

The relativity authorities J. Foster and J. D. Nightingale (1979) acknowledged this consequence of the Strong Equivalence Principle in the following words:

> *Newton's third law that 'to every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal and directed to contrary parts', looses its meaning in general relativity, at least as far as gravitational interaction is concerned.*

It may be noted, however, that all mechanical actions and reactions are equal, in perfect agreement with the Principle of Reciprocal Action.

It is also true that all electromagnetic actions and reactions are equal, in perfect agreement with the Principle of Reciprocal Action.

Finally, the recently published theory of gravitation called General Gravitation (Howusu 1991) is based on the Weak Equivalence Principle, which asserts that:

> *In all inertial reference frames the active mass of a particle is equal to its inertial mass.*

Thus, in this theory, gravitational action and reaction are equal for the two masses discussed here, as well as every other pair of masses—in perfect agreement with the Principle of Reciprocal Action.

It is, therefore, logical to conclude that, since there is no experimental evidence for the Strong Equivalence Principle, and since its predictions contradict the well established Principle of Reciprocal Action, the theory of gravitation based on it—General Relativity—may not be a sufficiently natural and complete physical theory.

In addition, it should be borne in mind that Einstein chose the Strong Equivalence Principle for General Relativity solely because it is the only alternative compatible with his fundamental postulate of relativity—the requirement of the covariant formulation of all laws of physics—as stated in the following words (Rindler 1979):

> *The appeal of the (Strong) Equivalence Principle is thus mainly theoretical.*

and (Bondi 1979):

> *However, there is no Lorentz invariant form in which any mass other than rest mass appears.*

Thus, it follows that the fundamental postulate of relativity, and hence all theories of relativity, not excluding Special Relativity, may not be physically natural and complete.

Since, on the other hand, there is overwhelming experimental evidence for the Weak Equivalence Principle (Rindler 1977) and since its predictions are in perfect agreement with the well established Principle of Reciprocal Action, the theory of gravitation based on it—General Gravitation—which also excellently duplicates all the experimentally established gravitational results, may be a physically more natural and complete theory deserving serious exploration and development.

Appendix

Consider a homogeneous spherical body of radius R_1 and nonzero rest mass M_{01} rotating about a diameter with a uniform angular speed w_1. Let S be the reference frame with origin O at the center of the sphere, but not rotating with it. Let $\mathbf{r}(r,\theta,\phi)$ be an arbitrary point within the sphere in the spherical coordinates of S. Then the distance of the point from the axis of rotation (Z axis) is $r\sin\theta$. Therefore the linear speed u of the element of mass located instantaneously at $\mathbf{r}$ is given by

$$u = w_1 \sin\theta \tag{A.1}$$

Hence, it follows from definition (1) that the instantaneous density of inertial mass ρ_{i1} of the sphere is given by

$$\rho_{i1}(r,\theta,\phi) = \left(1 - \frac{w_1^2 r^2 \sin^2\theta}{c^2}\right)^{-1/2} \rho_{01} \tag{A.2}$$

where ρ_{01}, is the corresponding density of rest mass, which is constant. Consequently, the inertial mass M_{i1} of the sphere when it is rotating with angular speed w_1 is given by

$$M_{i1}(w_1) = \int_0^R \int_0^\pi \int_0^{2\pi} \rho_{i1}(r,\theta,\phi) r^2 \sin\theta \, dr \, d\theta \, d\phi \tag{A.3}$$

But the sphere has a nonzero rest mass and hence

$$w_1^2 r^2 \sin^2\theta < c^2 \tag{A.4}$$

Consequently, substituting (A.2) into (A.3), expanding and rearranging, it follows that

$$M_{i1}(w_1) = \sum_{n=0}^{\infty} \binom{-1/2}{n} (-1)^n \rho_{01} \left(\frac{w_1}{c}\right)^{2n} \cdot \int_0^R r^{2n+2} \, dr \cdot \int_0^\pi \sin^{2n+1}\theta \, d\theta \cdot \int_0^{2\pi} d\phi \tag{A.5}$$

Thus the results (3), (4) and (5) follow after the integration and substitution

$$M_{01} = \frac{4}{3}\pi R^3 \rho_{01} \tag{A.6}$$

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