On the Absorption of Gravity

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We propose a modified Weber's potential for gravitation that takes into account the influence of intervening matter. Then we obtain equations of motion similar to Newton's first and second laws, and derive the proportionality between inertial and gravitational masses. We conclude that the gravitational absorption coefficient should be proportional to the square root of the density of the intervening medium, and that for solids its value is approximately $10^{-11} m^{-1}$. *All of this is accomplished supposing a limitless, homogeneous and stationary universe.*
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"For Nature is very consonant and conformable to her self."
Isaac Newton - Optics, 1704 (1979)

Introduction

The principle that all forces in nature have a single origin is a powerful one. A weaker statement of this idea is that all forces have the same structure and behave in a similar way. If we follow this line of reasoning it is natural to suppose that the gravitational attraction between two bodies is influenced by the nature of the medium between them. We know that the propagation of light is influenced by the kind of medium through which it flows, and we might expect the same to happen with gravitation. Another line of reasoning that leads to the same idea arises in electrostatics and magnetostatics. If we put a metal or a dielectric material between two stationary charges, this medium will be affected by the charges (due to induction of charges, or to polarization). Accordingly, the net force in any of the two charges when there is an interposed medium is different from the net force on the same charge when a medium is not present. The same happens if we consider the force between two magnets in the presence or absence of an interposed magnetizable medium like iron. In these two last examples there is an influence of the intervening medium and this may be similar to what happens with light. Yet there is one important difference: in electrostatics and magnetostatics we are not compelled to suppose that something flows from one body to the other, as happens with light, and what affects the bodies may be described by a simultaneous many-body interaction. Nevertheless, what is common to all these examples is the influence of the intervening medium. It is thus natural to suppose that the same should occur in gravitational interactions.

We were led to this insight when considering the size of the universe and related problems. In particular, after showing how to implement Mach's idea that the inertial forces are due to a gravitational interaction of any body with other bodies in the universe, (Assis 1989a), we began to analyse the distribution and extent of matter in nature.

As we will follow and extend the same model in this paper we begin by giving its main characteristics: (I) Primitive concepts: electrical charge, gravitational mass, distance between material bodies, time between physical events, force or interaction between bodies; (II) Postulates: (A) Force is a vectorial quantity (adds like a vector, etc.), (B) The force that a material body A exerts on a material body B is equal and opposite to the force that B exerts on A, (C) The sum of all forces (of any kind) on any material body is zero; (III) For electromagnetic and gravitational forces we introduced as a model an interaction of the Weber type. Following these ideas, and without introducing the notions

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of absolute space or of inertial mass, we were able to *derive* some interesting results: (1) equations of motion similar to Newton's first and second laws, (2) the proportionality between inertial and gravitational masses, (3) an implementation of Mach's principle, namely, the equivalence between the frame of the "fixed stars" (the frame defined by the mean distribution of galaxies around us, in which this mean distribution is at rest and without rotation, and which seems to be the same frame in which the cosmic background radiation is isotropic) and an inertial frame, and (4) the same value of the advance of the perihelion of the planets as that given by general relativity but through a different orbit equation.

As we pointed out in our earlier work , the first to give a particular form of the third postulate that the sum of all forces on any body is zero was Sciama (1953). But it should be emphasized here that in addition to postulating it only for gravitational forces—whereas we apply it to all kinds of force—he restricted his postulate as being valid only in the rest frame of the test body. We, on the other hand, postulate that *the sum of all forces on any body is always zero in all frames of reference, even when the test body is in motion and accelerated.*

Brown (1955 and 1982) also utilized this postulate, although only implicitly, because he did not state it clearly. He also derived $m\vec{a}$ as due to a gravitational interaction of any body with the remainder of the universe. Yet neither he nor Sciama utilized a Weber's law for gravitation. While Sciama utilized an equivalent to the Lorentz force law and Maxwell's equations, Brown constructed a force law of his own. We prefer to keep Weber's law, not only because of its many successes in electromagnetism (Coulomb's law, Ampère's force and Faraday's law of induction are particular cases of Weber's force), but also because it is a completely relational theory. By *relational* we mean any theory which depends only on the relative positions, velocities and accelerations of the interacting bodies. As a result, the meaningful quantities which appear in the potential energy or in the force law have the same values for all observers in all coordinate frames. An important discussion of a general theory of this kind and its relation to Mach's principle has been given by Edwards (1974).

Other arguments in favour of a Weber force are that it follows Newton's action and reaction law; the force is always along the line joining the particles; it can be derived from a velocity-dependent potential energy; and it complies with the conservation laws of energy, linear and angular momentum. Weber's theory can be found in his collected works (Weber 1892-4), part of which has been translated to English as his important papers of 1848 and 1871 (Weber 1966 and 1872, respectively). There are good descriptions of Weber's work in the last chapter of Maxwell's *Treatise* (1954), and in O'Rahilly's classic book (1965), which we highly recommend. A recent review of Weber's electrodynamics, with important extensions, is in Wesley's works (1987, 1990 and 1991).

There is also an important paper by Eby (1987) in which he applies Weber's law to gravitation and arrives at many results similar to our own with regard to Mach's principle, the precession of the perihelion of the planets, etc. A similar approach has been followed by Ghosh (1984, 1986 and 1991), although he utilized a force law of his own, which is not exactly like Weber's force. An important aspect of his theory is that he succeeded in deriving Hubble's law of redshift as a drag effect which appears naturally in his model.

We should mention also the fundamental work which is being developed by Treder and collaborators on Mach's principle and the equivalence principle, as well as the absorption of gravity and the retardation of the gravitational potential (Treder 1971 and 1972; Treder, von Borzeszkowski, van der Merwe and Yourgrau 1980; Treder and Mücket 1981; Steenbeck and Treder 1984). They analyse both the general theory of relativity and its relation with all these concepts, and some extensions of GR and their possible relevance and limitations. They make an historical and critical analysis of all these aspects, and we strongly recommend their works to anyone who wants to have a better knowledge of these subjects. Before continuing it should be mentioned that a modification of Newton's law of gravitation with an exponential term (not exactly the same as the modification we will propose here) is widely discussed nowadays due to problems related to Eötvös experiment, dark matter, flat rotation curves of galaxies, etc. (Fischbach et al. 1986 and 1988; Kuhn and Kruglyak 1987; Fujii 1975; Sanders 1986). Another model which considered a similar exponential term is the Lorentz-Dicke theory (for a good discussion and references see Clube 1980, 1989 and 1991).

Absorption of gravity

To the best of our knowledge, the first person to propose a modification of Newton's law of gravitation with an exponential term was Laplace (1880), who proposed:

$$F = \frac{Gm_1m_2}{r^2}e^{-\lambda r} \qquad (1)$$

where λ would be a kind of coefficient for the absorption of gravity. In order for his law to be compatible with the observations of the orbits of the planets Laplace obtained an upper limit for λ in the solar system: $\lambda < 10^{-6} / A.U.$, where $A.U.$ means the astronomical unit, the average distance between the Earth and the Sun ($1A.U. = 1.5\times10^{11} m$).

Following our work on Mach's principle, (Assis 1989a), we propose a modification originally in the gravitational energy between two material bodies and not initially in their force. As a model for the gravitational interaction energy we propose a modified version of Weber's potential. When there is a homogeneous medium filling all space between two point particles of gravitational masses m_1 and

m_2 separated by a distance r we assume that the generalized potential energy between them is given by

$$U = -H_g \frac{m_1 m_2}{r}\left(1 - \frac{\xi}{2}\frac{\dot{r}^2}{c^2}\right)e^{-\alpha r} \qquad (2)$$

where $r \equiv |\vec{r}_1 - \vec{r}_2|$, $\dot{r} \equiv dr/dt$, H_g we assume to be a constant, $\xi = 6$ to get the correct precession of the perihelion of the planets (see Assis 1989a), c is a constant with the same value and dimensions of the velocity of light in vacuum, and α we assume to be a constant characteristic of the medium in the straight line between m_1 and m_2 ($\alpha = 0$ in vacuum). We prefer to include the absorption coefficient already in the energy due to the pioneering work of Bouguer (1729) on the absorption of light, where he discovered the attenuation of light (energy) by translucent materials. Sometimes Bouguer's law is referred to as Lambert's law or even as Beer's law, although this does not seem to be a fair statement, (Mach 1953 and Curtis 1978). To our knowledge the first to propose an exponential term in the gravitational potential energy, and not in the force law, were Seeliger and C. Neumann, in 1895, working independently of one another (see for references and important discussions: Jaakkola 1987 and 1991; Kropotkin 1991; and Keys 1987).

The proposal of Seeliger and C. Neumann is equivalent to equation (2) with $\xi = 0$. More specifically, they added an exponential factor to the Newtonian potential, yielding $\phi = \iiint (\rho/r)e^{-\alpha r} dV$, where ρ is the density of matter.

We would like to emphasize here that our main assumption, *i.e.* equation (2), is not an *ad hoc* proposal. The physical rationale for the exponential term has already been described (analogy with optics, electrostatics and magnetostatics; the possibility of an absorption of gravity or the propagation of the gravitational interaction, etc). We might add two more ideas which lead to an exponential decay for gravity. The first is that in an infinite universe Newton's gravitational potential is indefinite. This means that we can obtain any value for the gravitational force or potential depending on where we choose to begin the integrations. Obviously this is not desirable, and an exponential term is the natural solution to avoid this problem. This was the point of view of Seeliger and C. Neumann. The second is that we know that the radiation of distant galaxies is decreased by the geometrical factor $1/4\pi r^2$, as well as another factor due to the redshift (which can be described by an exponential decay in the energy of the incoming light). Since there are many reasons for an interconnexion between gravitation and electromagnetism (Jaakkola 1991) the same decay would be expected for gravitation. This would link Hubble's law, Olbers' paradox and the gravitational paradox.

Using a standard procedure ($\vec{F} \equiv -\hat{r}dU/dr$) we can derive the force that m_2 exerts on m_1:

$$\vec{F} = -H_g \frac{m_1 m_2}{r^2}\hat{r}\left[1 - \frac{\xi}{2}\frac{\dot{r}^2}{c^2} + \xi\frac{r\ddot{r}}{c^2} + \alpha r\left(1 - \frac{\xi}{2}\frac{\dot{r}^2}{c^2}\right)\right]e^{-\alpha r} \qquad (3)$$

where $\hat{r} \equiv (\vec{r}_1 - \vec{r}_2)/r$ and $\ddot{r} \equiv d^2r/dt^2$. With $\alpha = 0$ we return to a Weber's law for gravitation, while if $\xi = 0$ (or when $\dot{r} = \ddot{r} = 0$) we get a force law not exactly similar to Laplace's expression, Eq. (1), but still with an exponential decay.

To proceed further we need to introduce another assumption, this time related to the nature of the universe. We know from detailed observations that the universe is remarkably isotropic when measured by the integrated microwave and X-ray backgrounds, by radio source and deep galaxy counts (Sciama 1973; Webster 1976; Gursky and Schwartz 1977; Peebles 1980; Raine 1981; Partridge 1988). This fact suggests that the universe is homogeneous on a very large scale. Our assumption is then that the universe extends uniformly in all directions without limits, not only in space but also in time. This means a limitless universe with a smoothed out finite mass density ρ_0, which is also constant in time. We also assume that the universe is in a steady state situation, without expansion and without creation of matter. In this idea we are following the insight of W. Nernst, who proposed a universe in a stationary state (Nernst 1937 and 1938; Monti 1987).

As in the previous work, we now analyse the interaction of any body with the remainder of the universe. Separating this into two parts (interaction with local bodies and with anisotropic distributions of matter around it like the Milky Way, represented by U_A; and interaction with isotropic distributions of matter surrounding it, represented by U_B) we get $U = U_A + U_B$, where U is the total interaction energy between this body and the remaining universe. Since we assume conservation of matter and energy, this means that U is a constant in time. In the above expression U_A is due to the usual interactions (electromagnetic, gravitational, elastic, nuclear, etc.), and U_B comes from the gravitational interaction with the "fixed stars" (the distant galaxies isotropically distributed around the body). Integrating Eq. (2) over the whole isotropic universe yields

$$U_B = A\left[\frac{m_1}{2}\left(\vec{v}_1\cdot\vec{v}_1 - 2\vec{v}_1\cdot(\vec{\omega}\times\vec{r}_1) + (\vec{\omega}\times\vec{r}_1)\cdot(\vec{\omega}\times\vec{r}_1)\right) - \frac{3}{\xi}m_1 c^2\right] \qquad (4)$$

where

$$A = \frac{4\pi}{3}H_g\frac{\xi}{c^2}\rho_0\int_0^\infty re^{-\alpha_0 r}dr = \frac{4\pi}{3}H_g\frac{\xi}{c^2}\frac{\rho_0}{\alpha_0^2} \qquad (5)$$

In this result $\vec{r}_1$ and $\vec{v}_1$, are the radius vector and velocity of body m_1 relative to an observer who sees the set of isotropic "fixed stars" at rest (that is, without an overall translational velocity), but rotating with $\vec{\omega}(t)$ relative to him. We designate the mean gravitational absorption coefficient of the universe by α_0. The main point to note in this expression (4) is that besides the constant value $-3Am_1c^2/\xi$, which arose from the Newtonian potential in (2), we obtained the kinetic energy as due to a gravitational interaction of any body with the isotropic distribution of galaxies around it. Also, the centrifugal potential energy $m_1\left(\frac{\omega^2 r_1^2}{2}\right)$, for instance, can be seen to be a real interaction energy (that is, a gravitational energy between m_1 and the spinning "fixed stars"). This shows once more how powerful is Mach's idea that an inertial frame is equivalent to the frame of the "fixed stars," namely, the frame in which $\vec{\omega}(t)=0$ (Mach, 1960). As Eqs. (2) and (3) involve only r, $\dot{r}$ and $\ddot{r}$ they are relational expressions and have the same value for any observer. Consequently, Eq. (4) is extremely general.

We now calculate the force of the rotating set of "fixed stars" on any body m_1. Integrating Eq. (3) over the whole universe yields

$$\vec{F}_B = -Am_1\left[\vec{a}_1 + \vec{r}\times\frac{d\vec{\omega}}{dt} + 2\vec{v}_1\times\vec{\omega} + \vec{\omega}\times(\vec{\omega}\times\vec{r}_1)\right], \quad (6)$$

where A is given by (5) and $\vec{r}_1$, $\vec{v}_1$ and $\vec{a}_1$ are the radius vector, velocity and acceleration of body m_1 relative to an observer to whom the homogeneous universe is rotating with $\vec{\omega}(t)$, but is not translating. The important advance in this paper relative to previous work (Assis 1989a) is that we have now extended the integration over the whole boundless universe, so that there is no need to introduce a cutoff in the universe "radius". Eq. (6) is the most important result of this work, when related to all the facts we will discuss. It is also valid in all coordinate frames, not just in inertial frames. For instance, to an observer O' to whom the universe is not spinning $\vec{\omega}' = d\vec{\omega}'/dt = 0$ but to whom body m_1 has acceleration $\vec{a}_1'$ and the "fixed stars" a linear acceleration $\vec{a}_s'$, Eq. (6) will read for the force of the "fixed stars" on m_1:

$$\vec{F}_B = -Am_1(\vec{a}_1' - \vec{a}_s') \quad (7)$$

We now obtain the equations of motion. We can represent the resultant force on any body m_1 by $\vec{F} = \vec{F}_A + \vec{F}_B$, where $\vec{F}_B$ is the force due to isotropic distributions of bodies (galaxies, etc.) around m_1 and $\vec{F}_A$ is the resultant force on m_1 due to local bodies (the Earth, a spring, a magnet, etc.) and anisotropic distributions of mass around m_1 (the Sun, the Milky Way, etc). Using postulate (C) yields $\vec{F}_A + \vec{F}_B = 0$. With Eq.(6), supposing that $\vec{\omega} = d\vec{\omega}/dt = 0$, and representing $\vec{F}_A$ by $\sum_{j=2}^{N}\vec{F}_{j1}$, where $\vec{F}_{j1}$ is the force exerted by a certain body j on body m_1, we get an equation of motion similar to Newton's second law, namely

$$\sum_{j=2}^{N}\frac{\vec{F}_{j1}}{A} = m_1\vec{a}_1, \quad (8)$$

where the dimensionless constant A is given by (5) and $\vec{a}_1$ is the acceleration of m_1, relative to the mean distribution of matter in the universe ($\vec{a}_1$ is the acceleration of m_1 in the frame of the "fixed stars"). If m_1 interacts only with isotropic distributions of matter, or if the resultant of the local forces on m_1 is zero, $\vec{F}_A = 0$, we recover an equation of motion similar to Newton's first law: As A and m_1 are different from zero we obtain $\vec{a}_1 = 0$. This means that m_1 will remain at rest or in rectilinear uniform motion relative to the frame of the fixed stars.

As we emphasized earlier, (Assis 1989a), the main value of Eq. (8) is that it explains the proportionality between inertial and gravitational masses. We can see this easily by observing that in our model the force $m_1\vec{a}_1$, or $-Am_1\vec{a}_1$, has its origin in the gravitational interaction between m_1 and the "fixed stars". As a result, in our derivation, m_1 in Eq. (8) is a gravitational mass (thus far we have not introduced the concept of inertial mass). When we identify Eq. (8) with Newton's second law of motion, we then derive the proportionality between inertial and gravitational masses.

The main change from our earlier work is that we can now include the gravitational interaction of the whole universe, as the integration in Eq.(5) extended to infinity. Eq. (6) also indicates that the "fictitious" forces (Coriolis, centrifugal, etc.) arise only when the set of fixed stars is rotating as a whole, as had been pointed out by Mach in 1883 (Mach 1960). This shows that an inertial frame is nothing more than a frame in which the "fixed stars," or the mean distribution of matter in the universe, are nonaccelerated. It is important to remark that we did not need to postulate the proportionality between inertial and gravitational masses at the outset, as was done in the general theory of relativity. This proportionality appears as a consequence of our model.

We can obtain the Newtonian gravitational constant G, as a function of the absorption coefficient α_0. Considering two bodies interacting gravitationally with one another and with the "fixed stars" we get from Eqs. (3) and (8), when $r\ddot{r} << c^2$, $\dot{r}^2 << c^2$ and $r\alpha_0 << 1$

$$-\frac{H_g}{A}m_1m_2\frac{\hat{r}}{r^2} = m_1\vec{a}_1 \quad (9)$$

From Eq. (9) we obtain, equating (or identifying) this result with Newton's law of gravitation

$$G = \frac{H_g}{A} = \frac{3}{4\pi}\frac{c^2}{\xi}\frac{\alpha_0^2}{\rho_0} \quad (10)$$

Keeping in mind the idea of the unity of nature, we propose that the coefficient for the mean absorption of gravity in the universe has the value $\alpha_0 = H_0/c$, where H_0 is Hubble's constant. Using $\alpha_0 = H_0/c$ in Eq. (10) yields $G = 3H_0^2/(4\pi\xi\rho_0)$. For the moment there are still some uncertainties in the determination of H_0 and ρ_0 (Sandage 1972 and 1983; Sandage and Tammann 1974 and 1984; De Vaucouleurs 1981 and 1982; van der Bergh 1982; Tully 1988; Börner 1988), but using the estimated value (Börner 1988; Binney and Tremaine 1987) of $\rho_0/H_0^2 \cong 4.5\times10^8\ kgs^2/m^3$ we find that the above estimate for G (that is, $3H_0^2/(4\pi\xi\rho_0)$, with ξ = 6) is 4/3 of the laboratory value 6.67 $\times10^{-11}\ Nm^2/kg^2$. This is a remarkable result, for it relates a universal constant, G, with cosmological quantities such as ρ_0 and H_0. The model presented here gives a strong argument that the very similar numerical values of $G = 6.67\times10^{-11}\ Nm^2/kg^2$ and $3H_0^2/(4\pi\xi\rho_0)$, with ξ = 6, are not a cosmological coincidence. This is another fact in favour of Mach's ideas.

Now a few remarks are in order. The first is that the value of G obtained in the previous paper (1989a) is twice the value obtained here. The correct value is presumably the one presented here, because we have now included the influence of the whole universe, while in the previous work we had utilized a cutoff in the universe radius. The second, and more important, remark is that in the model presented here of a universe in a stationary state not only G but also ρ_0, H_0, ε_0, etc. should in fact be constants. In our previous analysis we had arrived at a conclusion related to the temporal variation of G, ρ_0, etc. As this is still a controversial result, (van Flandern 1975; Reasenberg and Shapiro 1978; Damour, Gibbons and Taylor 1988; Nordtvedt 1990), the conclusion we draw here, *i.e.* that G, ρ_0, etc. have constant values in time, cannot be ruled out.

Another fact of great relevance which appears from Eq. (10) is that we can estimate the gravitational absorption coefficient of any material:

$$\left.\begin{aligned} \alpha_0 &= \left(\frac{4\pi\xi G}{3c^2}\rho_0\right)^{1/2} \\ \frac{\alpha}{\alpha_0} &= \left(\frac{\rho}{\rho_0}\right)^{1/2} \end{aligned}\right\} \qquad (11)$$

In this last expression α_0 and ρ_0, are the mean values for the universe, while α and ρ are the gravitational absorption coefficient and mass density, respectively, of any material. With the above estimate we obtain $\alpha_0/(\rho_0)^{1/2} \cong 1.6\times10^{-13}\,(m/kg)^{1/2}$. Table I lists some materials, their density and estimated absorption coefficients according to Eq. (11). Although there are many uncertainties surrounding the value of ρ_0, usually linked with uncertainties in the determination of the Hubble's constant, the value presented in Table I is a typical one. The value presented for the mean density of interplanetary space is that given in Gold (1964).

From Table I some interesting conclusions can be drawn. The first is that the value of the absorption coefficient for interplanetary space is compatible with the upper limit obtained by Laplace, namely, $\lambda < 7\times10^{-18}\,m^{-1}$. For solids and liquids the typical range of α is found to be 10^{-12}—$10^{-11} m^{-1}$. This is remarkably close to the value obtained by Bottlinger

TABLE I. The gravitational absorption coefficient for some media

Medium	$\rho(kgm^{-3})$	$\alpha(m^{-1})$
Universe	$\cong 10^{-27}$	$\cong 5\times10^{-27}$
Interplanetary space	$\leq 2\times10^{-20}$	$\leq 2\times10^{-23}$
Water	1×10^{3}	5×10^{-12}
Planet Earth	5.5×10^{3}	1×10^{-11}
Lead	1.1×10^{4}	1.7×10^{-11}

in 1912 (Bottlinger 1912a and b; Martins 1986) in order to explain some anomalies in the longitude of the Moon, described by Newcomb in 1895. These anomalies in the expected orbit of the Moon occur mainly during eclipses, and in his work Bottlinger supposed them to be due to an absorption of the Sun's gravity by the Earth. The value he obtained, to fit the astronomical data, was that a material with a density like that of water would have $\alpha \cong 3\times10^{-13}\,m^{-1}$. Although this is close to the value given in Table I, it should be mentioned that Bottlinger supposed the gravitational absorption coefficient to be proportional to the mass between the attracting bodies, while we obtained, from Eq. (11), that it should be proportional to the square root of the density.

To the best of our knowledge, the main experiments performed in order to detect directly the gravitational absorption are due to Q. Majorama (Majorana 1919, 1920, 1921 and 1930; Martins 1986; Dragoni 1988). Although he obtained positive results, indicating a weakening of gravitational attraction between the Earth and a test body when mercury or lead was interposed between them, the comparison with Table I cannot be done in a direct way. The first reason is that he supposed a law like Laplace's expression, Eq. (1), while our model for the force is Eq. (3), which differs from Eq. (1) by a factor $(1+\alpha r)$ even in a static situation when $\dot{r}=\ddot{r}=0$. In the second place, he supposed theoretically an absorption coefficient proportional to the density of the medium between the two particles, while we obtained in Eq. (11) that the absorption coefficient should be proportional to the square root of the density. The third and last point is that he surrounded the test body with an isotropic distribution of mass (the absorbing body). As we showed in the previous paper, (Assis 1989a), this would lead to a change in the inertial mass of the test body. The absorption of gravity could be masked by this effect, as was correctly indicated by Russell (1921). Since in our model the inertial mass is a real gravitational mass, with a proportionality coefficient, G, depending on the distribution of matter in the universe, any change in the inertial mass due to the absorption of gravity following, for instance, Majorana's interpretation, will be compensated by an analogous change in the gravitational mass. This eliminates the astronomical problems pointed out by Russell with regard to the absorption of gravity. At any event, it should be noted here that the absorption coefficient obtained experimentally by Majorana for liquid mercury, for instance, was $9\times10^{-11}\,m^{-1}$, only one order of magnitude greater than the value obtained by Eq. (11). Because the experiments of Majorana were never repeated, we strongly suggest an improved repetition of these and similar experiments with better apparatus.

Lastly, we should mention the experimental work of Eötvös *et al.* (1922). In a series of experiments to test the proportionality between inertial and gravitational masses, they also studied the absorption of gravity. Although they did not detect a positive result, the precision of their apparatus gave upper limits for absorption. In the case of lead, they obtained $\alpha \leq 2\times10^{-7}\, m^{-1}$ and in the case of the Earth $\alpha \leq 10^{-11}\, m^{-1}$, supposing the mean density in the upper layers of the Earth to be approximately $2\times10^{3}\, kgm^{-3}$. For lead, the value of Table I is compatible with the Eötvös *et al.* findings, while for the Earth they were within the limit of detection.

Discussion and conclusions

From a single idea (an exponential term in the gravitational law) we have been able to correlate many phenomena. In the first place, we introduced a weakening in Weber's potential energy applied to gravitation. This was supposed to be due to intervening matter, and in this way we included the influence of the whole universe in the origin of inertia, as was required by Mach's principle. However, we should mention here that Eqs. (2) and (3) can be only an approximation for two reasons: (A) Although we are including the influence of the medium, through $\exp(-\alpha r)$, where α depends on the nature of the intervening matter, this is still an action-at-a-distance theory. Proposals for ways to extend Weber's law and similar formulations to include radiation have been made (Moon and Spencer 1954 and 1959; Brown 1982; Wesley 1987, 1990 and 1991; Dicke 1964). Treder and collaborators, in particular, have given a very clear analysis of the retardation of gravitational potential in many theories and models (Treder 1975; Treder, von Borzeszkowski, van der Merwe and Yourgrau 1980). For the moment we will not consider these possibilities. It should, however, be mentioned that the action-at-a-distance aspect of the theory may be a positive characteristic of the model, instead of a limitation (Graneau 1990a, b, c and d). In fact, Sokols'kii and Sadovnikov (1987) have shown that Weber's action-at-a-distance theory, when applied to gravitation, models the delay in the propagation of the interaction. Historically, the first to derive the wave equation for the propagation of an electric disturbance in a wire of negligible resistance, showing that the signal travels with the light velocity in the wire, were Kirchhoff and Weber in 1857, working independently of one another (Whittaker 1951; Rosenfeld 1956; Jungnickel and McCormmach 1986). What is most remarkable is that both worked with Weber's action-at-a-distance theory, and this result was obtained before the advent of Maxwell's equations in their complete form, which only happened in 1860-4. An alternative way of getting finite velocity for the propagation of a signal in action-at-a-distance theories has been given by Graneau (1987).

It should be remarked that up to now there is no direct experimental evidence showing that gravitational effects are delayed in time. Although we would expect gravitation to behave like light in this respect as well, there is no experimental proof of this to date. For instance, stellar aberration has been known for quite a long time, yet a possible analogous effect, gravitational aberration, has never been observed. This shows that we need to be careful with analogies. For a discussion of all these aspects and further references see (Phipps 1987).

(B) The second possible limitation of Eqs. (2) and (3) is that they can be valid only up to second order in *v/c*. We showed this limitation in Weber's theory applied to electromagnetism elsewhere (Assis 1989b; Assis and Caluzi 1991), and it is natural to suppose that the same should happen with gravitation. In order to answer Helmholtz's criticisms to Weber's law, (Helmholtz 1872; Maxwell 1954), Phipps proposed (1990a and b) a modified potential given by the expression $U = q_1 q_2 \left(1-\dot{r}^2/c^2\right)^{1/2} / (4\pi\varepsilon_0 r)$. Since this potential is free of "negative mass behavior" for all velocities smaller than c, it overcomes Helmholtz's objection. Moreover it has a limit velocity c, while Weber's potential gives $\sqrt{2}c$. This may indicate that Phipps' potential is a better model, but we will not analyse it here. Following Phipps' proposal we could try, instead of Eq. (2), an expression like

$$U = -H_g \frac{m_1 m_2}{r}\left(1-\xi\frac{\dot{r}^2}{c^2}\right)^{1/2} e^{-\alpha r} \tag{12}$$

but we will reserve this discussion for another paper. A third possible limitation of Eqs. (2) and (3) is that a better model can include terms proportional to $\dddot{r}$, d^4r/dt^4, etc.

As we saw from Eq. (10), we could not determine the value of H_g because it cancels out in the expression for G. This is a general result that follows from our third postulate, $\sum \vec{F} = 0$. That is, we can multiply this equation by any constant and the result will be the same. This means that we can only obtain ratios of masses or of forces, m_1/m_2 or F_1/F_2; we cannot determine an absolute value of m_1 or of F_1. This is again in agreement with Mach's principle.

A further important result of this work is that the absorption coefficient for gravity was found to be proportional to the square root of the density of the intervening medium. We also obtained numerical estimates for this coefficient for different media and showed that they agree with the experimental findings in this area. An absorption coefficient proportional to the square root of the density is an unexpected result. Intuitively, we would expect it to be proportional to the density, and this was also the opinion and working hypothesis of all previous workers in this field. Even following the analogy of the absorption of gravity with that of light a step further would yield this result (α proportional to ρ), because Beer's law states that the absorption coefficient for light is directly proportional to the concentration or density of the absorbing substance.

On the other hand, our result is due to the fact that to obtain the second equality in (11) we utilized the first equality of (11) and another similar relation, namely

$$\alpha = \left(\frac{4\pi\xi G}{3c^2}\rho\right)^{1/2}$$

We thus implicitly assumed that G and c would remain with their present constant values in a medium or universe with a much larger density and coefficient of absorption. Of course other hypotheses could be utilized, yielding different relationships, but for the time being we will adhere to this approach.

Another point to bear in mind when comparing the experimental and theoretical works of earlier scientists with our model is that most of them utilized a model similar to Laplace's, which, when $\lambda r << 1$, yields a factor $(1-\lambda r)$ multiplying Newton's law of universal gravitation. On the other hand Eq. (3) gives a factor $(1-\alpha^2 r^2)$ when $\dot{r}=\ddot{r}=0$ and $\alpha r << 1$. This shows that we need to exercise care when discussing earlier experiments and their relation to the present model.

As in our previous work (1989a), we obtained the inertial mass of a body as being due to the isotropic distribution of matter in the universe; while the local bodies and anisotropic distributions give rise to the usual Newtonian forces. The term that enabled us to derive the inertia in Eq. (3) is proportional to $\hat{r}\ddot{r}/r$. In our earlier work we studied the problem of two bodies (for instance the Sun and a planet) interacting with one another and with the fixed stars, and showed the precession of the perihelion in this model. We only would like to remark here that the inertial mass we obtained earlier and now should in fact be scalar. In this problem of two bodies plus the fixed stars, for instance, the term $\hat{r}\ddot{r}/r$ between the Sun and the planet was used as a usual Newtonian force, not as an anisotropy of the inertial mass of the planet. This answers criticisms by Cocconi and Salpeter (1958), for instance, because we do not consider the inertia as being due to all matter in the cosmos, but as due only to the isotropically (or homogeneously) distributed matter.

For further discussion of Mach's principle and its relation with many gravitational theories we suggest that interested readers consult the important earlier works of Treder, van der Merwe, Mücket and collaborators, (Treder 1971, 1972 and 1975; Treder, von Borzeszkowski, van der Merwe and Yourgrau 1980; Steenbeck and Treder 1984; Treder and Mücket 1981; Mücket 1983; van der Merwe 1968). The book by Barbour (1989) is an excellent, up-to-date historical and critical review of the many ideas and concepts leading to Mach's principle.

Since we began with a quotation from Newton, we shall conclude with more of his words. Our closing remark is his first rule of reasoning in philosophy, published in the *Principia* (Newton 1952):

> *"We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances".*

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Appendix

To arrive at Eq. (4) from Eq. (2), we first note that in general $r = r(t)$, so that

$$\frac{d\dot{r}^2}{dr}=\frac{d\dot{r}^2}{dt}\frac{dt}{dr}=2\ddot{r}$$

The simplest way to integrate equations (2) and (3) is using spherical coordinates. Moreover, it is easier to perform the integrations after writing $\dot{r}$ and $\ddot{r}$ in vectorial notation. Since

$$r=\left[(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2\right]^{1/2}$$

we have

$$\dot{r}=\frac{dr}{dt}=\hat{r}\cdot(\vec{v}_1-\vec{v}_2)$$

where $\hat{r}$ has already been defined and

$$\vec{v}_1-\vec{v}_2\equiv\frac{d(\vec{r}_1-\vec{r}_2)}{dt}$$

Moreover

$$\ddot{r}=\frac{d^2r}{dt^2}=\frac{1}{r}\left[(\vec{v}_1-\vec{v}_2)\cdot(\vec{v}_1-\vec{v}_2)+(\vec{r}_1-\vec{r}_2)\cdot(\vec{a}_1-\vec{a}_2)-\dot{r}^2\right]$$

where

$$\vec{a}_1-\vec{a}_2\equiv\frac{d^2(\vec{r}_1-\vec{r}_2)}{dt^2}$$

It should be observed that in Eq. (4) the term which resemble's Einstein's rest mass energy, m_0c^2, *i.e.*

$$-\frac{3A}{\xi}m_1c^2$$

is due to the Newtonian-Seeliger term in Eq. (2), namely

$$-H_g\frac{m_1m_2}{r}e^{-\alpha r}$$

The Weberian contribution (proportional to $\dot{r}^2$) gives rise to the terms which depend on $\vec{v}_1$ and $\vec{\omega}$ in Eq. (4).

To perform the integrations of equations (2) and (3), it is better to begin with the simplest situation when $\vec{r}_1=0$. Then integrate again when $\vec{r}_1\neq 0$ but $\vec{\omega}=0$, and finally in the general situation discussed here.